DYNAMICS OF A BRIDGED CRACK IN A DISCRETE LATTICE

by GENNADY S. MISHURIS

(*Department of Mathematics, Rzeszów University of Technology, Rzeszów, Poland*)

ALEXANDER B. MOVCHAN†

(*Department of Mathematical Sciences, University of Liverpool, Liverpool, United Kingdom*)

and LEONID I. SLEPYAN

(*School of Mechanical Engineering, Tel Aviv University, Tel Aviv, Israel*)

[Received 9 April 2007. Revise 18 November 2007]

Summary

The paper addresses a problem of partial fracture of a lattice by a propagating fault modelling a crack bridged by elastic fibres. It is assumed that the strength of bonds within the lattice alternates periodically, so that during the dynamic crack propagation only weaker bonds break, whereas the stronger bonds remain intact. The mathematical problem is reduced to the functional equation of the Wiener–Hopf type, which is solved analytically. The load–crack speed dependence is presented, which also has implications on the stability analysis for the bridged crack propagating within the lattice. In particular, we address the evaluation of the dissipation rate, which is found to be strongly dependent on the crack speed. In this lattice model, our results also cover the case of the supercritical crack speed.

1. Introduction

We consider a Mode-III dynamic problem for a straight crack steadily growing in an infinite, square, mass-spring lattice (Fig. 1). The distances between neighbouring point masses at the lattice nodes are the same for the 'vertical' and 'horizontal' bond lines; this distance is taken as the natural unit of length.

In general, in this lattice, there are two alternating types of the normal-to-the-crack (vertical) bonds that may differ in their stiffness (μ_1 and μ_2 for the bonds of the first and the second type, respectively). These bonds may also differ in their strength (critical strain). The horizontal bond stiffness is denoted by μ_0. The lattice nodes have point masses M_1 and M_2 (placed on the vertical bonds of the stiffness μ_1 and μ_2, respectively), which also may be different. An elementary cell of periodicity containing two masses is shown in Fig. 1. The cell location is defined by two integers, m and n, and the normalized continuous coordinates are $x = 2m$, $y = n$. For the vertical bonds of the first type, $x = 0, \pm 2, \ldots$, whereas for the other vertical bonds, $x = \pm 1, \pm 3, \ldots$.

The intact lattice is uniformly strained at infinity, so that the internal forces in the μ_1, μ_2 and μ_0 bonds at infinity are equal to σ_0, $\sigma_0\mu_2/\mu_1$ and 0, respectively. The crack propagates with the speed v in such a manner that it breaks the μ_1 bonds between the lines $n = 0$ and $n = -1$ at $\eta = 0$,

†⟨abm@liverpool.ac.uk⟩

Q. Jl Mech. Appl. Math, Vol. 61. No. 2
Advance Access publication 23 January 2008. doi:10.1093/qjmam/hbm030

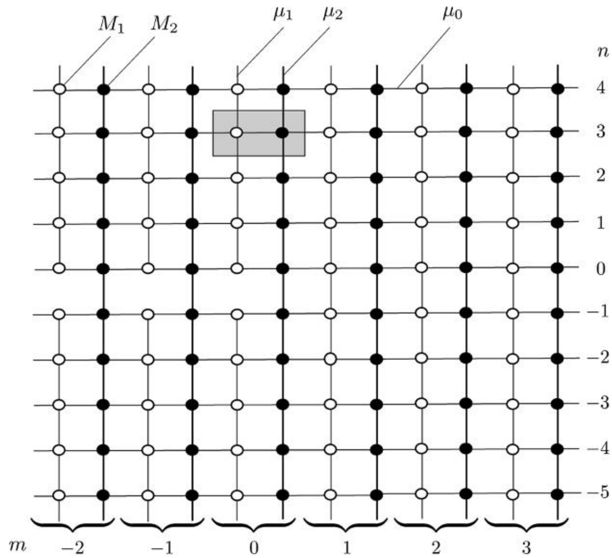

Fig. 1 The lattice structure with a bridged crack. The elementary cell is shown as a shaded rectangle. The horizontal and vertical coordinates of cells are denoted by m and n, respectively. The stronger bonds are shown by thicker lines

whereas the μ_2 bonds remain intact. So the time interval between the breakage of the neighbouring μ_1 bonds is equal to $2/v$.

