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Quasi-anomalous Doppler effect in a periodic-waveguide cyclotron maser

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Abstract

The *quasi-anomalous* Doppler effect is proposed in this paper as a new operating regime for cyclotron-resonance masers (CRMs). It combines features of the known normal and anomalous Doppler effects. A linear analysis of the CRM interaction shows that the quasi-anomalous Doppler effect may occur with fast em waves in inductive (low-impedance) periodic waveguides, and that it may produce gain without an initial rotation of the electron beam. A practical scheme of a slotted-waveguide CRM operating in the high frequency passband is proposed for a future experiment.

1. Introduction

The tuning condition of the known cyclotron-resonance maser (CRM) interaction is [1]

$$\omega \mp n\omega_c - k_z V_{0z} \sim 0\,, \tag{1}$$

where ω and k_z are the em wave angular frequency and axial wavenumber, respectively, V_{0z} is the initial axial electron velocity and n is the cyclotron harmonic number. The relativistic electron gyrofrequency is $\omega_c = eB_0/\gamma m_0$, where B_0 is the external axial magnetic field, and e, m_0 and γ are the electron charge, rest mass and relativistic factor, respectively. In terms of the em-wave phase velocity $V_{ph} = \omega/k_z$, Eq. (1) can be written as

$$V_{ph} \mp n\omega_c/k_z - V_{0z} \sim 0\,. \tag{2}$$

The minus and plus signs in Eqs. (1) and (2) correspond to the normal and anomalous Doppler effects, respectively. The normal Doppler effect requires $V_{ph} > V_{0z}$, and consequently it can be implemented in *fast-wave* or *slow-wave* devices ($V_{ph} \geq c$, or $V_{0z} < V_{ph} < c$, respectively). The slow-wave cyclotron maser may employ a dielectric loaded waveguide or a periodic waveguide, whereas the fast-wave CRM typically uses an empty waveguide. For both fast- and slow-wave cyclotron masers operating in the normal Doppler regime, the initial kinetic energy of the electron motion in the azimuthal direction is converted to radiation energy. Thus, for a zero initial transverse velocity ($V_{0\perp} = 0$), the cyclotron interaction results in radiation absorption rather than gain. For a non-zero initial transverse electron velocity ($V_{0\perp} \neq 0$), the azimuthal and the axial bunching effects oppose one another. The azimuthal bunching effect dominates in the normal Doppler operating regime.

The anomalous Doppler effect [3-9] may occur in *slow-wave* cyclotron devices in which $V_{ph} < V_{0z}$. Dielectric [3] or periodic [4,5] waveguides can be used in order to slow down the wave and to provide the low impedance ($Z_w < V_{0z}\mu_0$, where μ_0 is the vacuum permeability) needed for the anomalous Doppler interaction. For $V_{0\perp} = 0$, the kinetic energy of the longitudinal electron motion is converted to both an azimuthal rotation and to an em wave radiation energy. The dominant bunching effect in the anomalous Doppler regime is the axial bunching, known as the Weibel effect [7,9].

The dispersion diagram in Fig. 1a shows figuratively the normal Doppler cyclotron interaction with a fast wave in an empty waveguide (points A and B). Fig. 1b shows the anomalous Doppler effect with a slow wave in a dielectric loaded waveguide (point C).

The advantages of the CRM operation in the anomalous Doppler regime are (a) the possibility to obtain amplification without an initial transverse electron velocity, (b) a high efficiency operation [6], (c) a relatively small axial magnetic field, and (d) the absence of a backward wave interaction (and consequently of an absolute instability) [7]. On the other hand, the disadvantages of the CRM in the anomalous Doppler regime are (a) the weak coupling between the e beam and the evanescent em wave, and (b) the electric charge and damage to the dielectric material by the bombarding high-energy electrons.

