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# Towards Making Broadcast Encryption Practical

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**Abstract**

The problem we address is how to communicate securely with a set of users (the target set) over an insecure broadcast channel. In order to solve this problem, several broadcast encryption schemes have been proposed. In these systems, the parameters of major concern are the length of transmission and number of keys held by each user's set top terminal (STT). Due to the need to withstand hardware tampering, the amount of secure memory available in the STTs is quite small, severely limiting the number of keys each user holds. In such cases, known theoretical bounds seem to indicate that non-trivial broadcast encryption schemes are only feasible when the number of users is small.

In order to break away from these theoretical bounds, our approach is to allow a controlled number of users outside the target set to occasionally receive the multicast. This relaxation is appropriate for low-cost transmissions such as multicasting electronic coupons. For this purpose, we introduce *$f$-redundant* establishment key allocations, which guarantee that the total number of recipients is no more than $f$ times the number of intended recipients. We measure the performance of such schemes by the number of transmissions they require, by their redundancy, and by their opportunity, which is the probability of a user outside the target set to be part of the multicast. We first prove a new lower bound and discuss the basic trade-offs associated with this new setting. Then we present several new $f$-redundant establishment key allocations. We evaluate the schemes' performance under all the relevant measures by extensive simulation. Our results indicate that, unlike previous solutions, it seems possible to design practical schemes in this new setting.

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# 1 Introduction

## 1.1 The Problem

The domain we consider in this paper is that of broadcast applications where the transmissions need to be encrypted. As a primary example we consider a broadband digital TV network, broadcasting either via satellite or via cable, but other applications such as Internet multicasts are similar [MQ95].

In this context, the *head-end* occasionally needs to multicast an encrypted message to some subset of users (called the target set) using the broadcast channel. Each network user has a *set-top terminal* (STT) which receives the encrypted broadcast and decrypts the message, if the user is entitled to it. For this purpose the STT securely stores the user's secret keys, which we refer to as establishment keys. Because of extensive piracy [McC96], the STTs need to contain a secure chip which includes secure memory for key storage. This memory should be non-volatile, and tamper-resistant, so the pirates will find it difficult to read its contents. As a result of these requirements, STTs have severely limited secure memory, typically in the range of a few KB[1] [Gem98].

Earlier work on broadcast encryption (cf. [FN94]) was motivated by the need to transmit the key for the next billing period or the key for the next pay-per-view event, in-band with the broadcast, since STTs only had uni-directional communications capabilities. The implicit assumption was that users sign up for various services using a separate channel, such as by calling the service provider over the phone. In such applications it is reasonable to assume that the target set is almost all the population, and there are only small number of excluded users. Moreover, it is crucial that users outside the target set are not able to decrypt the message since it has a high monetary value, e.g., the cost of a month's subscription.

However, current STTs typically follow designs such as [CEFH95] which allow bi-directional communication, where the uplink uses an internal modem and a phone line, or a cable modem. These new STTs upload the users' requests and download next month's keys via a callback mechanism, and not through the broadcast channel. This technological trend would seem to invalidate the necessity for broadcast encryption schemes completely. We argue that this is not the case—there are other applications where broadcast encryption is necessary, such as multicasting electronic coupons, promotional material, and low-cost pay-per-view events. Such applications need to multicast short-lived, low value messages that are not worth the overhead of communicating with each user individually. In such applications, though, the requirements from the solution are slightly different. On one hand, it is no longer crucial that only users in the target set receive the message, as long as the number of free-riders is controlled. On the other hand, it is no longer reasonable to assume anything about the size of the target set.

## 1.2 Related Work

Fiat and Naor [FN94] were first to introduce broadcast encryption. They suggested methods of securely broadcasting key information such that only a selected set of users can decrypt this information while coalitions of up to $k$ other users can learn nothing, either in the information-

[1] Kilobyte.

theoretic sense, or under a computational security model. Their schemes, though, required impractical numbers of keys to be stored in the STTs. Extensions to this basic work can be found in [BC94, BFS98, SvT98].

Recently Luby and Staddon [LS98] studied the trade-off between the transmission length and the number of keys stored in the STTs. They assumed a security model in which encryptions cannot be broken, i.e., only users that have a correct key can decrypt the message. We adopt the same security model. Their work still addressed fixed-size target sets, which are assumed to be either very large or very small, and no user outside the target set is allowed to be able to decrypt the message. A main part of their work is a disillusioning lower bound, showing that either the transmission will be very long or a prohibitive number of keys need to be stored in the STTs.

A related line of work goes under the title of "tracing traitors" [CFN94, NP98]. The goal is to identify some of the users that leak their keys, once a cloned STT is found. This is achieved by controlling which keys are stored in each STT, in a way that the combination of keys in a cloned STT would necessarily point to at least one traitor.

Key management schemes for encrypted broadcast networks, which give the vendor the flexibility to offer program packages of various sizes to the users, can be found in [Woo98]. The problem of tracking the location of STTs in order to prevent customers from moving an STT from, e.g., a home to a bar, is addressed in [GW98].

## 1.3 Contributions

Our starting point is the observation that the requirement "no users outside the target set can decrypt the message" is too strict for some applications. For instance, for the purposes of multicasting electronic coupons, it may be enough to guarantee that the recipient set contains the target set, and that the total number of recipients is no more than $f$ times the size of the target set. Service providers can afford a small potential increase in the number of redeemed coupons, as long as this simplifies their operations and lowers their cost. We call establishment key allocation schemes that provide such guarantees "$f$-redundant broadcast encryption schemes". Relaxing the requirements in this way allows us to depart from the lower bounds of [LS98].

On the other hand, we have a more ambitious goal when it comes to possible target sets. Unlike earlier work, we require our schemes to be able to multicast to *any* target set, not just those target sets of very small or very large cardinality.

We concentrate on schemes which store only very few keys in each STT. As we mentioned before, STTs typically have only a few KB of key storage. With a key length of 128 bits this translates to around 100 keys. Thus, for systems with several million users, it is reasonable to require the maximum number of keys per user's STT to be $O(\log n)$, where $n$ is the total number of users, or at most $O(n^\epsilon)$, where, say, $\epsilon \leq 1/4$.

Subject to these constraints, we are interested in several measures of the quality of an establishment key allocation. The first is the number of transmissions $t$: we can always attain our requirements trivially if we assign each STT a unique key, but then we suffer a very high number of transmissions. The second parameter, which we call *opportunity*, is the proportion of free riders in the population outside the target set. The opportunity measures the incentive a customer has to avoid paying (in cheap pay-per-view type services). If the opportunity is very high, close to 1, there is no incentive for customers to pay, as they can almost surely get a free ride.

After discussing the basic trade-offs associated with the problem, we present some simple examples, that show the problem difficulty. We then prove a new lower bound on the tradeoff between transmission length and number of keys stored per STT, that incorporates the $f$-redundancy of our establishment key allocations. We show that the $f$-redundancy gives us a substantial gain: for the same number of transmissions $t$ we can hope for only $\exp(\Omega(n/tf))$ keys per STT, whereas the bound of [LS98] is $\exp(\Omega(n/t))$.