The propagating bridged crack causes additional displacements of the lattice nodes; they are denoted by $u_{1;m,n}$ and $u_{2;m,n}$ for the M_1 and M_2 masses, respectively. In the steady-state problem, considered here, the displacements $u_{1(2);m,n}(t)$ can be represented as $u_{1(2);n}(\eta)$, where $\eta = 2m - vt$. The total force within a bond is represented by a sum of the unperturbed stress and an additional stress corresponding to the displacements of the crack faces due to fracture. If the additional forces in the $\mu_{1(2)}$ bonds are $\sigma_{1(2)}$, then the total internal forces are

$$\sigma_{1\,\text{total}} = \sigma_1 + \sigma_0, \quad \sigma_{2\,\text{total}} = \sigma_2 + \sigma_0\mu_1/\mu_2. \tag{1}$$

Note that in the additional problem, the nodes in the layers $n = 0$ and $n = -1$ at $\eta < 0$, which were connected by the μ_1 bonds, are now loaded (to neutralize the initial stress) by the forces $\pm p = \mp\sigma_0$, respectively. The load, σ_0 (or p), driving the crack to propagate with the given speed, v, may depend on this speed. It is important that σ_0 is independent of time and the coordinate m.

The bridged crack static problems for homogeneous models were considered by several authors (**1** to **10**). Most of the bridged crack models, existing in the literature, refer to an elastic homogeneous continuum, and even in the static situation no analytical solution for lattice structures is known. Recently, a model of a crack in elastic continuum was considered for the case where the crack faces are bridged by discrete fibres (**11**). Note that a similar type of fracture was described in numerical simulation of the Mode-II crack dynamics in a regular triangular lattice where a ‘binary crack’ fracture can occur (**12**).

The purpose of this work is to analyse the load versus the crack speed together with the energy dissipation corresponding to a propagation of a bridged crack in a simple lattice model. We apply the Fourier transforms, continuous on η and discrete on n. In this way, the general problem is reduced to a Wiener–Hopf-type equation, and the required results are obtained in an explicit form. For the sake of simplicity, the final derivations are done for an isotropic and stiffness–mass uniform lattice where the alternating vertical bonds differ only by critical strains. In such a lattice, the crack propagation may be described as sequential breakage of the weaker bonds, while the stronger bonds remain intact by bridging the crack faces.

While the task is to find the crack speed as a function of the load, it is convenient to determine the inverse function considering v as a given parameter. In this case, we have a linear problem in hand unless a fracture criterion is introduced. Finally, we obtain the load as a single-valued function of the crack speed, and using the stable branches of the inverse function deduce the desired result.

2. Dynamic problem

The equations of motion are as follows:

$$M_1\ddot{u}_{1;m,n} = \mu_0(u_{2;m,n} + u_{2;m-1,n}) + \mu_1(u_{1;m,n+1} + u_{1;m,n-1}) - 2(\mu_0 + \mu_1)u_{1;m,n}$$
$$+ (2\mu_1 u_{1;m,0} + p)H(-\eta)(\delta_{n,0} - \delta_{n,-1}), \tag{2}$$

$$M_2\ddot{u}_{2;m,n} = \mu_0(u_{1;m+1,n} + u_{1;m,n}) + \mu_2(u_{2;m,n+1} + u_{2;m,n-1}) - 2(\mu_0 + \mu_2)u_{2;m,n}, \tag{3}$$

where $u_{1(2);m,n} = u_{1(2);n}(\eta)$, H is the Heaviside unit step function and $\delta_{n,\nu} = 1$ if $\nu = n$, otherwise it is zero. The H-term in (2) reflects the state on the crack faces. It neutralizes the action of the μ_1 bonds, which are broken in the crack area, and introduces the additional external forces to compensate the initial ones. The symmetry, $u_{1(2);m,-1-n} = -u_{1(2);m,n}$ is taken into account.