The quasi-anomalous Doppler effect presented in this paper combines the fast-wave features of the normal regime with the low-impedance waveguide needed for the anomalous Doppler effect. This combination is possible,

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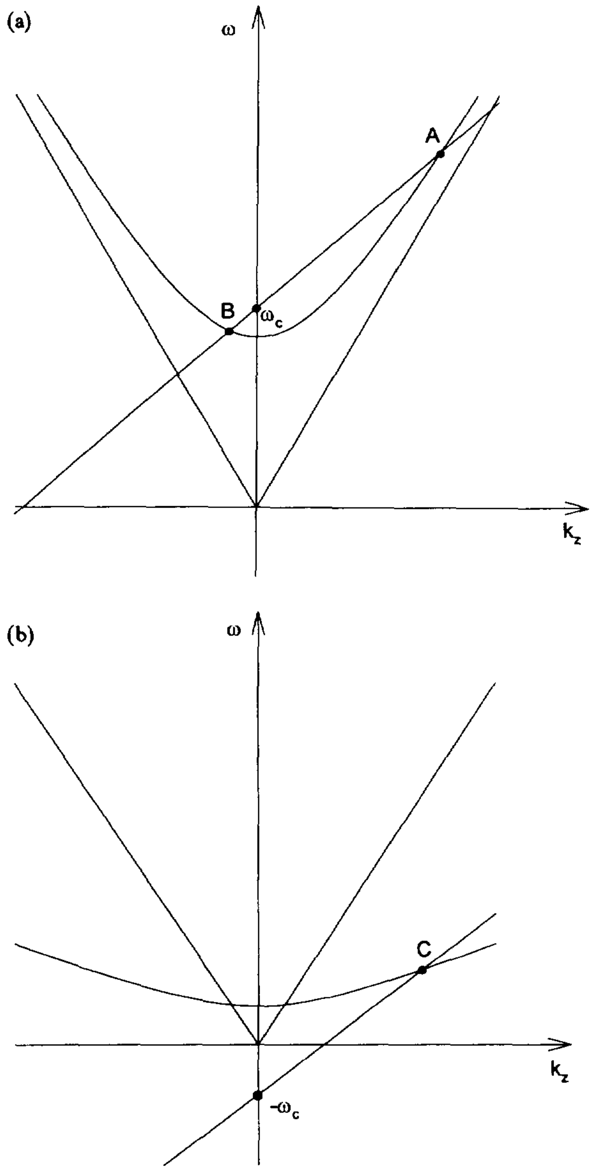

Fig. 1. Dispersion diagrams of the cyclotron interaction in the normal (a) and anomalous (b) Doppler regimes.

for instance, in a CRM interaction in a slotted periodic waveguide operating in a high frequency passband, as described below.

2. Linear analysis

A scheme of a CRM device in a slotted periodic waveguide [10,11] is shown in Fig. 2. The em wave in the periodic waveguide is given by a linear combination of spatial harmonics, as follows

$$H_y(\mathbf{r},t) = A(z)\sum_n h_{yn}\phi_n(x,y)\,e^{j\omega t - jk_{zn}z}, \tag{3a}$$

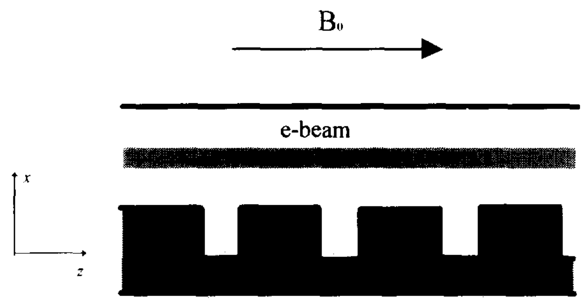

Fig. 2. Schematic of a slotted periodic waveguide CRM device.

$$E_x(\mathbf{r},t) = A(z)\sum_n e_{xn}\phi_n(x,y)\,e^{j\omega t - jk_{zn}z}, \tag{3b}$$

where h_{yn} and e_{xn} are the magnetic and electric field components of the nth harmonic, respectively; k_{zn} and $\phi_n(x,y)$ are its axial wavenumber and transverse profile, respectively; and $A(z)$ is a slowly varying amplitude. The normalized wave impedance of the nth harmonic is defined as

$$\hat{Z}_n = \frac{e_{xn}}{h_{yn}}\sqrt{\frac{\epsilon_0}{\mu_0}}. \tag{3c}$$

The gain-dispersion equation of the cyclotron interaction in a periodic waveguide, derived in Ref. [2], is given by