We then present several establishment key allocation constructions, and an approximation algorithm that finds a key cover with minimal number of transmissions, for any given target set of users. Since this problem is similar to the minimum set cover problem that is known to be NP-hard, we cannot expect to find an optimal solution efficiently. Instead we use a greedy approximation algorithm to find good key covers. We conducted an extensive simulation study of the problem, from which we present only the interesting results.

**Organization:** In the next section we formally define the problem and the various parameters we are interested in. In Section 3 we show some simple solutions. In Section 4 we prove our new lower bound on the trade-off between the number of keys per user, the redundancy factor, and the transmission length. In Section 5 we discuss how to find which keys to use given an establishment key allocation. In Section 6 we show our schemes and the results of their performance evaluation. We conclude in Section 7.

# 2 Definitions and Model

Let $\mathcal{U}$ be the set of all customers (STTs) connected to a head-end, with $|\mathcal{U}| = n$. We use $K$ to denote the *target set*, i.e., the set of paying customers, and denote its size by $|K| = k$.

We describe the allocation of the establishment keys by a collection $\mathcal{S} = \{S_1, S_2, \ldots\}$ of *key sets* such that $\cup S_i = \mathcal{U}$. We associate a unique establishment key $e_i$ with each set $S_i \in \mathcal{S}$. A key $e_i$ is stored in the secure memory of every STT $u \in S_i$. Hence the number of keys an STT $u \in \mathcal{U}$ stores is equal to the number of sets $S_i \in \mathcal{S}$ it belongs to. Formally,

**Definition 2.1** *Let $\mathcal{S}$ be an establishment key allocation. The* degree *of an STT $u$ is $deg(u) = |\{i : S_i \ni u\}|$. The degree of a collection $\mathcal{S}$ is $deg(\mathcal{S}) = \max_{u\in\mathcal{U}} deg(u)$.*

**Definition 2.2** *Given a target set $K$, a* key cover *of $K$ is a collection of sets $S_i \in \mathcal{S}$ whose union contains $K$:*

$$\mathcal{C}(K) \subseteq \mathcal{S} \text{ such that } K \subseteq \cup_{S_i \in \mathcal{C}(K)} S_i.$$

*The* minimal key cover *is $\mathcal{C}_{\min}(K) = \mathcal{C}(K)$ for which $|\mathcal{C}(K)|$ is minimal.*

Suppose the head-end needs to send a message $\mu$ to all the members of a target set $K$. Given any key cover $\mathcal{C}(K)$, the head end encrypts $\mu$ using the establishment keys $e_i$ corresponding to the sets $S_i \in \mathcal{C}(K)$, and broadcasts each encryption separately.[2].

[2] This method was called the OR protocol in [LS98].

| $\mathcal{S}$ | $deg(\mathcal{S})$ | $t_{\max}(\mathcal{S})$ | $f$ | $\eta$ |
|---|---|---|---|---|
| $\{\mathcal{U}\}$ | **1** | **1** | $n$ | 1 |
| $\{1,\ 2,\ \ldots,\ n\}$ | **1** | $n$ | **1** | **0** |
| $2^{\mathcal{U}}$ | $2^{n-1}$ | **1** | **1** | **0** |

Table 1: A summary of some simple examples. Bold numerals indicate an optimal parameter.

**Definition 2.3** *We denote the best possible number of transmissions that the head-end can use for a target set $K$ by $t_K = |\mathcal{C}_{\min}(K)|$. Thus the worst case number of transmissions is $t_{\max}(\mathcal{S}) = \max_K t_K$.*

In order to define the redundancy and opportunity measures we need the following technical definition.

**Definition 2.4** *We denote the set of recipients of a given key cover $\mathcal{C}(K)$ by $R_{\mathcal{C}}(K) = \cup\{S_i \in \mathcal{C}(K)\}$ and the total number of recipients by $r_{\mathcal{C}}(K) = |R_{\mathcal{C}}(K)|$.*

By the definition of a key cover $\mathcal{C}(K)$, every member of the target set $K$ has at least one of the keys used to encrypt $\mu$. However, other STTs outside $K$ usually exist, which are also capable of decrypting the message. All our establishment key allocations are constructed with a worst case guarantee that there are never too many of these free riders. Formally,

**Definition 2.5** *An establishment key allocation $\mathcal{S}$ is said to be $f$-*redundant *if*

$$\frac{r_{\mathcal{C}}(K)}{k} \leq f$$

*for every $K \subseteq \mathcal{U}$ with $|K| = k$.*

A variant measure of redundancy is the *actual redundancy* $f_a$, which is the proportion of non-paying customers in the recipient set $R_{\mathcal{C}}(K)$. We are interested in the average case $f_a$, so we define it as a function of the target set $K$. Formally,

**Definition 2.6** *For a target set $K$ with $|K| = k$ the* actual redundancy *is $f_a = \frac{r_{\mathcal{C}}(K)-k}{k}$.*

If $\mathcal{S}$ guarantees a worst case redundancy factor $f$, then $0 \leq f_a \leq f - 1$ for any target set $K$.

Finally, we define the opportunity, $\eta$, as the proportion of non-paying recipients (free riders) in the non-paying population ($0 \leq \eta \leq 1$). The opportunity measures the incentive a customer has to avoid paying (e.g., in cheap pay-per-view type services). Again, this is a function of the target set $K$.

**Definition 2.7** *For a target set $K$ with $|K| = k$ the* opportunity *is $\eta = \frac{r_{\mathcal{C}}(K)-k}{n-k}$.*

# 3 Simple Examples

To demonstrate our definitions and the trade-offs associated with the problem let us examine some simple solutions for the problem. See Table 1 for a summary of the examples.

**Example 3.1** *The "always broadcast" solution: $\mathcal{S} = \{\mathcal{U}\}$.*

Both the degree, $deg(\mathcal{S})$, and the number of transmissions, $t_{\max}(\mathcal{S})$, required to distribute the message are optimal and equal to 1 in this case. However, the redundancy is $f = n$ in the worst case and the opportunity, $\eta$, is always 1. The last two parameters are very bad since the system gives no incentive for a customer to pay for a program; a single paying customer enables the entire population a free ride.

**Example 3.2** *The "key per user" solution:* $\mathcal{S} = \{\{1\}, \{2\}, \ldots, \{n\}\}$.

Here the degree $deg(\mathcal{S}) = 1$ is optimal, and so are the redundancy $f = 1$, and the opportunity $\eta = 0$. However, the number of transmissions is a very poor $t_{\max}(\mathcal{S}) = n$.

**Example 3.3** *The "all possible sets" solution:* $\mathcal{S} = 2^{\mathcal{U}}$.

The degree here is an impractical $deg(\mathcal{S}) = 2^{n-1}$, however, all the other parameters are optimal: $t_{\max}(\mathcal{S}) = 1$, $f = 1$, and $\eta = 0$. This is because every possible target set $K$ has its own designated key.