For the steady-state regime, these equations become

$$M_1 v^2(d^2/d\eta^2)u_{1;n}(\eta) = \mu_0[u_{2;n}(\eta) + u_{2;n}(\eta - 1)] + \mu_1[u_{1;n+1}(\eta) + u_{1;n-1}(\eta)]$$
$$- 2(\mu_0 + \mu_1)u_{1;n}(\eta) + (2\mu_1 u_{1;0} + p)H(-\eta)(\delta_{n,0} - \delta_{n,-1}),$$
$$M_2 v^2(d^2/d\eta^2)u_{2;n}(\eta) = \mu_0[u_{1;n}(\eta + 1) + u_{1;n}(\eta)] + \mu_2[u_{2;n+1}(\eta) + u_{2;n-1}(\eta)]$$
$$- 2(\mu_0 + \mu_2)u_{2;n}(\eta).$$

The continuous Fourier transform with respect to η and the discrete transform in n with parameters k and q, respectively, lead to

$$\left(\alpha_1 - 2\mu_1\cos q - \frac{4\mu_0^2\cos^2 k}{\alpha_2 - 2\mu_2\cos q}\right)u_1^{\mathrm{FF}} = (2\mu_1 u_{1;-} + p^{\mathrm{F}})(1 - e^{-iq}), \tag{4}$$

$$(\alpha_2 - 2\mu_2\cos q)u_2^{\mathrm{FF}} = \mu_0(1 + e^{-2ik})u_1^{\mathrm{FF}}, \tag{5}$$

where $p^{\mathrm{F}} = p/(0 + ik)$ and $u_{1;-}$ is the left-side transform of $u_{1;0}(\eta)$ in η. If $u_{1;+}$ is the right-side transform, then $u_{1;-} + u_{1;+} = u_{1;0}^{\mathrm{F}}(k)$. The quantities α_1 and α_2 in (4) and (5) are

$$\alpha_1 = M_1(0 + ikv)^2 + 2(\mu_0 + \mu_1), \quad \alpha_2 = M_2(0 + ikv)^2 + 2(\mu_0 + \mu_2),$$

where we use the rule: ikv is replaced by $0 + ikv \equiv \lim_{\varepsilon\to+0}(\varepsilon + ikv)$, following from the causality principle for steady-state solutions which is considered as the limit, $t \to \infty$, in the corresponding transient problem (**13**, pp. 91–94).

The inverse Fourier transform with respect to q leads to the Wiener–Hopf-type equation for the function $u_1^{\mathrm{F}}(k)$ and to an explicit relation between the functions $u_2^{\mathrm{F}}(k)$ and $u_1^{\mathrm{F}}(k)$. As a result, these functions can be explicitly determined. In the following, for simplicity, we consider an isotropic and stiffness–mass uniform lattice ($\mu_1 = \mu_2 = \mu_0$, $M_1 = M_2$) where the alternating vertical bonds differ only by the critical strains. In this simplified case, the nodal mass and the bond stiffness are taken as natural units. It follows that

$$\alpha_1 = \alpha_2 = \alpha = 4 + (0 + ikv)^2$$

and we deduce

$$u_{1,0}^{\mathrm{F}}(k) = u_+ + u_- = \left[2u_- + \frac{p}{0+ik}\right]\frac{1}{\pi}\int_0^\infty \frac{(1-\cos q)(\alpha - 2\cos q)}{(\alpha - 2\cos q)^2 - 4\cos^2 k}dq.$$

