The Transshipment Fund Mechanism: Coordinating the Decentralized Multilocation Transshipment Problem

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Received 20 October 2008; revised 30 November 2009; accepted 17 January 2010
DOI 10.1002/nav.20405
Published online 1 March 2010 in Wiley InterScience (www.interscience.wiley.com).

Abstract: The multilocation replenishment and transshipment problem is concerned with several retailers facing random demand for the same item at distinct markets, that may use transshipments to eliminate excess inventory/shortages after demand realization. When the system is decentralized so that each retailer operates to maximize their own profit, there are incentive problems that prevent coordination. These problems arise even with two retailers who may pay each other for transshipped units. We propose a new mechanism based on a transshipment fund, which is the first to coordinate the system, in a fully noncooperative setting, for all instances of two retailers as well as all instances of any number of retailers. Moreover, our mechanism strongly coordinates the system, i.e., achieves coordination as the unique equilibrium. The computation and information requirements of this mechanism are realistic and relatively modest. We also present necessary and sufficient conditions for coordination and prove they are always satisfied with our mechanism. Numerical examples illustrate some of the properties underlying this mechanism for two retailers.
 Naval Research Logistics 57: 342–353, 2010

Keywords: transshipment; incentives and contracting; supply chain management

1. INTRODUCTION

Transshipments, the movement of stock in the same echelon level, are often used among retailers in practice as a means to increase their profits. Typically, when demands are realized, a retailer facing excess supply transships one or more units to another retailer who is facing excess demand, provided that the cost of doing so is not higher than that of incurring the holding and shortage costs, respectively. A transshipment policy refers to the set of rules used to determine how many units should be transshipped from one retailer to another in different scenarios, if at all. An associated replenishment policy refers to the set of decisions used to determine how many units should be ordered by each retailer from the supplier, while considering the possible use of future transshipments. We refer to the combined decision of determining the replenishment and transshipment policies as the transshipment problem.

Early studies of transshipment problems in the literature considered centralized systems in which a single decision maker acts so as to maximize the total profit of the entire system. This includes for example the work of Krishnan and Rao [13], Tagaras [18], Robinson [14], and, more recently, Herer et al. [10]. Other works have addressed decentralized inventory systems, either with transshipments, see for example, Rudi et al. [15], or structured with more general (mostly vertical) supply chains, see for example, the survey by Cachon [3]. In decentralized systems, each decision maker operates so as to maximize their own profit, a setting that often results in inefficiencies. It is then desirable to achieve coordination, i.e., motivate the decision makers to act as in the centralized system while still maximizing their own profit. One way to coordinate the system is to allow transfer payments among the decision makers based on their actions. Therefore, the question addressed is what set of transfer payments, referred to as a mechanism or a contract, will achieve this goal.

The work of Rudi et al. [15] focuses on direct transfer payments between two retailers in the form of unit transshipment prices. Hu et al. [11] show with a counter example that such coordinating prices do not always exist and demonstrate this limitation for a wide range of instances. They analyze conditions under which coordinating unit transshipment prices exist in a similar system with uncertain production capacity and conclude that “further research is needed to develop implementable transshipment pricing schedules that achieve coordination in a wider range of situations”. Taking this path, in this article, we successfully extend the domain of problems that the mechanism of Rudi et al. can coordinate.

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The contributions of this article are the following. First, we develop a new mechanism based on a contract between the retailers and a transshipment fund. The transshipment fund in this mechanism is owned by a third party, who manages an account according to the contract. The account is "opened" by the retailers and subsequently used for financing or rewarding the transshipment activity through payments between the fund and the retailers. The coefficients in these payments can be interpreted as buying and selling prices that depend on the pair of retailers involved in the transaction. There are in total $2n(n-1)$ such buying and selling prices, where n is the number of retailers. This is the first coordinating mechanism for all instances of two retailers and also for all instances of any number of retailers. Moreover, we show that our mechanism achieves coordination as the unique equilibrium, a notion we introduce as *strong coordination*. Second, we present necessary and sufficient conditions for coordination and sufficient conditions for strong coordination and prove that they are always satisfied with our mechanism. Third, this is the first model formalizing transshipments in a fully noncooperative setting while achieving coordination under realistic computation and information requirements, where the latter are less demanding than existing mechanisms. Our fully noncooperative mechanism does not suffer from the disadvantage of many cooperative allocation rules (mentioned below), which in general have to rely on a high degree of group rationality, particularly the complete information sharing between decision makers regarding order quantities and demand realizations. Finally, numerical illustration is provided for several instances of the problem with two retailers, showing the payments made between the transshipment fund and the retailers for each unit transshipped.

The use of a transshipment fund can be viewed as a third party financing arrangement, in the spirit of recent popular literature on supply chain finance (SCF). SCF refers to the set of solutions available for financing the specific goods and/or products as they move from origin to destination along the supply chain. An Aberdeen Group benchmark report [1] finds that SCF is strongly appealing to companies that seek to create a cost-advantaged supply chain. More than two-thirds of the surveyed companies reported that they are investigating or putting in place SCF programs to lower end-to-end costs. This is possible for example by "using third-party intermediaries to finance the pre- or post-shipment transaction or even in some cases to own the inventory" . In our mechanism, the third party is a financial entity executing the transshipment related monetary transactions specified in the contract. The third party could be either a subcontractor serving the system by operating the transshipments and profiting from the transshipment fees, or the supplier, in which case there are natural incentives for participation in order to motivate transactions among and with the retailers. Alternatively, the third party could be paid a (small) portion of system profits to motivate participation. We believe that the idea behind our innovative mechanism may be an important step toward achieving coordination in other decentralized systems in practice as well.

In the rest of the introduction, we review the literature on decentralized transshipment and related models. Anupindi et al. [2] develop a model in which N retailers hold stock locally and/or at one or more central locations. They analyze the transshipment problem in which the decision concerning the allocation of remaining stock is cooperative among the retailers while the inventory decision is noncooperative. They constructed an allocation rule that motivates the retailers to order the same amounts as in the centralized system. Granot and Sošić [8] and Sošić [17] extend the above work and consider scenarios for which, in the cooperative stage, not all remaining supply or demand has to be shared at the end of the period. Slikker et al. [16] proved the existence of a fully cooperative game centralized core allocation rule. Yan and Zhao [21] consider the two retailer model of Rudi et al., but with normal distributions and asymmetric information about these distributions. Zhao et al. [22] consider the transshipment problem in a decentralized system in which demand at the retailer follows a Poisson process, each retailer is modeled as a make-to-stock queue, and both requesting and filling transshipments are controlled by (possibly different) threshold levels.

Other forms of capacity trading are discussed in the literature. Kouvelis and Gutierrez [12] consider a two market stochastic inventory system with nonoverlapping selling seasons in which excess inventory from the primary market can be transshipped to the secondary market. They demonstrate with examples how a nonlinear pricing scheme, administered through an intermediate organizational unit, may be useful in the coordination of the production quantities of the two markets. Van Mieghem [19] studies production and investment coordinating contracts between a manufacturer and a subcontractor who decide separately on their capacity levels and may subcontract capacity after demand realization when deciding on production and sales. Chod and Rudi [5] consider resource reallocation under price differential and based on an updated forecast.

Golany and Rothblum [7] introduce a general framework for the use of incentive mechanisms in supply chains. Specifically, they show that linear rewards and penalties determined by the marginal influence of decision makers' actions on the profit of other decision makers, evaluated at a given optimal solution, induce the centralized solution. However, their approach cannot be directly applied here because of the multistage nature of transshipment problems and because they require common knowledge/verifiability of all decisions made, an assumption that is not obviously satisfied in our setting.