$$\tilde{A}(\hat{s}) = \frac{(\hat{s}-\hat{\theta}_n)^2}{\hat{s}(\hat{s}-\hat{\theta}_n)^2 - \frac{1}{2}\kappa_n(s)C_n(s)\hat{\theta}_{pr}^2}A_0, \tag{4}$$

where the coupling term between the em wave and the rotating electrons is

$$\kappa_n(s) = \bar{\beta}_{ez}(\hat{s}-\hat{\theta}_n)(\hat{Z}_n - \bar{\beta}_{ez}) + \frac{1}{2}\bar{\beta}_{e\perp}^2(\hat{k}_0\hat{Z}_n - \hat{s} - \hat{k}_{zn}), \tag{5}$$

and $\hat{s} = jsL$, $\hat{k}_0 = k_0L$ and $\hat{k}_{zn} = k_{zn}L$ are the normalized Laplace variable, the free-space wavenumber, and the nth harmonic wavenumber, respectively. L is the interaction length, and $\bar{\beta}_{ez} = \bar{V}_{0z}/c$ and $\bar{\beta}_{e\perp} = \bar{V}_{0\perp}/c$ are the normalized axial and azimuthal electron velocity components, respectively. The slowly varying amplitude in the Laplace space is $\tilde{A}(\hat{s}) = \int_0^\infty A(z)\,e^{-sz}\,dz$, and $A_0 = A(z=0)$ is its initial value. The cyclotron tuning parameter is

$$\hat{\theta}_n = (\omega \mp \omega_c - \bar{V}_{0z}k_{zn})\frac{L}{\bar{V}_{0z}}, \tag{6}$$

where the minus and plus signs correspond to the normal and anomalous Doppler cyclotron resonances, respectively. The other operating parameters in Eq. (4) are the space-charge parameter, $\hat{\theta}_{pr} = \omega_{p0}F_{f_n}L/\bar{V}_{0z}$ where ω_{p0} is plasma frequency, and F_{f_n} and $C_n(\hat{s})$ are the electron beam filling-factor and the power flow ratio for the nth harmonic, respectively. Eq. (4) is valid for the normal and the anomalous Doppler effects, and, consequently, to the quasi-anomalous effect as demonstrated below for $\bar{\beta}_{e\perp} = 0$.

Eq. (4) is reduced to a simple second order equation for $\bar{\beta}_{e\perp} = 0$. Its poles are found analytically as

$$\hat{s}_{1,2} = \frac{1}{2}\left(\hat{\theta}_0 \pm \sqrt{\hat{\theta}_0^2 + 4Q_0}\right), \tag{7}$$

where the coupling parameter, assuming $C_n(\hat{s}) \sim \frac{1}{2}$, is

$$Q_0 \approx \frac{1}{4}\bar{\beta}_{ez}(\hat{Z}_0 - \bar{\beta}_{ez})\hat{\theta}_{pr}^2. \tag{8}$$

In order to obtain amplification, the pole s_i should have an imaginary component. This is possible only if Q_0 is a complex or a negative number. For $Q_0 < 0$, the normalized wave impedance $\hat{Z}_n$ must be smaller than the normalized electron speed, i.e.

$$\hat{Z}_n < \bar{\beta}_{ez}. \tag{9}$$

The anomalous Doppler effect can be realized in a dielectric waveguide. Its impedance is $Z_w = \sqrt{\mu_0/\epsilon_{eff}\epsilon_0}$, where ϵ_{eff} is the relative effective permittivity of the waveguide. According to Eq. (9), the condition for amplification without initial rotation in this case is $\epsilon_{eff} > 1/\bar{\beta}_{ez}^2$.

In the periodic waveguide shown in Fig. 2, the impedance of the nth spatial harmonic is

$$\hat{Z}_n \cong \frac{\hat{k}_{zn}\hat{k}_0}{\hat{k}_z^2}, \tag{10}$$

where $\hat{k}_z = \sqrt{1 - k_{co}^2/k_0^2}$, and k_{co} is the cutoff wavenumber of the empty waveguide (without the periodic structure). Inequality (9) can be written in this case as

$$\hat{k}_{zn} < \frac{\bar{\beta}_{ez}\hat{k}_z^2}{\hat{k}_0}, \tag{11}$$

Consequently, the condition (9) can be satisfied in this periodic waveguide with a fast harmonic in higher pass-band. This effect is referred here as the *quasi-anomalous* Doppler effect.