We have arrived at the Wiener–Hopf-type equation

$$u_+ + L(k)u_- = \frac{[1 - L(k)]p}{2(0+ik)}$$

with

$$L(k) = \frac{\sqrt{\alpha^2 - 16\cos^4 k/2} + \sqrt{\alpha^2 - 16\sin^4 k/2}}{2\sqrt{(\alpha+2)^2 - 4\cos^2 k}}.$$

The function $L(k)$ satisfies the conditions required for the factorization

$$L(k) = L_+(k)L_-(k) \quad \text{with } L_\pm(k) = \exp\left[\pm\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln L(\xi)}{\xi - k}d\xi\right].$$

It follows that

$$\frac{u_+}{L_+} + L_- u_- = \left(\frac{1}{L_+} - L_-\right)p^{\mathrm{F}} = C_+ + C_-, \tag{6}$$

where

$$C_+ = \left[\frac{1}{L_+(k)} - \frac{1}{L_+(0)}\right]\frac{p}{2(0+ik)}, \quad C_- = \left[\frac{1}{L_+(0)} - L_-(k)\right]\frac{p}{2(0+ik)},$$

$L_+(0) = \sqrt{L(0)}R(v)$, $L_-(0) = \sqrt{L(0)}/R(v)$, $L_\pm(\pm i\infty) = 1$, $L(0) = 1/\sqrt{8}$ and

$$R(v) = \exp\left[\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Arg}L(k)}{k}dk\right].$$

The solution, written in terms of the one-sided Fourier transforms, is

$$u_+(k) = C_+(k)L_+(k), \quad u_-(k) = \frac{C_-(k)}{L_-(k)}. \tag{7}$$

It follows that

$$u_{1;0}(0) = \lim_{k\to i\infty}(-ik)u_+(k) = \left[\frac{8^{1/4}}{R(v)} - 1\right]\frac{p}{2},$$

$$u_{1;0}(-\infty) = \lim_{k\to 0}(ik)u_-(k) = [\sqrt{8} - 1]\frac{p}{2}.$$

Note that the solution is valid for the supercritical regime $v > 1$, as well as for $v \leqslant 1$. Clearly, $u_{1;0}(0) \to 0$ as $v \to \infty$ and hence

$$R(v) \to 8^{1/4} \quad \text{as } v \to \infty. \tag{8}$$

In addition, it can be found that $u_{2;0}(-\infty) = p/2$ (this result follows directly from the equilibrium relation valid at $\eta = -\infty$).

The global energy release rate can be calculated as follows. The resulting normalized displacement relative to the initial state is $u_{1;0}(-\infty)$, and it is equal to one half of the additional bond strain. For a single broken bond, the energy required to restore the initial state is $pu_{1;0}(-\infty)$. In addition, the initial energy, $p^2/2$, must be taken into account, and the sum gives the total energy release per elementary cell. In turn, the local energy release is defined as the strain energy of the bond at the moment when it breaks. So the global energy release rate, G, the local energy release rate, G_0, and the dissipation, $D(v)$, are

$$G = \frac{p}{2}\left[u_{1;0}(-\infty) + \frac{p}{2}\right] = \frac{p^2}{\sqrt{2}}, \tag{9}$$

$$G_0 = \frac{1}{4}[p + 2u_{1;0}(0)]^2 = \frac{p^2\sqrt{2}}{2R^2(v)}, \tag{10}$$

$$D(v) = G - G_0, \tag{11}$$

and the normalized global energy release rate, G/G_0, and dissipation, $D(v)/G_0$, are

$$\frac{G}{G_0} = R^2(v), \quad \frac{D(v)}{G_0} = R^2(v) - 1. \tag{12}$$

The latter is plotted as a function of the crack speed in Fig. 2. Note that, in accordance with (8), $D/G_0 \to 2\sqrt{2} - 1 \approx 1.8284$.