The rest of the article is organized as follows: in Section 2, we present the transshipment model considered in this article and our proposed transshipment fund mechanism; in Section 3, we provide a particular payment scheme within our mechanism and show that it always satisfies the conditions for strong coordination; we further demonstrate the analysis for two retailers, including numerical examples; in Section 4, we discuss other possible mechanisms, including the relation to existing mechanisms in the literature, and conclude.

2. MODEL

Our model extends the model of Rudi et al. [15] to a general number of retailers and to a fully noncooperative game both in the replenishment and transshipment stages. In Section 2.1, we present the general framework and model assumptions and notation. In Section 2.2, we propose our new coordinating mechanism.

2.1. Framework

We consider n retailers, denoted by $i \in \{1, \ldots, n\}$, facing random demand for the same item at n distinct markets in a single period. Customers at a certain market may purchase the item only from the retailer serving that market. However, each retailer may transship items to other retailers. The sequence of events is as follows. At the beginning of the period, all retailers decide simultaneously on their individual order quantity from the supplier, without knowing the demand. Then, demand at all markets is realized, and subsequently units may be transshipped between the markets. Finally, part or all of the demand at each market is satisfied; unsatisfied demand incurs shortage costs and leftover inventory is salvaged. Our notation follows and extends the notation of Rudi et al. [15]. For each retailer and corresponding market i,

c_i is the unit purchase cost from the supplier,

r_i is the unit selling price to the customer ($r_i > c_i$),

p_i is the unit shortage cost, representing reputation loss due to unsatisfied customer demand,

s_i is the unit salvage value of remaining inventory at the end of the period ($s_i < c_i$), and

D_i is the random variable representing the retailer's demand.

In addition, for markets $i \neq j$,

τ_{ij} is the unit transshipment cost from retailer i to retailer j, incurred by retailer i.

For each retailer i, the following quantities must be determined:

Q_i is the replenishment quantity ordered and purchased from the supplier at the beginning of the period (before demand is realized), and

T_{ij} is the quantity transshipped to each retailer $j \neq i$.

The expected profit of the system, given feasible order and transshipment quantities, is

$$\sum_{i=1}^{n} E_{\mathbf{D}} \left\{ \begin{array}{l} r_i \min\left\{Q_i^T, D_i\right\} - p_i\left(D_i - Q_i^T\right)^+ \\ +s_i\left(Q_i^T - D_i\right)^+ - \sum_{j=1}^{n} \tau_{ij} T_{ij} \end{array} \right\} - c_i Q_i, \quad (1)$$

where $\mathbf{D} = (D_1, \ldots, D_n)$ is the vector of random demands, and $Q_i^T \equiv Q_i - (\sum_j T_{ij} - \sum_j T_{ji})$ is retailer i's available inventory after demand realization and feasible transshipments.

Note that if transshipments were not allowed, each retailer i would face an independent newsvendor problem, whose solution, denoted by Q_i^{NB}, satisfies $\Pr(D_i \leq Q_i^{NB}) = \frac{v_i - c_i}{v_i - s_i}$, where $v_i = r_i + p_i$ represents the value to retailer i of each unit sold. Note that $v_i \geq r_i > c_i > s_i$.

It is assumed in this model that all retailers have common knowledge of the problem parameters $c_i, r_i, p_i, s_i, \tau_{ij}$ for all i and $j \neq i$ and of the joint demand distribution function. For computational convenience, we also assume a continuous joint demand distribution with strictly increasing marginals. Finally, to avoid uninteresting solutions, we assume that

$$v_i > v_j - \tau_{ij},\ s_i > s_j - \tau_{ij},\ c_j < c_i + \tau_{ij} \text{ for all } i \neq j,$$
$$\text{and } v_j - \tau_{ij} > s_i, \text{ for at least one pair } i \neq j. \quad (2)$$

Correspondingly, these assumptions mean that it is better to sell a unit at a certain market than to transship and sell it at any other market, better to salvage a unit at a certain market than to transship and salvage it at any other market, better to purchase a unit at a certain market than to purchase it at any other market and transship, and for at least one pair of markets, it is better to transship and sell a unit at one market than to salvage it at the other market. As a result of the above conditions, a transshipment of each unit from retailer i to retailer j increases both retailers' profits only if retailer i has excess inventory and retailer j has excess demand.

As mentioned in the introduction, there is no known mechanism in the literature that always coordinates the system even for two retailers. In the next subsection, we address this problem for the multilocation model presented earlier and show that coordination can be achieved, and moreover in the strong sense explained in the introduction, i.e., as the unique equilibrium.

2.2. A mechanism with a transshipment fund

We propose a mechanism that builds on a transshipment fund, a third party financial entity that contracts with the retailers on transshipment payments. According to the contract, there are initial payments made before demand realization from each retailer to the fund in order to participate in the transshipment contract. Then the retailers simultaneously and independently order and purchase inventory from the supplier. After demand realization, transshipment quantities between the retailers and payments from the fund to the retailers are specified according to a rule predetermined in the contract. In this rule, transshipment quantities and payments depend on the announcements of the retailers about their excess supply or demand. In turn, the announcements may not necessarily be truthful, depending on whether the payments prescribed in the contract make the transshipments beneficial. The contract is designed so that the transshipment quantities, while constrained by the announcements, are best to the system. It is understood that, when signing the contract, each retailer agrees separately and in advance to follow all the rules of the contract.

Denote by $\mathbf{m} = (m_1, \ldots, m_n)$ the initial payments made before demand realization from the retailers to the fund, where m_i is retailer i's initial payment which may be positive, negative, or zero. Denote by $\bar{\mathbf{a}} = (\bar{a}_1, \ldots, \bar{a}_n)$ the vector of announcements by all retailers of their excess supply/demand, interpreted as excess supply when $\bar{a}_i > 0$ and excess demand when $\bar{a}_i < 0$. The actual excess supply/demand is denoted by $\mathbf{a} = (a_1, \ldots, a_n)$, where $a_i \equiv Q_i - d_i$ and d_i is the demand realization of D_i. The announcement $\bar{a}_i$ may depend on the actual a_i observed privately by retailer i but not on a_j for $j \neq i$, which are unknown to retailer i. The random variables corresponding to $\mathbf{a}$ and $\bar{\mathbf{a}}$ before demand realization are denoted by $\mathbf{A} = (A_1, \ldots, A_n)$ and $\bar{\mathbf{A}} = (\bar{A}_1, \ldots, \bar{A}_n)$, respectively, where $A_i \equiv Q_i - D_i$. Denote by $\mathbf{T}(\bar{\mathbf{a}}) = [T_{ij}(\bar{\mathbf{a}})]$, the transshipment quantities specified in the contract as a function of the announcements $\bar{\mathbf{a}}$. The quantities T_{ij} must satisfy, for each i and $\bar{\mathbf{a}}$, nonnegativity and the feasibility constraints

$$\sum_{j=1}^{n} T_{ij} \le (\bar{a}_i)^+ \text{ and } \sum_{j=1}^{n} T_{ji} \le (-\bar{a}_i)^+. \tag{3}$$

Denote the payments from the fund to the retailers by $\mathbf{C}(\bar{\mathbf{a}}) = [C_1(\bar{\mathbf{a}}), \ldots, C_n(\bar{\mathbf{a}})]$, where $C_i(\bar{\mathbf{a}})$ is the payment to retailer i. These payments are nonnegative for announced excess supply and nonpositive for announced excess demand, i.e., $\bar{a}_i \ge 0$ implies $C_i(\bar{\mathbf{a}}) \ge 0$ and $\bar{a}_i \le 0$ implies $C_i(\bar{\mathbf{a}}) \le 0$. For simplicity, we assume for all i that $\frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}})$ exists almost everywhere.