# 4 The Lower Bound

## 4.1 Tools

Before presenting our lower bound on the degree of an $f$-redundant establishment key allocation, we need to introduce some definitions and results which we use in the proof.

We start with *covering designs*, which are a class of combinatorial block designs. A succinct description of covering designs can be found in [CD96, Ch. IV.8]. A more detailed survey is [MM92].

**Definition 4.1** *A $k$-$(n, d)$* covering design *is a collection of $d$-sets* (blocks) $\mathcal{D} = \{D_1, \ldots, D_\ell\}$ *over a universe of $n$ elements, such that every $k$-set of elements is contained in at least one block.*

**Definition 4.2** *The* covering number $C(n, d, k)$ *is the minimum number of blocks in any $k$-$(n, d)$ covering design.*

**Theorem 4.3 (Schönheim bound)** [Sch64] $C(n, d, k) \geq L(n, d, k)$, *where*

$$L(n, d, k) = \left\lceil \frac{n}{d} \left\lceil \frac{n-1}{d-1} \cdots \left\lceil \frac{n-k+1}{d-k+1} \right\rceil \right\rceil \right\rceil \geq \binom{n}{k} \Big/ \binom{d}{k}.$$

We also rely on the following result of Luby and Staddon, which addresses *strict* broadcast encryption protocols.

**Definition 4.4** *An establishment key allocation $\mathcal{S}$ is called* strict *for a collection of target sets $\mathcal{D}$ if the sets in $\mathcal{D}$ can be covered without redundancy. Formally, $R_{\mathcal{C}}(D) = D$ for all $D \in \mathcal{D}$.*

**Theorem 4.5** [LS98] *Let $\mathcal{D} = \{D_1, \ldots, D_\ell\}$ be a collection of target sets, with $|D_i| \geq d$ for all $D_i \in \mathcal{D}$. Then any establishment key allocation $\mathcal{S}$ which is strict for $\mathcal{D}$, and which can transmit to any $D_i \in \mathcal{D}$ using at most $t$ transmissions, must have*

$$deg(\mathcal{S}) \geq \left( \frac{\ell^{1/t}}{t} - 1 \right) \Big/ (n-d).$$

**Remark:** The precise statement we use here is a generalization of [LS98, Theorem 12]. In their original formulation the target sets $D_i$ all have a cardinality of exactly $d$, and the collection $\mathcal{D}$ consists of all $\binom{n}{d}$ possible $d$-sets. However, their proof can be easily extended to any arbitrary collection of target sets, of cardinality $d$ or larger.

## 4.2 The Bound

**Theorem 4.6** *Let $\mathcal{S}$ be an $f$-redundant establishment key allocation over a universe $\mathcal{U}$ of size $n$, for which $t_{\max}(\mathcal{S}) = t$. Then*

$$deg(\mathcal{S}) \geq \max_{1\leq k\leq n/f} \left( \frac{1}{t} \left[ \binom{n}{k} \Big/ \binom{kf}{k} \right]^{1/t} - 1 \right) \Big/ (n-k).$$

*Proof:* For a target set $K$ of size $|K| = k$, let $R(K)$ be the minimal possible recipient set for $K$ (or one such set if many minimal recipient sets exist). Consider the collection of minimal recipient sets

$$\mathcal{D} = \{R(K) : |K| = k\}.$$

Note that covering $K$ $f$-redundantly, using the $t' \leq t$ key sets that define $R(K)$, is precisely equivalent to covering $R(K)$ *strictly* with (the same) $t'$ key sets. Therefore we see that $\mathcal{S}$ is an establishment key allocation which is strict for $\mathcal{D}$, and can transmit to any $R(K) \in \mathcal{D}$ using at most $t$ transmissions. Note also that, trivially, $|R(K)| \geq k$ for any $|K| = k$. Thus we can apply Theorem 4.5 to obtain

$$deg(\mathcal{S}) \geq \left( \frac{|\mathcal{D}|^{1/t}}{t} - 1 \right) \Big/ (n-k). \tag{1}$$

By definition $|R(K)| \leq kf$ for all $K$, however, some sets $R(K) \in \mathcal{D}$ may have fewer than $kf$ elements. Define a modified collection $\mathcal{D}'$ in which each $R(K) \in \mathcal{D}$ is replaced by some superset $\hat{R}(K) \supseteq R(K)$ with $|\hat{R}| = kf$. Note that $|\mathcal{D}'| \leq |\mathcal{D}|$ since $\hat{R}(K_1) = \hat{R}(K_2)$ is possible when $R(K_1) \neq R(K_2)$. But now $\mathcal{D}'$ is a $k$-$(n, kf)$ covering design. Thus we can lower-bound its size by the Schönheim bound, Theorem 4.3, to obtain

$$|\mathcal{D}| \geq |\mathcal{D}'| \geq L(n, kf, k) \geq \binom{n}{k} \Big/ \binom{kf}{k}. \tag{2}$$

Plugging (2) into (1) and maximizing the expression over the choice of $k$ yields our result. ∎

Using standard estimations of binomial coefficients, and maximizing over $k$, we can obtain the following asymptotic estimate.

**Corollary 4.7** *Let $\mathcal{S}$ be an $f$-redundant establishment key allocation over a universe $\mathcal{U}$ of size $n$, for which $t_{\max}(\mathcal{S}) = t$. Then $deg(\mathcal{S}) \geq \exp(\Omega(n/tf))$.* ∎

We therefore see that the $f$-redundancy gives us a substantial gain in the degree: the bound of [LS98] for strict establishment key allocations is $deg(\mathcal{S}) = \exp(\Omega(n/t))$. In other words, if we allow a redundancy factor of $f$ we can hope to use only an $f$'th root of the number of keys required per STT in a strict establishment key allocation for the same number of transmissions.

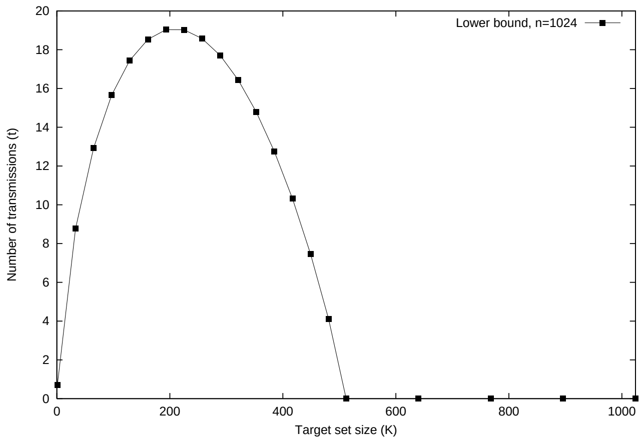


Figure 1: The lower bound for the number of transmissions ($t$) as a function of the target set size $k$, with $n = 1024$, $f = 2$, and $deg(\mathcal{S}) = \log_2 n$.

Theorem 4.6 and Corollary 4.7 give a lower bound on the required number of keys an STT needs to store. As we said before, this is typically a small fixed value which we can reasonably model by $\log_2 n$ or $n^\epsilon$. Thus we are more interested in the inverse lower bound, on the number of transmissions $t$. Asymptotically we can obtain the following bound.