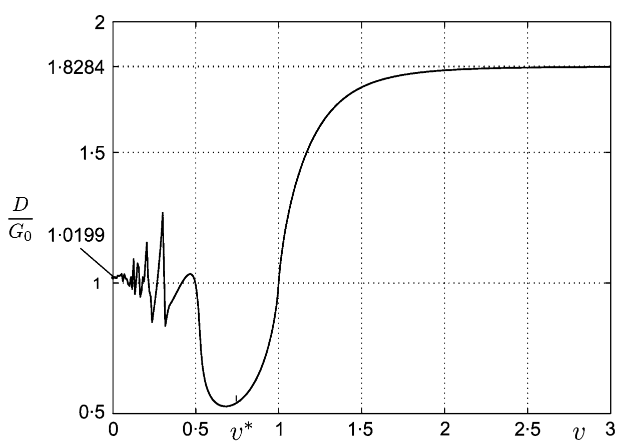

Fig. 2 The normalized dissipation rate versus the normalized crack speed

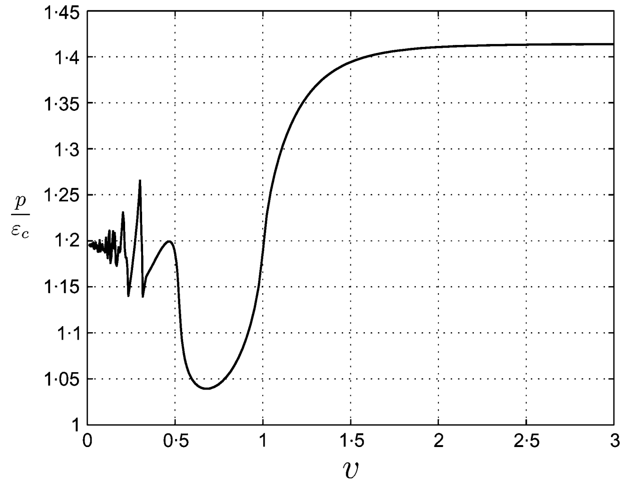

Fig. 3 The ratio p/ε_c of load and the limiting strain versus the crack speed v

At first glance, in the above-obtained solution, G_0 appears to be a crack speed-dependent function. However, if we require the bonds to obey the time-independent elasticity, the limiting strain, ε_c, must be fixed to be crack speed independent. In this case, $G_0 = \varepsilon_c^2/2$ and we obtain a specific dependence of the load, p, on the crack speed. From (10), we obtain

$$p = 2^{1/4}\sqrt{G_0}R(v) = 2^{-1/4}\varepsilon_c R(v), \tag{13}$$

while the energy release and the dissipation ratios (12) are still valid. The ratio p/ε_c as a function of v is shown in Fig. 3.

3. Quasi-static problem

For the quasi-static case, we use the discrete Fourier transform in m and the corresponding periodic version of the Cauchy-type integral. The following representations hold:

$$p^{\mathrm{F}} = \sum_{-\infty}^{-1} pe^{2ikm} = \frac{pe^{-2ik}}{1-e^{-2ik}} \quad \text{for } \operatorname{Im} k < 0,$$

$$L_\pm = \exp\left[\pm\frac{1}{4\pi i}\int_{-\pi}^{\pi} \ln L(\xi)\cot\frac{\xi-k}{2}d\xi\right] \quad \text{for } \pm\operatorname{Im} k > 0, \tag{14}$$

$$L_\pm(\pm i\infty) = \exp\left[\frac{1}{2\pi}\int_0^{\pi} \ln L(k)dk\right], \quad L_\pm(0) = \sqrt{L(0)}, \quad \alpha = 4.$$

Within the period, the function $p^{\mathrm{F}}(k)$ has a single pole at $k = i0$ and $p^{\mathrm{F}}(k) \sim p/(0+ik)$ in the vicinity of this point. So (6) is valid here, where p^{F} is defined by (14), and

$$C_+ = \left[\frac{1}{L_+(k)} - \frac{1}{L_+(0)}\right]\frac{pe^{-2ik}}{2(1-e^{-2ik})}, \quad C_- = \left[\frac{1}{L_+(0)} - L_-(k)\right]\frac{pe^{-2ik}}{2(1-e^{-2ik})}. \tag{15}$$