There are several motivations/advantages for such a mechanism, all related to transshipment incentive problems that prevent coordination in all cases when attention is restricted to linear direct payments between the retailers. As mentioned in the Introduction, these problems exist even for two retailers [11]. First, because each of the retailers cares about the transshipment payment arrangement with the transshipment fund and not about arrangements with the other retailers, this separation is potentially useful in solving transshipment incentive conflicts. Second, the transshipment fund serves as a joint account "opened" by the retailers with the amount $\sum_i m_i$ when signing the contract, and "used" after demand realization. As shown in the next section, this account is advantageous, because it makes the mechanism beneficial for the retailers in terms of expectation when signing the contract as well as in terms of transshipment incentives for every demand realization when implementing it. Third, the additional degrees of freedom provided by this mechanism create a large set of feasible transshipment payments that are sufficient to solve the incentive problem and achieve coordination in all cases.

Given the mechanism, specified by $\mathbf{C}$ and $\mathbf{m}$, retailer i's expected profit is

$$\begin{aligned} &\pi_i^d(\mathbf{Q}, \bar{\mathbf{A}}) \\ &\quad\equiv E_{\mathbf{D}} \left\{ \begin{array}{l} r_i \min\left\{Q_i^T(\bar{\mathbf{A}}), D_i\right\} - p_i\left[D_i - Q_i^T(\bar{\mathbf{A}})\right]^+ \\ +s_i\left[Q_i^T(\bar{\mathbf{A}}) - D_i\right]^+ + C_i(\bar{\mathbf{A}}) - \sum_{j=1}^{n} \tau_{ij} T_{ij}(\bar{\mathbf{A}}) \end{array} \right\} \\ &\qquad\qquad - c_i Q_i - m_i, \end{aligned} \tag{4}$$

where $\mathbf{Q} = (Q_1, \ldots, Q_n)$ is the vector of order quantities for all retailers, $\mathbf{D} = (D_1, \ldots, D_n)$ is the vector of random demands, $Q_i^T(\bar{\mathbf{a}}) \equiv Q_i - [\sum_j T_{ij}(\bar{\mathbf{a}}) - \sum_j T_{ji}(\bar{\mathbf{a}})]$ is retailer i's available inventory after transshipments given the announcements $\bar{\mathbf{a}}$, and $Q_i^T(\bar{\mathbf{A}}), T_{ij}(\bar{\mathbf{A}}), C_i(\bar{\mathbf{A}})$ are the random variables corresponding to $Q_i^T(\bar{\mathbf{a}}), T_{ij}(\bar{\mathbf{a}}), C_i(\bar{\mathbf{a}})$ before demand realization, respectively.

3. COORDINATION

In this section, we develop a payment scheme under which the transshipment fund mechanism achieves coordination. Moreover, coordination is achieved in the strong sense, i.e., as the unique equilibrium. To this end, we provide necessary and sufficient conditions for coordination on the payment functions $\mathbf{C}$ and $\mathbf{m}$. We also provide sufficient conditions for strong coordination. We then show that the particular payment scheme we propose satisfies these conditions and additionally has several advantages over other possible coordinating payment functions.

To introduce our coordinating payment scheme, we first summarize results concerning optimal transshipment quantities in the centralized system. The centralized transshipment quantities depend on the entire vector $\mathbf{a}$ of actual excess supply/demand. The optimal transshipment quantities T_{ij} are obtained for each $\mathbf{a}$ by solving the LP,

$$\begin{aligned} &\max \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} (v_j - s_i - \tau_{ij}) T_{ij} \\ &\text{such that} \\ &\textstyle\sum_{j=1}^{n} T_{ij} \le (a_i)^+ \qquad \forall i \\ &\textstyle\sum_{j=1}^{n} T_{ji} \le (-a_i)^+ \qquad \forall i \\ &T_{ij} \ge 0 \qquad \forall i, j \end{aligned}$$

adapted to a maximization problem from Herer et al. [10]. Fix an optimal solution to this LP and denote it by $\mathbf{T}^{cp}(\mathbf{a})$, standing for complete pooling transshipment quantities given the vector $\mathbf{a}$, an extended interpretation of the complete pooling term for two retailers coined by Tagaras [18]. When multiple optimal solutions exist, any tie breaking rule may be applied.

Now we present the main result of our analysis. The following theorem presents strongly coordinating payment functions, which additionally have the advantage of easy computation. The proof of the theorem appears later in this section, after presenting results on necessary and sufficient conditions on the payment functions $\mathbf{C}$ for achieving centralized order quantities and sufficient conditions for achieving them as the unique equilibrium (Theorem 2) and necessary and sufficient conditions for achieving centralized transshipment quantities (Theorem 3). In particular, we will show that with the proposed payment functions of Theorem 1, each retailer is motivated to announce truthfully their own excess supply/demand, thus achieving centralized transshipment quantities. Moreover, given the other retailers' order quantities, each retailer's expected profit is the same as the centralized expected profit, up to a constant. Thus, the retailer is motivated to order the centralized quantity given that the others do the same. Beneficial participation of the retailers is ensured by setting the initial fixed payments m_i, charged by the transshipment fund, so that each retailer's expected profit is higher than when operating alone. Moreover, the expected profit of the transshipment fund can be set to zero (or a small positive amount), implying that the total expected profit of the centralized system is fully achieved.

THEOREM 1: Using the payment functions

$$\hat{C}_i(\bar{\mathbf{a}}) \equiv \sum_{\substack{j=1\\ j\neq i}}^{n} \left[v_j T_{ij}^{cp}(\bar{\mathbf{a}}) - (s_j + \tau_{ji}) T_{ji}^{cp}(\bar{\mathbf{a}})\right] + \sum_{\substack{j=1\\ j\neq i}}^{n} \sum_{\substack{k=1\\ k\neq i}}^{n} (v_j - s_k - \tau_{kj})\left[T_{kj}^{cp}(\bar{\mathbf{a}}) - T_{kj}^{cp}(0, \bar{\mathbf{a}}_{-i})\right] \quad (5)$$

for all i and each $\bar{\mathbf{a}}$, where $(0, \bar{\mathbf{a}}_{-i})$ is the vector $\bar{\mathbf{a}}$ with $\bar{a}_i$ replaced by 0, there always exist initial payments m_i, that together with $\hat{C}_i(\bar{\mathbf{a}})$, strongly coordinate the system.

We refer to the payment functions $\hat{C}_i(\bar{\mathbf{a}})$ as the *centralized gain* payments. These functions are linear in the transshipment quantities between each pair of retailers; thus, they are piecewise linear in the excess supply/demand announcements. The linear coefficients in these payment functions can be interpreted as buying and selling prices from/to the fund that depend on the pair of retailers involved in the transaction. The first sum in $\hat{C}_i(\bar{\mathbf{a}})$ involves the direct contribution to the system due to transshipments to/from retailer i, whereas the second sum compensates retailer i for the indirect contributions to the system, associated with transshipments among other retailers caused by this retailer's participation. More specifically, in the first, direct contribution sum, retailer i is paid by the fund the selling price v_j gained by retailer j per unit received from retailer i, and pays to the fund the buying price $s_j + \tau_{ji}$, which is the sum of costs incurred by retailer j per unit transshipped to retailer i. Thus, when transshipping a unit from i to j, the net direct contribution gain to each of these retailers is equal to the system gain, $v_j - s_i - \tau_{ij}$. The indirect contribution is caused by the difference between the system's optimal transshipments with and without the participation of retailer i (the quantity $T_{kj}^{cp}(\bar{\mathbf{a}}) - T_{kj}^{cp}(0, \bar{\mathbf{a}}_{-i})$). Retailer i is compensated by the fund with the price difference $v_j - s_k - \tau_{kj}$ gained by the system per each unit, in the above difference, transshipped from retailer k to retailer j. Thus, there are in total $2n(n-1)$ buying and selling prices involved in the payment functions. The computation of the centralized gain payments involves solving $n+1$ LP problems to find $\mathbf{T}^{cp}(\bar{\mathbf{a}})$ and $\mathbf{T}^{cp}(0, \bar{\mathbf{a}}_{-i})$ for all i. A simple IT system can be used to submit and post the announcements as well as to perform the above computation.