**Corollary 4.8** *Let $\mathcal{S}$ be an $f$-redundant establishment key allocation over a universe $\mathcal{U}$ of size $n$. Then*

$$t_{\max}(\mathcal{S}) \geq \begin{cases} \Omega\left(\frac{n}{f \log\log n}\right), & \text{when } deg(\mathcal{S}) = O(\log n), \\ \Omega(\frac{n}{f \log n}), & \text{when } deg(\mathcal{S}) = O(n^\epsilon). \end{cases}$$

■

The asymptotic bound of Corollary 4.8 hides the constants, and inverting Theorem 4.6 gives a rather unwieldy expression for the lower bound on $t$. Therefore, we choose to invert Theorem 4.6 numerically and to plot the result, as a function of the target set size $k$, in Figure 1. As we shall see in the sequel, the highest point on this curve ($t \approx 19$ for $n = 1024$) is significantly lower than our best constructions, which suffer from a worst case of $t_{\max}(\mathcal{S}) = 3n/8 = 384$ when $n = 1024$.

# 5 Finding a Good Key Cover

An $f$-redundant establishment key allocation guarantees that an $f$-redundant cover exists for every target set $K$. In particular, singleton target sets $K = \{u\}$ need to be addressed. Thus, $\mathcal{S}$ must include enough sets $S_i$ with $|S_i| \leq f$ so that every user is contained in one of them. For simplicity, we shall assume that $\mathcal{S}$ contains the singletons themselves as sets, i.e., every STT is assumed to hold one key that is unique to it.

Once we decide upon a particular $f$-redundant establishment key allocation $\mathcal{S}$, we still need to show an efficient algorithm to find an $f$-redundant key cover $\mathcal{C}(K)$ for every target set $K$. Among all possible $f$-redundant key covers that $\mathcal{S}$ allows, we would like to pick the best one. By

Input: Target set $K$, establishment key allocation $\mathcal{S} = \{S_1, \ldots, S_m\}$.

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0. $R \leftarrow \emptyset$; $\mathcal{C} \leftarrow \emptyset$
1. **Repeat**
2. $\mathcal{A} \leftarrow \{S_i : \frac{|S_i \setminus R|}{|(K \cap S_i) \setminus R|} \leq f\}$. (* Candidate sets *)
3. $A \leftarrow S_i \in \mathcal{A}$ which maximizes $|(K \cap S_i) \setminus R|$.
4. $R \leftarrow R \cup A$; $\mathcal{C} \leftarrow \mathcal{C} \cup \{A\}$.
5. **until** the candidate collection $\mathcal{A}$ is empty.
6. **return** $R$, $\mathcal{C}$.

Figure 2: Algorithm *f-Cover*

"best" we mean here a cover that minimizes the number of transmissions $t$. Trying to minimize the actual redundancy $f_a$ would lead to trivialities: Since we assumed that $\mathcal{S}$ contains all the singletons we can always achieve the optimal $f_a = 0$. Thus, for every target set $K$, we obtain the following optimization problem:

**Input:** A collection of sets $\mathcal{S} = \{S_1, \ldots, S_m\}$ and a target set $K$.

**Output:** A sub-collection $\mathcal{C}_{\min}(K) \subseteq \mathcal{S}$ with minimal cardinality $|\mathcal{C}_{\min}(K)|$ such that $K \subseteq \cup\{S_i \in \mathcal{C}_{\min}(K)\}$ and $| \cup \{S_i \in \mathcal{C}_{\min}(K)\}|/|K| \leq f$.

This is a variation of the Set Cover problem [GJ79], and thus an NP-hard optimization problem. We omit the formal reduction proving this. Moreover, it is known that no approximation algorithm exists for Set Cover with a worst case approximation ratio[3] better than $\ln n$ (unless NP has slightly super-polynomial time algorithms) [Fei98].

On the positive side, the Set Cover problem admits a greedy algorithm, which achieves the best possible approximation ratio of $\ln n$ [Joh74, Lov75]. Moreover, the greedy algorithm is extremely effective in practice, usually finding covers much closer to the optimum than its approximation ratio guarantees [GW97]. For this reason, our general algorithm *f-Cover* for choosing a key cover is an adaptation of the greedy algorithm. See Figure 2 for the details.

**Theorem 5.1** *If $\{\{1\}, \ldots, \{n\}\} \subseteq \mathcal{S}$ then algorithm f-Cover returns an f-redundant key cover of $K$ for any target set $K$.*

*Proof Sketch*: The set $R$ maintains the current cover in the algorithm. In every iteration, when a set $S_i$ is added to the cover, $|S_i \setminus R|$ new users are covered, and $|(K \cap S_i) \setminus R|$ of them are target set members that were not included in the cover before. Note that we only add a set $S_i$ if $\frac{|S_i \setminus R|}{|(K \cap S_i) \setminus R|} \leq f$ and that the sets $(S_i \setminus R)$ are disjoint for the $S_i$'s chosen in different iterations. From these observations it is easy to prove that the $f$-redundancy is kept throughout the algorithm execution, and in particular, when the algorithm terminates. ∎

**Remarks:**

- The candidate set $\mathcal{A}$ needs to be re-calculated in each iteration, since a non-candidate set $S_i$ may become a candidate (or vice versa) after some other $S_j$ is added to the cover.

[3] [Hoc95] contains a good discussion of approximation algorithms and in particular a chapter on Set Cover.

- We have not analyzed the worst case approximation ratio of algorithm *f-Cover* since we are mainly interested in its average case behavior, which we evaluated by simulations.
- It is easy to see that the time complexity of algorithm *f-Cover* is $O(m^2)$ where $m$ is the number of sets in $\mathcal{S}$.

In order to make the algorithm even more efficient, we do not use it in its most general form. Instead, we split the establishment key allocation $\mathcal{S}$ into *levels*, each containing sets of the same size. Formally, we break $\mathcal{S}$ into $\mathcal{S} = \mathcal{S}^1 \cup \mathcal{S}^2 \cup \cdots$, such that $|S_i^\ell| = k_\ell$ for some $k_\ell$ and for all $S_i^\ell \in \mathcal{S}^\ell$. The algorithm is performed in phases, where only sets belonging to level $\mathcal{S}^\ell$ are considered in the candidate set $\mathcal{A}$ during in phase $\ell$. The algorithm starts at the highest level, the one containing of the largest sets in $\mathcal{S}$. When $\mathcal{A}$ is empty at a certain level, the cover so far, $R$, and the covering sets, $\mathcal{C}$, are fed to the execution phase of the algorithm in the next (lower) level.

# 6 Practical Solutions

## 6.1 Overview

Our basic goal is to construct an $f$-redundant establishment key allocation, namely to construct an $\mathcal{S}$ that will satisfy the following requirements: (i) the number of establishment keys per user (degree) is low; and (ii) $|R_{\mathcal{C}}(K)|/|K| \leq f$ for every target set $K \subseteq U$. Given such an establishment key allocation, we evaluate its performance with respect to the number of transmissions $t$, the actual redundancy $f_a$, and the opportunity $\eta$, using computer simulations.