Further, from the relations (7), it follows that

$$u_{1;0}(0) = u_+(i\infty) = \left\{8^{1/4} \exp\left[\frac{1}{2\pi}\int_0^\pi \ln L(k)dk\right] - 1\right\}\frac{p}{2},$$
$$u_{1;0}(-\infty) = \lim_{k\to 0}(2ik)u_-(k) = [\sqrt{8}-1]\frac{p}{2}, \quad u_{2;0}(-\infty) = \frac{p}{2}. \tag{16}$$

In this quasi-static case, the global energy release rate, G, is still the same as in the dynamic case (9), whereas the local one obtained from (10) and (16) is

$$G_0 = \left[\frac{p}{2} + u_{1;0}(0)\right]^2 = \frac{\sqrt{2}}{2}\frac{p^2}{\mathcal{R}^2}, \quad \mathcal{R} = \exp\left[-\frac{1}{2\pi}\int_0^\pi \ln L(k)dk\right] \approx 1.42125. \tag{17}$$

So the normalized global energy release rate, G/G_0, and dissipation, $D(0)/G_0$, are

$$\frac{G}{G_0} = \mathcal{R}^2 \approx 2{\cdot}0200, \quad \frac{D(0)}{G_0} = \mathcal{R}^2 - 1 \approx 1.0200. \tag{18}$$

The latter value coincides with that predicted by the dynamic dependence as can be seen in Fig. 2.

4. Related continuous models

In this section, we give the comparative analysis of a related continuum formulation. First, we consider the steady-state Mode-III problem for an elastic plane where the crack faces are connected by uniformly distributed linearly elastic ‘springs’. In other words, instead of the discrete bonds, we introduce a continuous elastic foundation between the crack faces. So we deal with the wave equation with respect to the displacement $\mu\Delta u(x,y,t) = \varrho\partial^2 u(x,y,t)/\partial t^2$, with the following conditions at the upper half-plane boundary, $y = +0$:

$$u = 0 \ \text{ for } \eta = x - vt > 0 \quad \text{and} \quad \sigma = \mu\frac{\partial u}{\partial y} = 2\varkappa u - p \ \text{ for } \eta < 0, \tag{19}$$

where $\varkappa$ is the foundation stiffness and $\pm p$ is the external distributed load applied to the crack faces $y = \pm 0$. Taking the shear modulus, μ, and the density, ϱ, as natural units, we have the relation at $y = +0$

$$\sigma^{\mathrm{F}}(k) = \sigma_+ + \sigma_- = -\sqrt{(0+ikv)^2 + k^2}\,u_-, \tag{20}$$

which leads to the Wiener–Hopf equation

$$\sigma_+ + L(k)u_- = \frac{p}{0+ik}, \quad L(k) = 2\varkappa + \sqrt{(0+ikv)^2 + k^2}. \tag{21}$$

In contrast with the lattice model, in the continuum problem, the supercritical crack speed is forbidden (the crack does not gain energy if $v > 1$). So we assume here that the crack speed is subcritical, $0 \leqslant v < 1$.

The main objective of analysis of the solution to this problem is to find the energy release rate. In the steady-state regime for a continuous medium, all the released energy disappears through the moving crack tip, and the local energy release is the same as the global one. The displacement at minus infinity and hence the energy release rate depend on p only since the static state is realized far away on the left from the moving crack tip. We obtain

$$\sigma(-\infty) = 2\varkappa u(-\infty) = p, \quad G = 2\varkappa u^2(-\infty) = p^2/(2\varkappa). \tag{22}$$

Thus, in this case, the energy release rate is crack speed independent if the critical one is such.