The proof of the main result in Theorem 1 relies on several important results that we develop next. The following immediate lemma shows that for a given $\mathbf{Q}$, the system's centralized profit from (1) can be written, under optimal transshipments, as the sum of newsvendor expected profits plus the expected contribution of transshipments to all retailers. This result is useful subsequently in deriving first-order conditions for the optimal order quantities in both the centralized and the decentralized systems.

LEMMA 1: The system profit under optimal transshipments equals

$$\pi^t(\mathbf{Q}) \equiv \sum_{i=1}^{n} E_{\mathbf{D}}\big\{r_i \min\{Q_i, D_i\} - p_i(D_i - Q_i)^+ + s_i(Q_i - D_i)^+ + L_i^*(\mathbf{A}) - c_i Q_i\big\},$$

where $L_i^*(\mathbf{A})$ is the random contribution of the optimal transshipment quantities to retailer i's profit, defined given realization $\mathbf{a}$ of $\mathbf{A}$ by

$$L_i^*(\mathbf{a}) \equiv \begin{Bmatrix} \sum_{j=1}^{n}(-s_i - \tau_{ij}) T_{ij}^{cp}(\mathbf{a}) & \text{if} \quad a_i \geq 0 \\ \sum_{j=1}^{n} v_i T_{ji}^{cp}(\mathbf{a}) & \text{if} \quad a_i < 0 \end{Bmatrix}.$$

PROOF: The system expected profit under optimal transshipments is the expression in (1) with $Q_i^T \equiv Q_i - \sum_j T_{ij}^{cp}(\mathbf{A}) + \sum_j T_{ji}^{cp}(\mathbf{A})$, given by

$$\sum_{i=1}^{n} E_{\mathbf{D}} \left\{ \begin{array}{l} r_i \min\left\{Q_i - \sum_j T_{ij}^{cp}(\mathbf{A}) + \sum_j T_{ji}^{cp}(\mathbf{A}), D_i\right\} \\ -p_i\left[D_i - Q_i + \sum_j T_{ij}^{cp}(\mathbf{A}) - \sum_j T_{ji}^{cp}(\mathbf{A})\right]^+ \\ +s_i\left[Q_i - \sum_j T_{ij}^{cp}(\mathbf{A}) + \sum_j T_{ji}^{cp}(\mathbf{A}) - D_i\right]^+ \\ -\sum_{j=1}^{n} \tau_{ij} T_{ij}^{cp}(\mathbf{A}) \end{array} \right\} - \sum_{i=1}^{n} c_i Q_i.$$

Since $a_i \geq 0$ implies $\sum_j T_{ij}^{cp}(\mathbf{a}) \leq a_i$ and $\sum_j T_{ji}^{cp}(\mathbf{a}) = 0$, and similarly since $a_i \leq 0$ implies $\sum_{j=1}^{n} T_{ji}^{cp}(\mathbf{a}) \leq -a_i$ and $\sum_j T_{ij}^{cp}(\mathbf{a}) = 0$, this profit simplifies to

$$\sum_{i=1}^{n} E_{\mathbf{D}} \left\{ \begin{array}{ll} r_i D_i + s_i\left[Q_i - \sum_j T_{ij}^{cp}(\mathbf{A}) - D_i\right] & \\ \quad -\sum_{j=1}^{n} \tau_{ij} T_{ij}^{cp}(\mathbf{A}) & \text{if } A_i \geq 0 \\ r_i\left[Q_i + \sum_j T_{ji}^{cp}(\mathbf{A})\right] & \\ \quad -p_i\left[D_i - Q_i - \sum_j T_{ji}^{cp}(\mathbf{A})\right] & \text{if } A_i < 0 \end{array} \right\} - \sum_{i=1}^{n} c_i Q_i$$

$$= \sum_{i=1}^{n} E_{\mathbf{D}} \left\{ \begin{array}{ll} r_i D_i + s_i(Q_i - D_i) & \\ \quad +\sum_{j=1}^{n}(-s_i - \tau_{ij}) T_{ij}^{cp}(\mathbf{A}) & \text{if } A_i \geq 0 \\ r_i Q_i - p_i(D_i - Q_i) & \\ \quad +\sum_{j=1}^{n} v_i T_{ji}^{cp}(\mathbf{A}) & \text{if } A_i < 0 \end{array} \right\} - \sum_{i=1}^{n} c_i Q_i,$$

establishing the result because $a_i \equiv Q_i - d_i$. □

Note that while π_i^d in (4) is a function of the order quantities vector $\mathbf{Q}$ as well as the retailers' announcement vector $\bar{\mathbf{A}}$, π^t in Lemma 1 is not a function of the actual supply/demand vector $\mathbf{A}$, because $\mathbf{A}$ is computed away by the expectation. Thus, π^t is a function of only the order quantities vector $\mathbf{Q}$. One can show, using arguments similar to Herer et al. [10], that π^t is a concave function of $\mathbf{Q}$.

The optimal centralized order quantities $\mathbf{Q}^t = (Q_1^t, \ldots, Q_n^t)$ always exist uniquely. They simultaneously solve for all i the necessary and sufficient (due to concavity of π^t) first order conditions

$$\frac{\partial \pi^t}{\partial Q_i} = (v_i - c_i) + (s_i - v_i)\Pr(D_i \leq Q_i) + \sum_{j=1}^{n} E_{\mathbf{D}} \frac{\partial L_j^*}{\partial a_i}(\mathbf{A}) = 0, \quad (6)$$

where the derivatives in (6) and everywhere else in the article are right-sided.

In the decentralized system, because the contract always specifies the best transshipments, the LP presented at the beginning of this section is solved given $\bar{\mathbf{a}}$. Thus, given Lemma 1, when $0 \leq \bar{a}_i \leq a_i$ or $a_i \leq \bar{a}_i \leq 0$ (this condition applies also to Lemma 2 below), retailer i's expected profit can be rewritten as[1]

$$\pi_i^d(\mathbf{Q}, \bar{\mathbf{A}}) = E_{\mathbf{D}} \left\{ \begin{array}{c} r_i \min\{Q_i, D_i\} - p_i(D_i - Q_i)^+ + s_i(Q_i - D_i)^+ \\ +L_i^*(\bar{\mathbf{A}}) + C_i(\bar{\mathbf{A}}) \end{array} \right\} - c_i Q_i - m_i. \quad (7)$$

Each retailer i's best response function $Q_i(\mathbf{Q}_{-i})$ is defined by the retailer's optimal order quantity given the other retailers' order quantities $\mathbf{Q}_{-i}$. To coordinate the system, the transshipment quantities must be equal to $\mathbf{T}^{cp}(\mathbf{a})$. These quantities are achieved if and only if each retailer i announces the actual excess supply/demand, i.e., $\bar{a}_i = a_i$, which we refer to as truth announcements. This leads to the following lemma.

LEMMA 2: Under truth announcements and concavity of $\pi_i^d(\mathbf{Q}, \mathbf{A})$ in Q_i, $\mathbf{Q}$ are equilibrium order quantities if and only if they simultaneously solve the first order conditions for all i,[2]

$$\frac{\partial \pi_i^d}{\partial Q_i} = (v_i - c_i) + (s_i - v_i)\Pr(D_i \leq Q_i) + E_{\mathbf{D}}\left[\frac{\partial L_i^*}{\partial a_i}(\mathbf{A}) + \frac{\partial C_i}{\partial \bar{a}_i}(\mathbf{A})\right] = 0. \quad (8)$$

PROOF: Under the assumptions of the lemma, retailer i's best response is derived from the first order condition with respect to Q_i. The derivative in (8) follows immediately because $\frac{\partial a_i}{\partial Q_i} = 1$. □

Denoting the truth announcements Nash equilibrium order quantities by $\mathbf{Q}^d = (Q_1^d, \ldots, Q_n^d)$, we have the following theorem (its assumptions of truth announcements and concavity will be verified later for our proposed centralized gain payments).

