

Useful Inequalities

1. For all $0 < x < 1$ and $n \geq 1$:

$$1 - x \leq \left(1 - \frac{x}{n}\right)^n < e^{-x} < 1 - x + \frac{x^2}{2}$$

or

$$x - \frac{x^2}{2} < \ln(1 + x) < x$$

2. If $0 < x < 1 < y$ and $xy \leq 1$:

$$1 - xy < (1 - x)^y < e^{-xy}$$

3. If $0 \leq x \leq 1 - \epsilon$ for some $\epsilon > 0$, then

$$\frac{\ln \epsilon}{1 - \epsilon} \cdot x \leq \ln(1 - x) \leq -x$$