We are interested in "average" performance, although we do not want to assume any particular probability distribution over the choice of target sets. To avoid this contradiction to some extent, we evaluate the performance measures separately for each target set size $k$, and show the results as functions of $k$. Thus, if something is known about target set size (e.g., that sets of size $> n/4$ never occur in some application), only portions of the graphs need to be consulted.

Each data point for a target set size $k$ in the graphs represents the mean of the relevant measure, averaged over $r$ samples of $k$-sets chosen uniformly at random. We show the 95% confidence intervals[4] for each data point, unless the graphical height of the confidence intervals is very close to the size of the symbols depicted on the curves. We typically use $r = 25$ samples per data point.

Unless stated otherwise, we assume that the redundancy is $f = 2$. We also conducted experiments with other values of $f$ but they showed qualitatively similar results.

[4] A 95% confidence interval means that the population mean appears within the specified interval with probability 0.95. See [Jai91] for a precise definition of confidence interval.

## 6.2 The Tree Scheme

### 6.2.1 The Scheme's Description

A simple multi-level establishment key allocation is a balanced tree, that is built by recursively partitioning the sets of a high level into equally-sized, disjoint sets in the next level. Sets that form a partition of a single set, one level above them, are considered children of this set in the tree. The number of keys each STT holds in this scheme is only $1 + \log_a n$, where $a$ is the arity of the tree. In the sequel we always assume a binary tree ($a = 2$).

An important advantage of a tree scheme (besides its simplicity) is that the greedy algorithm of Figure 2 can easily be made to run in time linear in the size of the cover set, rather than in the total number of sets in the collection. The idea is to start at the root of the tree (the set $\mathcal{U}$) and then traverse it either in a DFS or in a BFS order. Whenever an $f$-redundant set is found, select it and ignore the subtree under it.

The problem with the tree scheme is its worst case behavior. Consider the case where $f = 2$ and the collection is a full binary tree. If the target set comprises $k = n/4$ users such that no two of them belong to a common set of size 4 or less, then we are forced to use $t = n/4$ transmissions. It is easy to see that this is the worst possible configuration.

The average behavior of the basic tree is substantially better than the worst case. Figure 3 shows the average number of transmissions on several variants of a tree for a population of $n = 1024$ users. We see from the "threshold at sets of size 2" curve in the figure that the peak of the average $t$ is 164, which is 36% less than the worst case of 256. We explain this threshold and discuss the different variants of the tree in Section 6.2.2.

We conducted the same tests for larger populations and noticed that the qualitative behavior does not change significantly, so we omit the details. Here we focus on simulations of small populations for another reason. We shall see in Section 6.4 that we can capitalize on the detailed understanding of small populations when we discuss partitioning large populations. Our results show that breaking a large population into small subgroups and solving the problem independently for each subgroup results in a good performance trade-off.

### 6.2.2 "$<$" or "$\leq$"?

A subtle issue in the execution of algorithm *f-Cover* is whether the inequality in step 2 is strict ($<$) or not ($\leq$). Assume that $f = 2$ and that the collection $\mathcal{S}$ is a full binary tree. If a set of size $S_i$ with $|S_i| = 2$ is tested using non-strict inequality, and only one member of $S_i$ is in the target set $K$, then $S_i$ is selected as a candidate and may be part of the cover. However, using a strict inequality gives a better choice, which is to select the singleton containing that user, thereby reducing the actual redundancy without increasing the number of transmissions. On the other hand, using strict inequality for larger set sizes tends to increase the number of transmissions. So, intuitively, we would like to use "$<$" in the lowest levels of the tree, and use "$\leq$" for sets of size $T$ or larger, for an appropriate threshold $T$. Figures 3, 4 and 5 compare the performance of a tree scheme when the threshold is varied. Note that the $T = 2$ curve, which we commented on before, represents using "$\leq$" everywhere.

The most striking graph is that of the actual redundancy (Figure 4). We see that when we use strict inequality in the level of the tree corresponding to sets of size 2 (i.e., the "$\leq$"

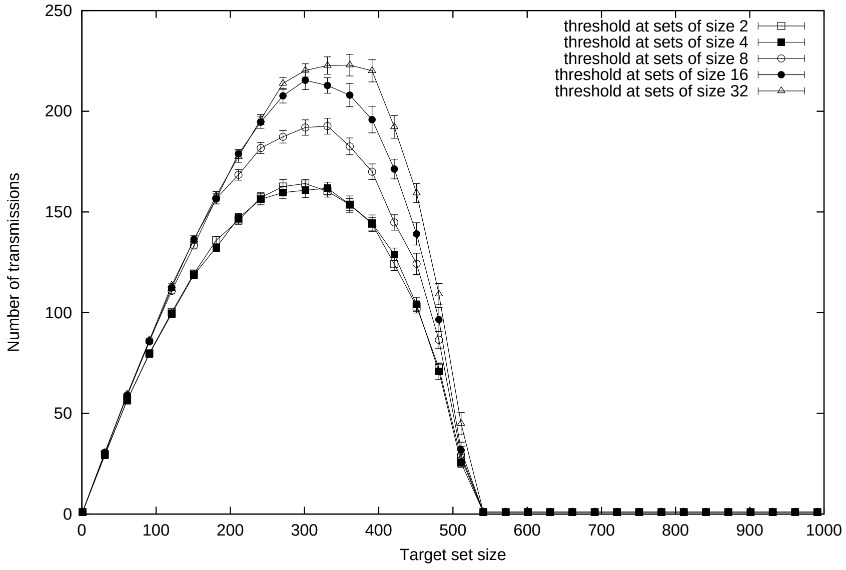


Figure 3: The effect of the "$\leq$" threshold $T$ on the number of transmissions ($t$), for a tree with $n = 1024$.

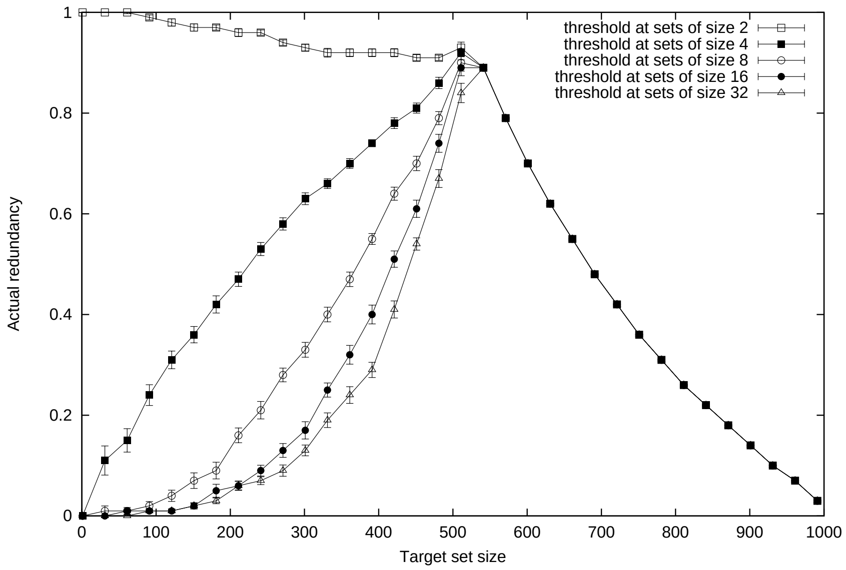


Figure 4: The effect of the "$\leq$" threshold $T$ on the Actual redundancy ($f_a$) for a tree with $n = 1024$.