THEOREM 2: Under truth announcements and concavity of $\pi_i^d(\mathbf{Q}, \mathbf{A})$ in Q_i, a necessary and sufficient condition

[1] The general case is contained in the proof of Theorem 3.

[2] The derivatives $\frac{\partial L_i^*}{\partial a_i}$ and $\frac{\partial C_i}{\partial \bar{a}_i}$ are each taken with respect to the i'th argument of the variable appearing in the functions L_i^* and C_i, i.e., a_i and $\bar{a}_i$, respectively, and then evaluated at $\mathbf{a}$.

on the payment functions $\mathbf{C}$ for achieving centralized order quantities is

$$E_{\mathbf{D}}\frac{\partial C_i}{\partial \bar{a}_i}(\mathbf{A}^t) = \sum_{\substack{j=1\\ j\neq i}}^{n} E_{\mathbf{D}}\frac{\partial L_j^*}{\partial a_i}(\mathbf{A}^t) \text{ for all } i, \tag{9}$$

where the expectation is computed at $\mathbf{A}^t \equiv \mathbf{Q}^t - \mathbf{D}$. Moreover, if (9) holds for all $\mathbf{A}$, instead of just for $\mathbf{A}^t$, then ordering the centralized quantities is the unique equilibrium.

PROOF: By Lemma 2 and the assumptions of the theorem, a necessary and sufficient condition for achieving centralized order quantities is $\frac{\partial \pi_i^d}{\partial Q_i}(\mathbf{Q}^t, \mathbf{A}^t) = 0$ for all i. This is a condition on the transshipment payments $\mathbf{C}$, because $\mathbf{Q}^t$ and consequently $\mathbf{A}^t$ are given. Since $\frac{\partial \pi^t}{\partial Q_i}(\mathbf{Q}) = 0$ for all i exactly identifies $\mathbf{Q}^t$, the necessary and sufficient condition is that $\frac{\partial \pi^t}{\partial Q_i}(\mathbf{Q}) = 0$ for all i implies $\frac{\partial \pi_i^d}{\partial Q_i}(\mathbf{Q}, \mathbf{A}) = 0$ for all i. Thus, substituting (6) and (8) with $\mathbf{A} = \mathbf{A}^t$ and cancelling terms, the proof of the first part of the theorem is established. Applying the same substitution and cancellation of terms to the reverse implication, i.e., that $\frac{\partial \pi_i^d}{\partial Q_i}(\mathbf{Q}, \mathbf{A}) = 0$ for all i implies $\frac{\partial \pi^t}{\partial Q_i}(\mathbf{Q}) = 0$ for all i, establishes the second part of the theorem concerning the unique equilibrium. □

Condition (9) makes the expected marginal payment to retailer i due to an additional unit of $\bar{a}_i$, resulting from ordering an additional unit of Q_i under truth announcements, equal to the expected contribution of this unit to the other retailers' total profit. Consequently, the expected contribution of the unit to retailer i's profit, including L_i^* and the payment C_i, is equal to its expected contribution to the system. Note that payment functions satisfying condition (9) are not unique, essentially because the condition is stated in expectation. Note also that satisfying (9) for all $\mathbf{A}$ is possible, as the payment functions $\hat{\mathbf{C}}$ of Theorem 1 satisfy this condition (this is shown in the proof of Theorem 1).

It remains to verify that the centralized gain payments satisfy (9) for all $\mathbf{A}$, and that they are truth motivating and make $\pi_i^d(\mathbf{Q}, \bar{\mathbf{A}})$ concave in Q_i. Then, we show that they also make participation beneficial for all retailers, given an appropriate choice of the initial payments, m_i. We first establish conditions for truth announcements.

THEOREM 3: The conditions

$$\bar{a}_i \geq 0 \Longrightarrow 0 \leq \frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) \leq v_i - s_i, \text{ and} \tag{10}$$
$$\bar{a}_i < 0 \Longrightarrow 0 \geq \frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) \geq -(v_i - s_i)$$

for all $\bar{\mathbf{a}}$ and i, are necessary and sufficient for truth announcements and thus for achieving centralized transshipment quantities.

PROOF: Equation (7) describes retailer i's profit $\pi_i^d(\mathbf{Q}, \bar{\mathbf{A}})$ when $0 \leq \bar{a}_i \leq a_i$ or $a_i \leq \bar{a}_i \leq 0$, i.e., when announcing lower excess supply/demand than actual amounts. In other cases, when announcing higher excess supply/demand than the actual amounts or when announcing excess supply instead of excess demand or vice versa, there are additional losses of $v_i - s_i$ for each transshipped unit, presumed not salvaged, but in fact not sold. This is because $L_i^*(\bar{\mathbf{a}})$ over evaluates the transshipment gains as it assumes that all transshipped units are sold rather than salvaged. If a unit is not transshipped, then (7) applies with no additional losses.

To prove sufficiency of (10), consider an actual excess supply/demand vector $\mathbf{a}$ with $a_i \geq 0$. By the argument regarding retailer i's profit at the beginning of the proof, the marginal profit gain from announcing an additional unit which is actually not transshipped is zero, and when announcing an additional unit which is transshipped the marginal profit gain equals

$$\begin{array}{ll}
\dfrac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \dfrac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) & \text{if} \quad 0 \leq \bar{a}_i < a_i \\
\dfrac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \dfrac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) - (v_i - s_i) & \text{if} \quad \bar{a}_i \geq a_i \\
\dfrac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \dfrac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) + (v_i - s_i) & \text{if} \quad \bar{a}_i < 0.
\end{array}$$

Note that the additional loss of $v_i - s_i$ reduces the nonnegative $\frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}})$ when $\bar{a}_i \geq a_i \geq 0$ and increases (reduces in absolute value) the nonpositive $\frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}})$ when $\bar{a}_i < 0$ (thus it has a plus sign). Because (10) implies that this marginal profit gain is nonnegative for $\bar{a}_i < a_i$ and nonpositive for $\bar{a}_i \geq a_i$, announcing $\bar{a}_i = a_i$ is optimal. Similarly, announcing $\bar{a}_i = a_i$ is optimal when considering an actual excess supply/demand vector $\mathbf{a}$ with $a_i < 0$.

To prove necessity of (10), consider an equilibrium with truth announcements for all demand realizations. Suppose that (10) is violated for some $\bar{\mathbf{a}}$ with $\bar{a}_i \geq 0$. Consider demand realizations that result in $\mathbf{a} = \bar{\mathbf{a}}$. The above expression for the marginal profit gain from announcing an additional unit implies that if $\frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) > v_i - s_i$, then it is beneficial to announce $\bar{a}_i > a_i$, and if $\frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) < 0$ then it is beneficial to announce $\bar{a}_i < a_i$. This contradicts the equilibrium assumption. A similar argument applies when (10) is violated for some $\bar{\mathbf{a}}$ with $\bar{a}_i < 0$. □

Now we are ready to prove the main result in Theorem 1 presented at the beginning of this section.