The complete solution (written in terms of the Fourier transforms) can be obtained in a standard way. We have $L(k) = L_+(k)L_-(k)$, where

$$L_\pm(k) = (1-\upsilon^2)^{1/4}\sqrt{0 \mp ik}\exp\left[\pm\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln\mathcal{L}(\xi)}{\xi - k}d\xi\right] \quad \text{for } \pm\mathrm{Im}\,k > 0 \tag{23}$$

with

$$\mathcal{L}(k) = L(k)\{(1-\upsilon^2)(0+ik)(0-ik)\}^{-1/2}. \tag{24}$$

It follows that $L_\pm(\pm i\infty) \sim (1-\upsilon^2)^{1/4}\sqrt{0 \mp ik}$ and $L_\pm(0) = \sqrt{2\varkappa}$. Equation (21) being rearranged in the form

$$\frac{\sigma_+(k)}{L_+(k)} + L_-(k)u_-(k) = \left[\frac{p}{L_+(k)} - \frac{p}{L_+(0)}\right]\frac{1}{0+ik} + \frac{1}{(0+ik)L_+(0)} \tag{25}$$

yields

$$\sigma_+(k) = \left[1 - \frac{L_+(k)}{L_+(0)}\right]\frac{p}{0+ik}, \quad u_-(k) = \frac{p}{(0+ik)L_+(0)L_-(k)}. \tag{26}$$

In addition, consider the formulation where the elastic foundation connects not only the crack faces but also is introduced between the crack-continuation half-plane boundaries (in this case, $u_+(k) > 0$ and $\sigma_+ = \varkappa_1 u_+$). This formulation looks more similar to the lattice case. The problem under this 'improved' formulation can be solved in the same way as the above considered, and the energy release rate relation (22) is still valid for this case. Moreover, the foundation stiffness ahead of the crack does not influence the energy release. Indeed, the energy release, in the considered bridge problems, is uniquely defined by the external load and the foundation stiffness on the crack.

5. Discussion and concluding remarks

We have obtained a new dynamic solution for a bridged crack in a lattice structure. Using an independent analytical procedure, we have also derived a solution of the corresponding static problem, and indeed the static limit of the dynamic solution fully agrees with the static result for the bridged crack in the lattice.

The results of numerical computation, based on the formulas (12) and (13) for the dissipation rate and the load–crack speed relation, are shown in Figs 2 and 3. These diagrams suggest that there are two regions, stability or otherwise, for different values of the crack speed υ. This conclusion is consistent with earlier work by Marder and Gross (**14**) dealing with the modelling of free-face cracks in lattices. When the crack speed is less than the critical value υ^*, steady propagation is impossible and the crack is likely to accelerate from 0 to $\upsilon \geqslant \upsilon^*$, corresponding to the stable branch where the dissipation rate increases with the increase of υ.

We have obtained that the non-dimensional critical crack speed (the long wave speed is taken as the speed unit) corresponding to the minimum of the dissipation rate is $\upsilon^* \simeq 0.7$. The corresponding values of the internal force in the 'vertical' bonds are $\sigma_1(0) \approx 0.35590p$ and $\sigma_2(-\infty) = p$. This suggests that the strength of the non-breaking bonds must be about three times greater than the strength of the breaking bonds.

To compare the lattice and the continuum model solutions, it is reasonable to take in the latter $\varkappa = 1/2$ as the averaged stiffness of the bonds connecting the lattice crack faces. In this case, under the same crack face load, the external forces in the lattice solution, $p = p_{\text{lattice}}$, must be twice as much as $p = p_{\text{hom}}$ in the continuous one. As a result, the global energy release in the lattice $G_{\text{lattice}} = 2\sqrt{2}p^2_{\text{hom}}$ (11), whereas $G_{\text{hom}} = p^2_{\text{hom}}$ ($\varkappa = 1/2$) (22). The main difference is that in the continuous case, the local energy release rate is the same as the global one, whereas in the lattice a part (which depends on the crack speed) of the global energy release rate is dissipated (see (12) and Fig. 2).