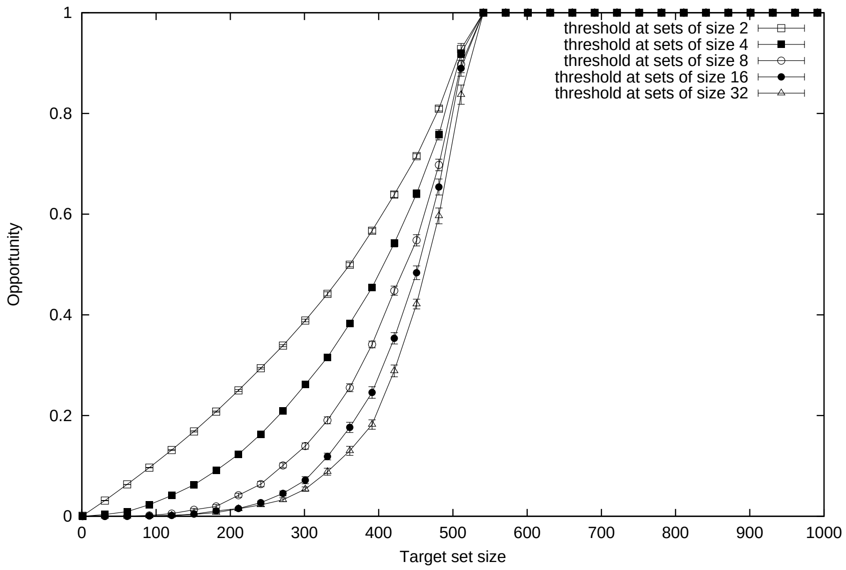


Figure 5: The effect of the "$\leq$" threshold $T$ on the Opportunity ($\eta$), for a tree with $n = 1024$.

threshold is $T = 4$) the actual redundancy, $f_a$, drops dramatically for target set sizes below $n/2$. At the same time, the number of transmissions, $t$, remains unchanged. There is also an improvement in the opportunity, $\eta$. Moving the threshold further up improves $f_a$ and $\eta$ at the cost of increasing $t$. We found out that, in most cases, and especially when extra keys are added (see below), it pays to set the threshold at $T = 8$ since the increase in $t$ is very small while the gain in $f_a$ and $\eta$ is substantial. Thus, in all the following simulations we only use strict inequality for sets of size 4 and below.

Note that choosing $T = 8$ has an effect on the worst case performance since now $k = 3n/8$ users can be selected such that no four of them belong to a common set of size 8 and no three of them belong to a common set of size 4. As a result, we would be forced to use $t = 3n/8$ transmissions, all at the level corresponding to singleton sets.

When $T = 8$, the peak number of transmissions $t$ is $193 \approx n/5$ (see Figure 3), which means a 50% improvement over the worst case performance of 384, and achieves actual redundancy that is always lower than 0.9. However, in most of the range the results are much better. In particular, if the interesting target set size range is below $k = n/5$, we get $t < n/6$, $f_a < 0.16$, and $\eta < 0.04$.

## 6.3 Where Extra Keys are Effective

The basic tree scheme requires only $\log_2 n$ keys to be stored in each STT. Therefore it is reasonable to consider schemes with slightly more keys: For populations of several millions, we can afford to keep twice or four times as many keys in an STT.

In this section, we study schemes in which a tree is augmented by additional sets. The motivation for doing so is clear: by increasing the number of sets (and thereby keys), the

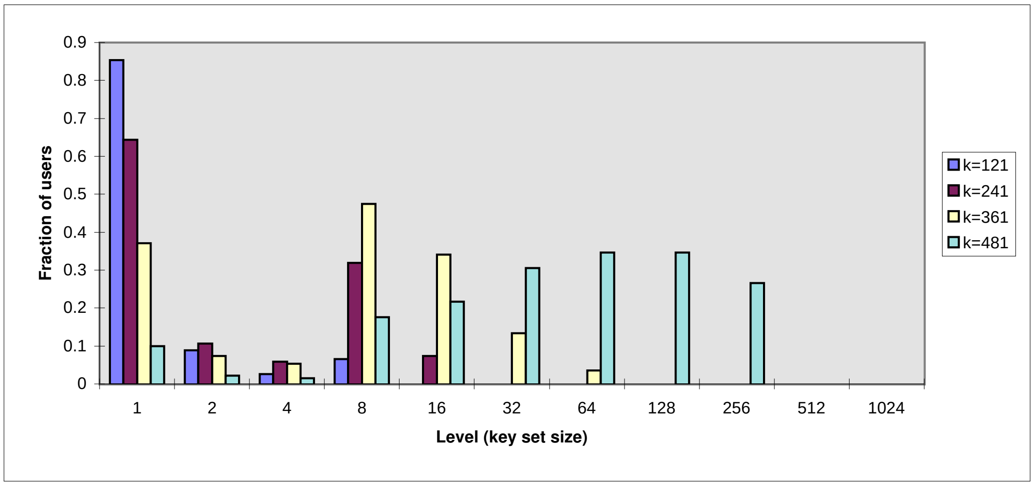


Figure 6: A histogram of the key sizes used for several target set sizes $k$, for $n = 1024$.

probability of finding a smaller cover increases. We are interested in locating the levels where it best pays to add sets, subject to the constraints on the number of keys per STT.

In order to generate the extra key sets, we start with a "level-degree" profile, which specifies how many keys each user should hold at each level. For a level with set size $k$, a degree of $d$ implies that each user should belong to $d-1$ extra sets, in addition to the one basic tree set it belongs to at this level. Thus we need to be able to generate $nd/k$ sets of size $k$, such that each user belongs to exactly $d$ of them. We achieve this by randomly permuting the $n$ users $d-1$ times, and for each random permutation we add the users in positions $(i-1)k+1, \ldots, ik$ as a set, for $i = 1, \ldots, n/k$.

A vivid explanation for the preferred placement of the extra keys can be found in the histogram in Figure 6. The histogram depicts the fraction of users covered by keys from each level of sets, for target sets of four sizes. We used a population of $n = 1024$ users and a basic tree scheme with 11 levels. The histogram clearly shows that the small sets are the ones used most often. As the target set size grows, some larger key sets are also used. However, even when the target sets are $k = 241$ and $k = 361$, i.e., target sets requiring the highest number of transmissions, relatively few keys are used for sets of size 32 and up. Therefore it seems that adding key sets at the low levels of the tree is the right approach.