PROOF OF THEOREM 1: First note that (5) can be rewritten as

$$\hat{C}_i(\bar{\mathbf{a}}) \equiv \sum_{\substack{j=1\\j\neq i}}^{n} \left[v_j T_{ij}^{cp}(\bar{\mathbf{a}}) - (s_j + \tau_{ji}) T_{ji}^{cp}(\bar{\mathbf{a}})\right] + \sum_{\substack{j=1\\j\neq i}}^{n} \sum_{\substack{k=1\\k\neq i}}^{n} (v_j - s_k - \tau_{kj}) \left[T_{kj}^{cp}(\bar{\mathbf{a}}) - T_{kj}^{cp}(0, \bar{\mathbf{a}}_{-i})\right] = \sum_{\substack{j=1\\j\neq i}}^{n} \left[L_j^*(\bar{\mathbf{a}}) - L_j^*(0, \bar{\mathbf{a}}_{-i})\right].$$

The functions $\hat{C}_i(\bar{\mathbf{a}})$ satisfy (10). To see this note that for these payments, the derivatives in (10) become $\frac{\partial L_i^*}{\partial a_i}(\bar{\mathbf{a}}) + \frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) = \sum_{j=1}^n \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}})$. Thus $\bar{a}_i \sum_{j=1}^n \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}}) \geq 0$ because an additional unit of excess supply/demand weakly improves the LP optimal objective value, $\sum_{j=1}^n L_j^*(\bar{\mathbf{a}})$. Furthermore, the remainder of (10) follows immediately when $\sum_{j=1}^n \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}}) = 0$ because $v_i > s_i$. It also follows when $|\sum_{j=1}^n \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}})| > 0$, because then in the LP, there is an increase in the quantity transshipped from i to some $k \neq i$, thus $|\sum_{j=1}^n \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}})| \leq v_k - s_i - \tau_{ik} < v_i - s_i$. To see this, note that the first inequality follows from concavity of the optimal LP objective value (because the r.h.s. of this inequality is the magnitude of the derivative when $\bar{\mathbf{a}}$ is changed to zero everywhere except for i, k) and the last inequality follows from $v_k - \tau_{ik} < v_i$ by the initial conditions (2).

Given that we established truth announcements using the functions $\hat{C}_i(\bar{\mathbf{a}})$, we may replace everywhere $\bar{\mathbf{a}}$ by $\mathbf{a}$ in the remainder of this proof. It is immediate to verify that $\frac{\partial \hat{C}_i}{\partial \bar{a}_i}(\mathbf{a}) = \sum_{j=1, j\neq i}^n \frac{\partial L_j^*}{\partial a_i}(\mathbf{a})$ for all i and $\mathbf{a}$, thus $\hat{C}_i$ satisfy (9) for all $\mathbf{A}$. They also imply concavity of $\pi_i^d(\mathbf{Q}, \mathbf{A})$ in Q_i because $\pi^t(\mathbf{Q}) - \pi_i^d(\mathbf{Q}, \mathbf{A}) = \sum_{j=1, j\neq i}^n E_{\mathbf{D}}\{r_j \min\{Q_j, D_j\} - p_j(D_j - Q_j)^+ + s_j(Q_j - D_j)^+ - c_j Q_j + L_j^*(0, \mathbf{A}_{-i})\} + m_i$ is constant with respect to Q_i.

Finally, we show the existence of initial payments m_i ensuring beneficial participation. To this end, we extend the notation $\pi_i^d(\mathbf{Q}, \mathbf{A})$ to $\pi_i^d(\mathbf{Q}, \mathbf{A}; \mathbf{C}, \mathbf{m})$, thus explicitly expressing the dependence on the mechanism design parameters $\mathbf{C}$ and $\mathbf{m}$. Start by disregarding the initial payments m_i. Since the centralized actions form a truth announcements Nash equilibrium, retailer i's expected profit $\pi_i^d(\mathbf{Q}^t, \mathbf{A}^t; \hat{\mathbf{C}}, \mathbf{0})$ under the centralized action, given the opponents' centralized actions, is at least as high as the expected profit under any other action. In particular, it is at least as high as $\pi_i^d(\mathbf{Q}^{NB}, \mathbf{A}^{NB}; \hat{\mathbf{C}}, \mathbf{0})$, i.e., under the optimal order quantity when operating alone, $\mathbf{Q}^{NB}$, but allowing for transshipments, where $\mathbf{A}^{NB} \equiv \mathbf{Q}^{NB} - \mathbf{D}$. In turn, transshipment benefits make this expected profit at least as high as $\pi_i^d(\mathbf{Q}^{NB}, \mathbf{0}; \mathbf{0}, \mathbf{0})$, the expected profit when operating alone without transshipments. Moreover, note that $\sum_{i=1}^n \pi_i^d(\mathbf{Q}^t, \mathbf{A}^t; \hat{\mathbf{C}}, 0) - \sum_{i=1}^n E_{\mathbf{D}}\hat{C}_i(\mathbf{A}^t) = \pi^t(\mathbf{Q}^t) \geq \sum_{i=1}^n \pi_i^d(\mathbf{Q}^{NB}, \mathbf{0}; \mathbf{0}, \mathbf{0})$, again due to transshipment benefits. Thus, denoting by $\delta_i \equiv \pi_i^d(\mathbf{Q}^t, \mathbf{A}^t; \hat{\mathbf{C}}, \mathbf{0}) - \pi_i^d(\mathbf{Q}^{NB}, \mathbf{0}; \mathbf{0}, \mathbf{0})$, the expected profit gain over operating alone and disregarding the initial payments, we have $\delta_i \geq 0$. Denoting by $\lambda \equiv \frac{\sum_{i=1}^n E_{\mathbf{D}}\hat{C}_i(\mathbf{A}^t)}{\sum_{i=1}^n \delta_i}$, the ratio of expected fund payments to total expected profit gain, we have $\lambda \leq 1$. One possible set of initial payments is $m_i = \lambda\delta_i$ for each i. Since $\pi_i^d(\mathbf{Q}^t, \mathbf{A}^t; \hat{\mathbf{C}}, \mathbf{m}) = \pi_i^d(\mathbf{Q}^t, \mathbf{A}^t; \hat{\mathbf{C}}, \mathbf{0}) - \lambda\delta_i \geq \pi_i^d(\mathbf{Q}^{NB}, \mathbf{0}; \mathbf{0}, \mathbf{0})$, retailer i's expected profit including the initial payment is higher than when operating alone. Moreover, the expected net payment to the fund $\sum_{i=1}^n m_i - \sum_{i=1}^n E_{\mathbf{D}}\hat{C}_i(\mathbf{A}^t)$ is zero. □

In addition to establishing strong coordination, the centralized gain payments of Theorem 1 have the advantage that their computation is immediate in two aspects: (a) it involves solving $n+1$ LP problems and computing differences between their optimal values, and (b) the payments are independent of (and therefore insensitive to) almost all the parameters of the model, including demand distribution (in particular, the possibly difficult to compute centralized solution probabilities in (6), e.g. β_i and γ_i below for two retailers, are not required). Although there is a continuum of payment functions satisfying (9) and the remaining necessary and sufficient conditions for coordination, the payment functions (5) are unique in the sense that they do not depend on the demand distribution. This is because when omitting the expectations in (9) and instead requiring this condition for each $\bar{\mathbf{a}}$ when $C_i(0, \bar{\mathbf{a}}_{-i}) = 0$ for some continuous and almost everywhere differentiable function, one arrives uniquely at the centralized gain payments. These payment functions are piecewise linear in $\bar{\mathbf{a}}$ because they are derived from a maximization LP solution. Also note that although the transshipment payments $\mathbf{C}$ solve the incentive problems, the initial payments m_i for all i allow a range of possible allocations of the total expected profit gain from transshipment to the retailers.

3.1. Two retailers

Consider the special case of two retailers. Since only two scenarios for excess supply/demand are applicable for two retailers, depending on the transshipment direction between them, we can use a linear price payment:

> c_{is} is the price paid to i per unit of excess supply, and
> c_{si} is the price paid by i per unit of excess demand.