3. Numerical simulation and discussion

Fig. 3 shows the radiation power gain computed for a CRM in a slotted periodic-waveguide. The operating conditions in this case are a zero initial transverse electron velocity, and an operating frequency in the second pass-band.

The numerical parameters used in this illustrative example are the following. The rectangular waveguide cross-section is 0.4 in. × 0.9 in. The depth of the periodic grooves is 0.2 in., and their width is 1 mm. The waveguide length is 0.5 m, and its period is 2 cm. The external axial magnetic field B_0 is 3.5 kG. The electron axial velocity $\bar{V}_{0z}$

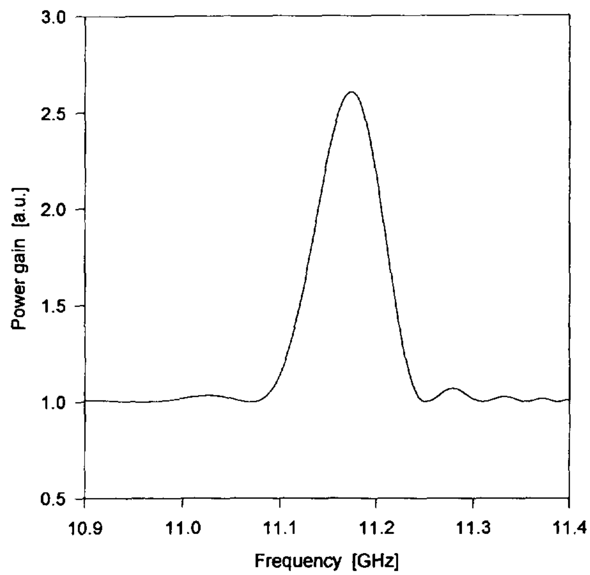

Fig. 3. Amplification in the quasi-anomalous Doppler regime for $V_{0\perp} = 0$.

is 0.3c, and the e-beam current is 0.5 A. The resulting em frequency is 11.2 GHz.

The corresponding Brillouin diagram is shown in Fig. 4. It is noted that although $V_{ph} > \bar{V}_{0z}$, the fast-wave normal Doppler interaction resembles the slow-wave anomalous Doppler interaction in the sense that it produces gain without an initial rotation of the electrons. Additional advantages of the quasi-anomalous operating regime are the fast-wave interaction which alleviates the proximity

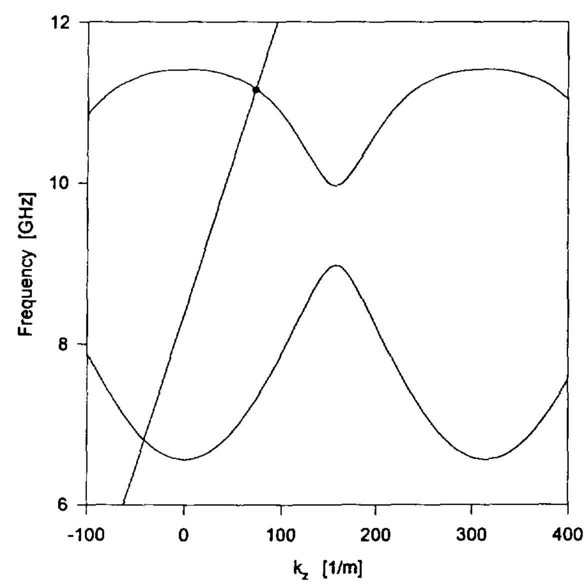

Fig. 4. A Brillouin diagram of slotted periodic waveguide which shows the quasi-anomalous CRM interaction.

required in dielectric-loaded waveguides between the electron beam and the waveguide surface. The quasi-anomalous operation can be implemented with low-energy electron beams.

A further experimental and theoretical study of the CRM interaction in the quasi-anomalous regime are needed. In particular, we plan to construct a slotted periodic waveguide, as proposed in this paper, in our CRM experimental setup [12] at Tel Aviv University.

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