Figures 7, 8, and 9 depict the performance of an 11-level tree ($n = 1024$) augmented tree with 9 extra keys. This choice allows us to double the number of keys per level in all the intermediate levels ($1 < |S_i| < n$). Following the conclusions we draw from the key usage histogram in Figure 6, these extra keys are distributed as uniformly as possible among the levels from the bottom (couples) level up to some level $\ell$. We varied $\ell$ in order to find the most effective distribution.

We first note that regardless of how the extra keys are distributed, the peak number of transmissions drops by at least 23% (from 193 down to 147 for the "up to sets of size 2" distribution) in comparison to a non-augmented tree.

Figure 7 shows that the best $t$ is achieved by distributing the extra keys at the three lowest levels, i.e., adding couples, quadruplets, and octets. Adding sets of size 16 as well resulted in

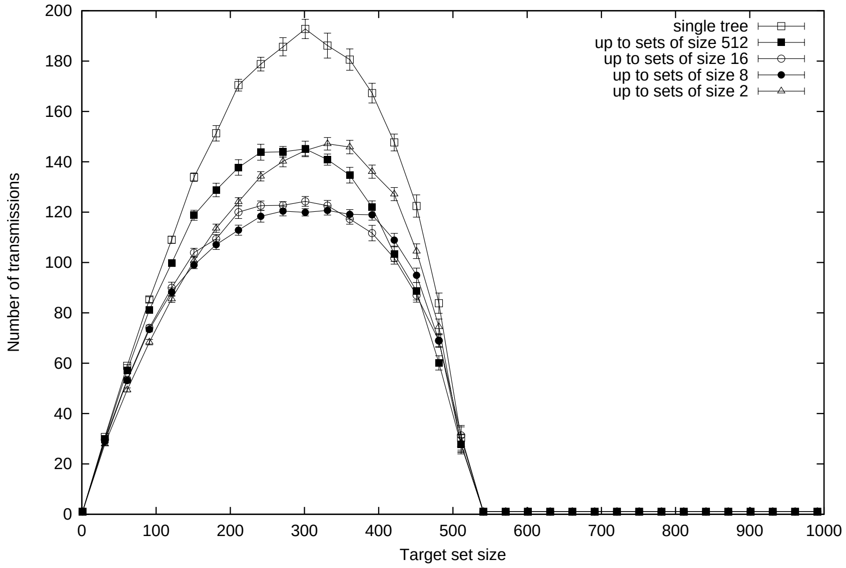


Figure 7: Number of transmissions ($t$) as a function of the target set size $k$, with $n = 1024$, $f = 2$, 11 levels, and 9 extra keys.

an almost identical performance. However, adding even larger sets gave significantly inferior performance. Figures 8 and 9 show that this improvement comes at the expense of an increase in $f_a$ for small target set sizes, although the actual redundancy is still well below the guaranteed worst case of $f_a \leq f - 1$ ($= 1$ when $f = 2$).

In a similar experiment with 38 extra keys ($= 11 + 3 \times 9$), the best $t$ was achieved by spreading the keys among the lowest 4 levels (up to sets of size 16); the peak $t$ for this experiment was about 94 transmissions, for target sets of size 271 ($= n/3.8$), which is 22% lower than the 121 achieved in Figure 7 by the "up to sets of size 8" distribution. We also ran the same experiments for larger and smaller values of $n$, with similar results. We omit the details.

Our conclusions from this set of experiments are that (a) adding a few extra keys substantially reduces the number of transmissions $t$, and (b) it pays to add these extra keys at the lower levels of the tree rather than to distribute them at higher levels as well.

## 6.4 Partitioning

The results in the previous sections suggest that keys are more "valuable" at the lower levels of the tree than at the higher levels. Thus, it seems reasonable to discard the keys of the largest sets (highest levels) altogether, and to use the additional key space for more lower level keys. We achieve this by partitioning the population, $n$, into $\nu$ disjoint partitions of size $n/\nu$. The space occupied by the $\log_2 \nu$ deleted keys per user is then used to increase the number of low level sets in each partition.

In this section we concentrate on larger, more realistic user population sizes. However, since each individual partition is small, we can apply the insight we have gained from our earlier

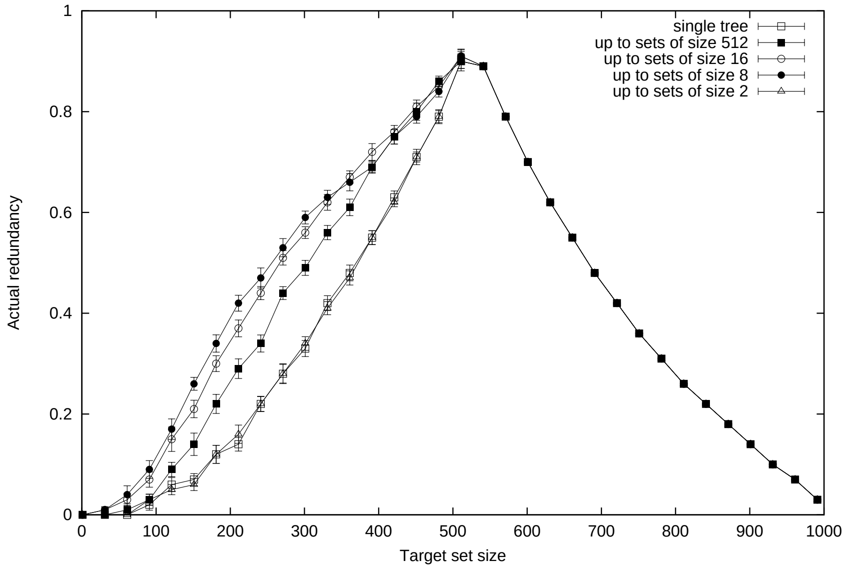


Figure 8: Actual redundancy ($f_a$) as a function of the target set size $k$, with $n = 1024$, $f = 2$, 11 levels, and 9 extra keys.

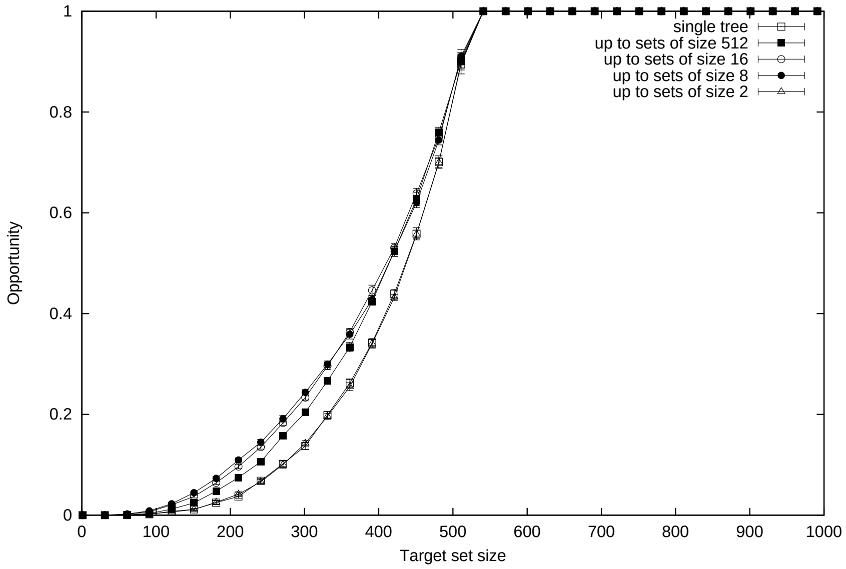


Figure 9: Opportunity ($\eta$) as a function of the target set size $k$, with $n = 1024$, $f = 2$, 11 levels, and 9 extra keys.