Thus, we replace for both i the derivative $\frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{a}})$, and correspondingly the random variable $\frac{\partial C_i}{\partial \bar{a}_i}(\bar{\mathbf{A}})$, by prices c_{is} and c_{si}. The complete pooling transshipment quantities are $T_{ij}^{cp}(\mathbf{a}) \equiv \min\{(a_i)^+, (-a_j)^+\}$ for $j \neq i$. The centralized first order condition (6) reduces to the one in Tagaras [18] and Robinson [14], which can be written for both i as

$$
\begin{aligned}
0 = \frac{\partial \pi^t}{\partial Q_i} =& (v_i - c_i) + (s_i - v_i)\alpha_i(Q_i) \\
&+ [(-s_i - \tau_{ij})\beta_i(Q_i, Q_j) - v_i\gamma_i(Q_i, Q_j)] \\
&+ [v_j\beta_i(Q_i, Q_j) + (s_j + \tau_{ji})\gamma_i(Q_i, Q_j)]
\end{aligned}
$$

where

$$
\begin{aligned}
\alpha_i(Q_i) &= \Pr(D_i \le Q_i), \\
\beta_i(Q_i, Q_j) &= \Pr(Q_i + Q_j - D_j \le D_i \le Q_i), \\
\gamma_i(Q_i, Q_j) &= \Pr(Q_i \le D_i \le Q_i + Q_j - D_j).
\end{aligned}
$$

The decentralized first order condition (8) reduces for both i to

$$
\begin{aligned}
0 = \frac{\partial \pi_i^d}{\partial Q_i} =& (v_i - c_i) + (s_i - v_i)\alpha_i(Q_i) \\
&+ [(-s_i - \tau_{ij})\beta_i(Q_i, Q_j) - v_i\gamma_i(Q_i, Q_j)] \\
&+ [c_{is}\beta_i(Q_i, Q_j) + c_{si}\gamma_i(Q_i, Q_j)],
\end{aligned}
$$

and is similar to Equation (10) in Rudi et al. (Ref. 15, p 1674), with c_{ij}, c_{ji} replaced by c_{is}, c_{si}, respectively. In this condition there are four prices involved, with just two equations, resulting in the additional degrees of freedom mentioned in Section 2.2 when discussing the motivation for the transshipment fund mechanism. Condition (9) becomes

$$
\begin{aligned}
&c_{is}\beta_i\left(Q_i^t, Q_j^t\right) + c_{si}\gamma_i\left(Q_i^t, Q_j^t\right) \\
&\quad = v_j\beta_i\left(Q_i^t, Q_j^t\right) + (s_j + \tau_{ji})\gamma_i\left(Q_i^t, Q_j^t\right). \quad (11)
\end{aligned}
$$

Note the linear relation between the two prices c_{is}, c_{si} corresponding to each retailer i, demonstrating the fact that these prices are not unique. Moreover, the values c_{is}, c_{si} are independent of the values c_{js}, c_{sj}.

For two retailers, conditions (10) are rewritten as

$$
\begin{aligned}
&\bar{a}_i \ge 0 \Longrightarrow s_i + \tau_{ij} \le c_{is} \le v_i + \tau_{ij}, \text{ and} \qquad (12) \\
&\bar{a}_i < 0 \Longrightarrow s_i \le c_{si} \le v_i.
\end{aligned}
$$

These inequalities imply that truth announcements are dominating actions, because they ensure for retailer i that transshipment is beneficial if and only if there is excess supply. This holds because the net payment received $c_{is} - \tau_{ij}$ is (weakly) better than the salvage value s_i and (weakly) worse than the sale value v_i. Similarly, for retailer j, transshipment is beneficial if and only if there is excess demand, because the payment c_{sj} is (weakly) better than the salvage value s_j and (weakly) worse than the sale value v_j.

Computing the centralized gain payments in (5) for two retailers, when $\bar{a}_i > 0$ and $\bar{a}_j < 0$, $\hat{c}_{is} = \frac{\partial \hat{C}_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) = \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}}) = v_j$. Similarly, when $\bar{a}_i < 0$ and $\bar{a}_j > 0$, $\hat{c}_{si} = \frac{\partial \hat{C}_i}{\partial \bar{a}_i}(\bar{\mathbf{a}}) = \frac{\partial L_j^*}{\partial a_i}(\bar{\mathbf{a}}) = s_j + \tau_{ji}$. Note that $\hat{c}_{is}, \hat{c}_{si}$ satisfy (11) for all Q_i and Q_j, thus achieving strong coordination. These prices can be interpreted as follows: when transshipping from i to j, retailer i charges the transshipment fund the maximal value that the fund can receive from j, and the transshipment fund charges retailer j the minimal value that the fund may be asked to pay i. Using these prices, both retailers gain an equal amount

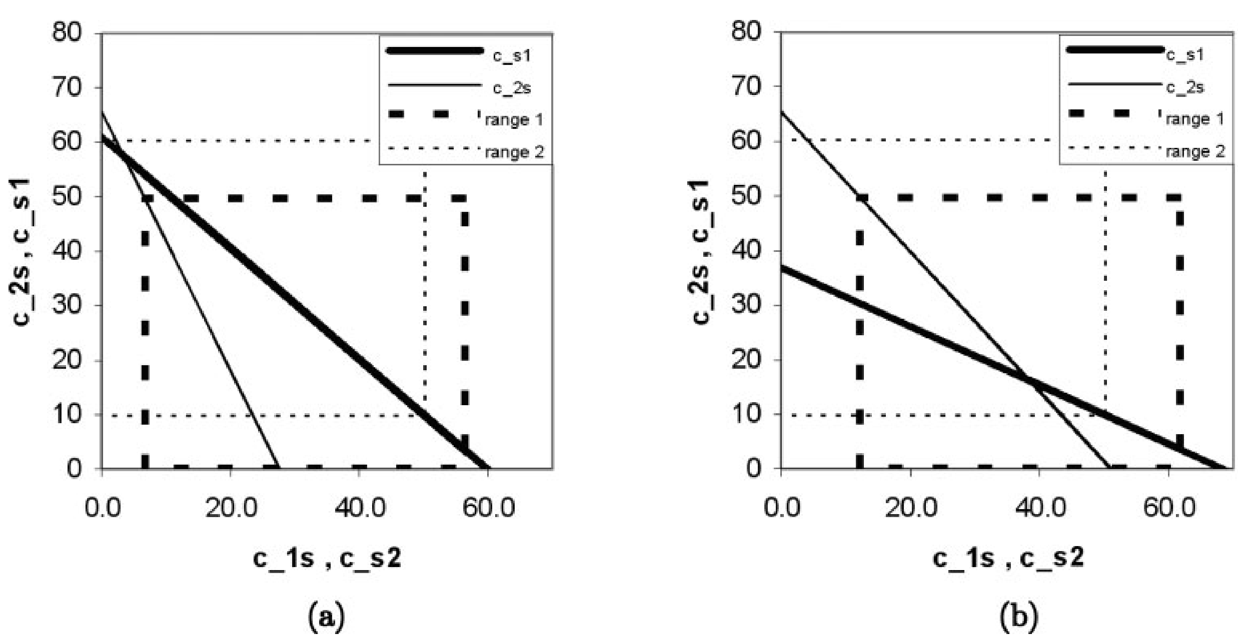

Figure 1. Transshipment fund vs. direct payments. (a) The mechanism of Rudi et al. [15] is not applicable. (b) The mechanism of Rudi et al. [15] is applicable.

of $v_j - (s_i + \tau_{ij})$ from each unit transshipped from i to j, which is the same as the system's gain from transshipping a unit from i to j in the centralized system. For the transshipment fund, these prices correspond to a net payment of $v_j - (s_i + \tau_{ij})$ to the retailers for each unit transshipped, a payment financed (in expectation) by the positive account, $m_1 + m_2$, deposited by them at the transshipment fund when signing the contract.