We emphasize that in the analysis of bridged crack problems, the discrete or a discrete/continuous lattice model looks preferable to the continuous one in both physical and mathematical contexts. The discrete model provides more information concerning the main features of the process; this includes the energy release rate, dissipation, stresses and the load versus the crack speed. In particular, lattice models can be used for analysis of the bridged crack fracture of multiple component fabrics and cellular materials.

The Wiener–Hopf technique can be successfully used to solve such problems in more general case than the ones considered above. Recall that different bond stiffnesses, $\mu_{1,2}$, and different masses, $M_{1,2}$, are acceptable. Furthermore, a symmetric lattice (with respect to the crack line, x) can be y-non-uniform, as the one considered in (**15**) for a free-face crack. Also the cell of periodicity can contain more than two masses and bonds if only one of the bonds in the cell breaks.

Acknowledgements

This paper has been written during the academic visit of Professor Slepyan to Liverpool University supported by the research grant EP/D079489/1 from the UK Engineering and Physical Sciences Research Council. Professor Mishuris was supported by the Marie Curie Transfer of Knowledge Fellowship of the European Community's Sixth Framework Programme, grant reference MTKD-CT-2004-509809.

References

1. B. Budiansky, J. W. Hutchinson and A. G. Evans, Matrix fracture in fiber-reinforced ceramics, *J. Mech. Phys. Solids* **34** (1986) 167–189.

2. L. R. F. Rose, Crack reinforcement by distributed springs, *ibid.* **35** (1987) 383–405.

3. W. E. Olmstead and A. K. Gautesen, Asymptotic solution of some singularly perturbed Fredholm integral equations, *Z. Angew. Math. Phys.* **40** (1989) 230–244.

4. J. R. Willis, Asymptotic analysis of crack bridging by ductile fibres, *Composites* **24** (1993) 93–97.

5. G. Meda and P. S. Steif, A detailed analysis of cracks bridged by fibers—I. Limiting cases of short and long cracks, *J. Mech. Phys. Solids* **42** (1994) 1293–1321.

6. A. B. Movchan and J. R. Willis, Asymptotic analysis of the reinforcement of a brittle crack by bridging fibres, *Q. Jl Mech. Appl. Math.* **46** (1993) 331–350.

7. N. V. Movchan and J. R. Willis, Critical load for a mode-I crack reinforced by fibres, *ibid.* **49** (1996) 545–564.

8. N. V. Movchan and J. R. Willis, Asymptotic analysis of reinforcement by frictional fibres, *Proc. R. Soc.* A **453** (1997) 757–784.

9. N. V. Movchan and J. R. Willis, Influence of spatial correlations on crack bridging by frictional fibres, *Engng. Fract. Mech.* **58** (1997) 571–579.

10. N. V. Movchan and J. R. Willis, Penny-shaped crack reinforced by fibres, *Q. Appl. Math.* **56** (1998) 327–340.
11. K. Bertoldi, D. Bigoni and W. J. Drugan, A discrete-fibers model for bridged cracks and reinforced elliptical voids, *J. Mech. Phys. Solids* **55** (2007) 1016–1035.
12. L. I. Slepyan and M. V. Ayzenberg-Stepanenko, Some surprising phenomena in weak-bond fracture of a triangular lattice, *ibid.* **50** (2002) 1591–1625.
13. L. I. Slepyan, *Models and Phenomena in Fracture Mechanics* (Springer, Berlin 2002).
14. M. Marder and S. Gross, Origin of crack tip instabilities, *J. Mech. Phys. Solids* **43** (1995) 1–48.
15. G. S. Mishuris, A. B. Movchan and L. I. Slepyan, Waves and fracture in an inhomogeneous lattice structure, *Waves Random Complex Media* **17** (2007) 409–428.