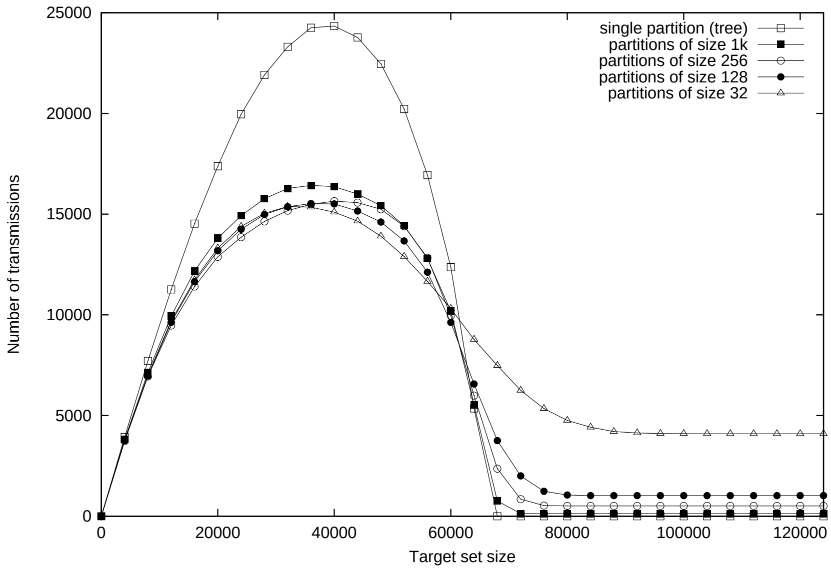


Figure 10: Number of transmissions ($t$) as a function of the target set size $k$, with $n = 128K$, $f = 2$, and 18 keys in total.

small-population experiments.

Figures 10 and 11 compare the performance of the a single tree scheme for a population of 128K customers with the performance of schemes that employ the same number of keys (18) but with $\nu$ partitions. Within each partition we distribute the $\log \nu$ extra keys to achieve the lowest peak $t$; as we have seen before, this means that the extra keys are distributed among the lowest levels in the tree, thus adding key sets of sizes between 2 and 32. For each value of $\nu$ we ran the equivalent of the experiment we discussed in Section 6.3. We report only the results of the best (lowest peak $t$) extra-key distribution for each value of $\nu$.

Figure 10 shows that the decrease in $t$ is dramatic for a large range of target set sizes. In particular, the peak $t$ drops by about 36%, from 24337 for a single partition to 15526 for $\nu = 1024$ partitions of size 128 each. Increasing the $\nu$ further reduces $t$ for some values of $k$. However, for large target set sizes, and especially those with $k > n/2$, we pay a penalty in the number of transmissions. For such large target sets we have to use $t = \nu$ transmissions instead of one. We argue that as long as $\nu$ is substantially smaller than the peak $t$, the savings in $t$ for smaller target sets far outweighs the penalty incurred for large target sets. Moreover, dealing with targets with $k > n/2$ can be done by maintaining a single additional broadcast key together with the partitions' keys.

Figure 11 shows that partitioning the users increases $f_a$ for target sets with $k < n/2$. However, the peak $f_a$ actually drops since we no longer use the very large key sets, e.g., those with size $n/2$ or $n/4$. Partitioning also improves the opportunity for $k \sim n/2$ (graph omitted).

We conclude that partitioning the users is an effective method for designing establishment key allocations. It is better to discard the large high-level key sets in favor of extra sets at the low levels. As a rule of thumb we suggest to use at least $\nu \approx \sqrt{n}$ partitions, and possibly more for larger values of $n$.

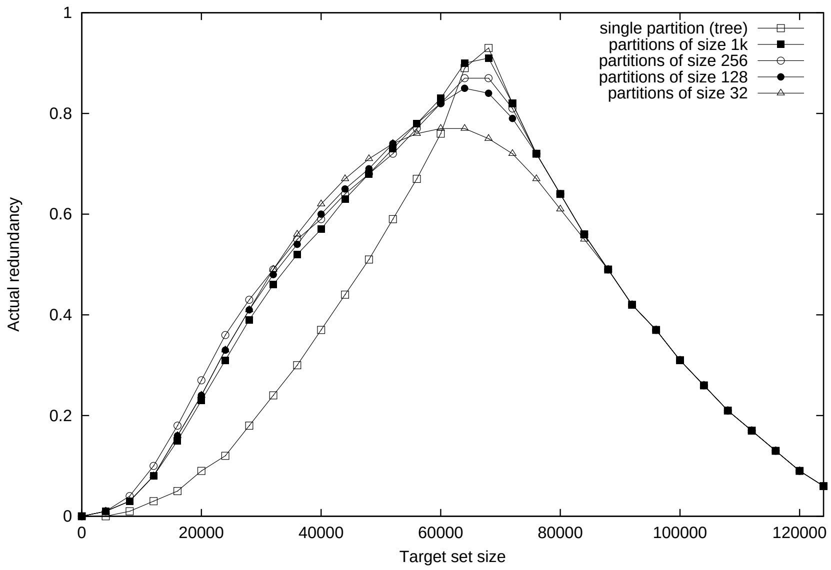


Figure 11: Actual redundancy ($f_a$) as a function of the target set size $k$, with $n = 128K$, $f = 2$, and 18 keys in total.

# 7 Conclusions and Future Work

We have demonstrated that by allowing a controlled number of free-riders we are able to design establishment key allocations that meet the hard limitations placed on secure key storage by current technology. We do this while addressing the ambitious goal of allowing *every* possible subset of users to be a target set (rather than only sets of a small fixed cardinality). We showed that despite these constraints, our schemes use substantially fewer transmissions than the naive designs. Moreover, although our schemes guarantee that the ratio between the numbers of free riders and intended receivers is at most $f-1$, the achieved redundancy ratio $f_a$ is typically much better than the guarantee. We conclude that our schemes are quite practical for applications where some free riders may be tolerated.

We have also identified some general design principles for such systems. We found that adding extra establishment key sets helps, provided that they are added at the low levels. We also found that partitioning the population into many small partitions is more effective than handling the whole population at once, since by eliminating the very large key sets we can add extra keys in each partition without exceeding the key storage limitations.

We believe that more can be done in this area. Our best constructions use five or ten times more transmissions than our lower bound suggests. Although this may seem like a small gap asymptotically, it is important in realistic scenarios. Therefore finding either better schemes or better lower bounds is still interesting.

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