

Problem Set 2

Due: March 23, 2010.

1. For a given graph G , let $\text{rad}(G)$ and $\text{diam}(G)$ denote the radius and the diameter of G , respectively. Prove that $\text{rad}(G) \leq \text{diam}(G) \leq 2 \cdot \text{rad}(G)$. Give examples (graphs) where equality holds for each of the inequalities.
2. Suppose that the Bellman-Ford algorithm is initiated with a few sources, i.e., a few nodes start out with $d = 0$. What will be the final output at each node? What will be the effect if different sources are started at different times?
3. In class we have considered BFS trees, which are shortest path trees when all edges are unweighted. Give algorithms to compute shortest-path trees in the weighted case, for both the synchronous and asynchronous models, and analyze their time and message complexities.
4. Suppose that the weight of edges may be negative. Will the Bellman-Ford algorithm work correctly? Explain.
5. Give a parametric example (description of the graph and schedule) where the asynchronous version of the Bellman-Ford algorithm generates $2^{\Omega(n)}$ messages in a graph of n nodes.