To illustrate the transshipment fund mechanism, we provide plots of the transshipment prices c_{is}, c_{si} in the case of two retailers for several instances of the problem. Consider the model parameters $c_1 = c_2 = 20$, $r_1 = r_2 = 50$, $p_1 = p_2 = s_1 = s_2 = 0$, $\tau_{21} = 10$, each retailer's demand distribution is uniformly and independently distributed between 0 and 10 and τ_{12} takes two possible values, 6.5 or 12. The two instances are depicted in Figs. 1a and 1b, respectively. In each graph, the downward sloping lines represent Eq. (11) for both retailers. The dotted rectangles in the graphs represent the range of the transshipment fund prices given in (12). Any pair of points, one on each downward sloping line, that lie inside the appropriate rectangle, correspond to a set of coordinating transshipment fund prices. The graphs demonstrate that such a set of prices is not unique. The intersection point of these lines represent the case $c_{is} = c_{sj}$ for both i and $j \neq i$, which is the proposed direct payments solution denoted in Rudi et al. [15] by c_{ij} (note that $m_i = 0$ for both i in this case). The relevant range in which the direct payments solution exists is the intersection of the two rectangles. Figure 1a plots an instance for which these prices are not applicable because they violate the inequalities in (12), whereas they are applicable for the instance plotted in Fig. 1b. The centralized gain prices $\hat{c}_{is}, \hat{c}_{sj}$ are the two corner points of the intersection of the rectangles, each lying on one downward sloping line. For these prices, the sum of initial payments $m_1 + m_2$ is 35.1 for $\tau_{12} = 6.5$ and 31.4 for $\tau_{12} = 12$, which is approximately, in this example, the total gain of the two retailers above operating alone, in which case their total profit is 180. Thus, this sum can be split in this example almost in any way, including equally, while still ensuring each retailer a positive gain above operating alone.

The main feature of the transshipment fund mechanism for two retailers is the possibility that $c_{is} \neq c_{sj}$, i.e., the transshipment price received by the transshipment fund from one retailer may be different from the transshipment price paid to the other retailer. To illustrate this point, we consider again the parameters used in Fig. 1 for varying values of τ_{12}. For each value of τ_{12}, Fig. 2 plots the range of possible net transshipment prices $c_{is} - c_{sj}$ paid by the transshipment fund, as implied by the necessary and sufficient conditions (11) and (12). The figure shows the maximal and minimal net transshipment prices for various sets of coordinating transshipment payments. The net price may be positive, zero, or negative.

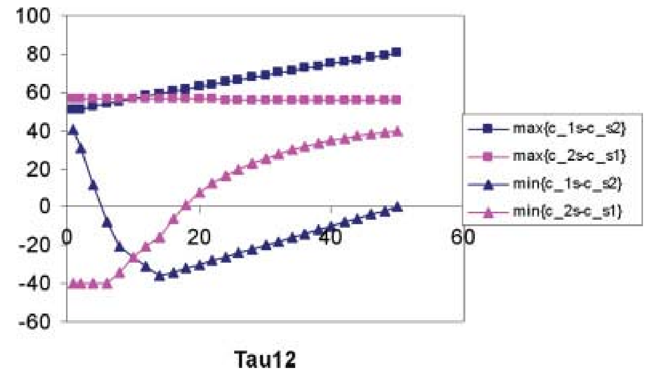

Figure 2. Net transshipment prices paid by the fund. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

4. DISCUSSION AND CONCLUDING REMARKS

We now discuss the properties of the transshipment fund mechanism. First, Theorem 1 shows that the mechanism strongly coordinates the system for all instances. Second, the retailers are motivated to announce the actual excess supply/demand, thus allowing for the centralized quantities to be transshipped. Note that the previous literature on decentralized transshipment problems did not discuss the information requirements for implementing the centralized transshipments. Moreover, the transshipment incentive problem that prevents coordination even in a two-retailers decentralized transshipment problem is not solved when the actual transshipments are assumed to be verifiable/common knowledge. The main informational advantage of our mechanism is that no further common knowledge is required on order quantities or on demand realizations. Third, the retailers benefit from the mechanism both in expectation when signing the contract and when implementing it after observing demand realization. In particular, if the fund contracts with several supply chains, or when the contract is implemented repeatedly in several periods, the average net payments from/to the fund, including the initial payments m_i, will approach the expected net payments.

To coordinate the system, it is necessary to enforce the required transshipment payments between the retailers and the transshipment fund. Enforcement is attainable because the contract and excess supply/demand announcements can be easily made common knowledge using a simple IT system. For two retailers for example, consider the case where $c_{is} < c_{sj}$, e.g., when choosing alternative prices to those suggested in Theorem 1. Without enforcement, the retailers could improve upon the mechanism by paying each other directly for each unit transshipped from i to j a price c_{ij} satisfying $c_{is} < c_{ij} < c_{sj}$ instead of via the transshipment fund. When $c_{is} > c_{sj}$, as is the case of the prices in Theorem 1, such improvements are not possible.

A natural mechanism that may be suggested to resolve the transshipment incentive problem is profit sharing. If each retailer gains proportionally from the total profit, the behavior will be as in the centralized system. We emphasize the informational disadvantage of the profit sharing mechanism compared with the transshipment fund mechanism. To implement such profit sharing, it is necessary that the order quantities and demand realization are common knowledge/verifiable. Such an assumption in the context of real life arrangements may be questionable.

The transshipment fund mechanism is close to the VCG mechanisms [6, 9, 20], which also achieve coordination by making the marginal effect of each player's decisions on their own profit equal to their effect on the system. However, our approach is not a direct implementation of these mechanisms because of the multistage nature of the transshipment problem. Only the last stage of our mechanism is based on VCG and is used to motivate truthful announcements of excess supply/demand. As the literature on the decentralized transshipment problem has shown, even under full cooperation at this last stage of the game, there are inefficiencies caused by nonoptimal first stage order quantities. Our mechanism is able to solve these issues as well. Moreover, VCG requires an assumption of private values, i.e., each player cares only about private parameters, whereas our game is of common values, because the transshipment gain for one retailer depends on the private information of other retailers about their excess supply/demand. Furthermore, we derive necessary and sufficient conditions for coordination, allowing for a range of coordinating payment schemes, only one of which is related to VCG mechanisms. Our mechanism also provides a practical operational method of coordination in the spirit of financial supply chain.

Another mechanism that is arguably less problematic than profit sharing in terms of informational requirements may be defined similar to Cachon and Zipkin [4]. They consider a two-stage supply chain and suggest transfers that are linear in observed variables like inventory levels and backorders. A similar idea in the context of a horizontal system, as the one analyzed in this article, may result in a mechanism based on sharing the overage cost incurred by the retailers after demand realization and after exhausting all possible overage elimination using transshipments. To this end, in the two retailers case, one may add to the direct transshipment prices the parameters $l_i \in [0, 1)$ for both i representing the proportion of retailer j's overage cost, $c_j - s_j$, paid by retailer i, as well as a net initial payment m_{12} between the retailers. We conjecture, based on some numerical experiments, that such a mechanism is comparable to the transshipment fund mechanism in terms of achieving coordination. It is inferior in terms of information requirements because more information is required to be verified between the retailers. In particular, it is necessary that the quantity of remaining inventory after transshipments is verified; otherwise, the retailers have incentive to distort upward their reported quantities in order to receive larger compensation from the other retailer.

In sum, we introduced in this article a new mechanism, based on a transshipment fund, in order to solve the incentive problems in the two-location transshipment problem for all instances and extend it to many retailers. Providing benefits to each retailer compared with operating alone, this transshipment fund mechanism motivates them to act as in the centralized system while still maximizing their own profits. Our model can also be extended to many periods with no essential changes. This is because our setting does not include fixed cost or lead time for replenishment; thus, there is no incentive to ration inventory for future use, because replenishment decisions can be repeated exactly in the same way in every period. We believe that the approach proposed by this kind of mechanism is useful for achieving coordination in other decentralized systems.

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