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Control of wave propagation in a dielectric medium by tailoring its dispersive properties

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Abstract

In recent studies we have developed a space–frequency model for the propagation of a high frequency signal in an arbitrary dispersive medium. The model can be solved analytically under certain conditions for a Gaussian pulse, revealing the conditions under which pulse compression or expansion occurs. In this work we have utilized previously obtained results to calculate analytically the medium resonance parameters for manipulating a signal with a given width for a carrier frequency that is on resonance. This enables tailoring materials for certain pulse characteristics in order to achieve an a-priori-defined amount of compression, expansion and delay.

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1. Introduction

In recent works [1,2] we have studied the effect that a general dispersive medium in which the complex dielectric permittivity depends on the frequency, on a pulse traveling through this medium. Previous literature includes several theoretical papers that dealt with the problem of distortion occurring when a short pulse is propagating in absorptive and dispersive media, including gases and plasmas [3–9]. They also studied the delay and pulse shape evolution along the path of propagation. Gibbins [8] extended earlier investigations [4,6] and examined distortions of short Gaussian pulses, modulating millimeter waves and propagating in the atmosphere. An approximation of the wave propagation factor was used to derive analytical expressions for the pulse shape. Conditions for pulse broadening and compression were identified.

The aim of this study is two fold: to understand how a naturally occurring medium affects electro-magnetic pulses traveling inside this medium, this is a typical problem in applications such as communications, radar systems and energy transfer. Another purpose is to describe a method for which one can affect the properties of the electromagnetic pulse such as its power, width and time of propagation (delay time) by introducing a dielectric medium in its path. In particular in this paper we study how a material with a resonant absorption line can be used to obtain the desired modifications. This of course will be shown to depend on the intrinsic properties of the absorption line that include its strength, resonance frequency and width. As a model for such an absorption line a Lorentzian curve was utilized. It will be shown that in order to realize useful features such as pulse compression a non-trivial choice of medium parameters should be considered.

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2. Results

2.1. The medium

We assume a dielectric medium, the effect of the medium on the electro-magnetic pulse is described by a complex dielectric permittivity that depends on the frequency. We further assume that the permittivity contains resonant like features. Since the most interesting effects occur near resonance we choose to study the behavior of the pulse at resonance frequencies. In this case the susceptibility χ_e is given by the Lorentzian function

$$\chi_e(f) = \frac{\chi}{1 - (\frac{f}{f_r})^2 + j(\frac{f}{f_r})Q^{-1}}. \tag{1}$$

In which f_r is the resonant frequency and χ is a measure of the 'strength' of the resonance, Q is the quality factor and $j = \sqrt{-1}$. Q also measures the Lorentzian width since for large enough Q, $|\chi_e(f)|$ drops to about 44% of its value for $\frac{f}{f_r} = 1 \pm \frac{1}{Q}$. $\chi_e(f)$ is related to the complex propagation factor $k(f)$ used in the transfer function given in [1,2] by the formula

$$k(f) = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_r} = \frac{2\pi f}{c} \cdot \sqrt{1 + \chi_e(f)}. \tag{2}$$

In what follows we make the approximation: $\chi \cdot Q = \varepsilon_1 \ll 1$ this will enable us to write the propagation factor as a sum of two terms

$$k(f) = \frac{2\pi f}{c} \cdot \sqrt{1 + \chi_e(f)} \cong \frac{2\pi f}{c} \cdot \left(1 + \frac{1}{2}\chi_e(f)\right) = k_s(f) + k_r(f). \tag{3}$$

One is the free space propagation factor given by

$$k_s(f) = \frac{2\pi f}{c} = \beta_s(f) \quad (\alpha_s = 0), \tag{4}$$

(α, β are the imaginary and real parts of k see [1,2]). And the other is the 'resonant' propagation factor given by

$$k_r(f) = \frac{\pi f}{c} \cdot \chi_e(f) = \beta_r(f) - j\alpha_r(f). \tag{5}$$

This allows the factorization of the transfer function (see [1,2]) as follows:

$$H = \frac{d_0}{d + d_0} e^{-jkd} = H_S H_r. \tag{6}$$

This will result in a simpler numerical scheme in which we only need to integrate H_r in order to calculate $\overline{A}$, from which it is trivial to calculate the pulse shape function (defined in [1,2]) A_{out}

$$\begin{aligned} \overline{A}(t) &= \int_{-\infty}^{+\infty} A_{in}(f) \cdot H_r(f + f_0) \cdot e^{+j2\pi f t} df, \\ \Rightarrow A_{out}(t) &= \frac{d_0}{d + d_0} \overline{A}\left(t - \frac{d}{c}\right) e^{-j\frac{2\pi f_0}{c} d}. \end{aligned} \tag{7}$$

2.2. Analytical calculations

The analytical calculations of the various quantities such as delay time, pulse width and pulse power include the calculations of derivatives up to second order of α and β (see [1,2]). But first we will differentiate between trivial 'free propagation' effects and non-trivial 'resonant' effects. Following the definition of t_d in [1,2] we can write the contributions to t_d as

$$t_d = \frac{\beta'_S}{2\pi} d + t_r = t_S + t_r = \frac{d}{c} + t_r. \tag{8}$$

In which t_r is the 'resonant' delay time give only in term of resonant quantities

$$t_r = \frac{1}{2\pi} \left[\beta'_r - \frac{\alpha'_r \beta''_r d}{(2\pi\sigma_{in})^2 + \alpha''_r d}\right] \cdot d. \tag{9}$$

The other quantities of interest that can be dissected in this way are the pulse width (see [1,2])

$$\begin{aligned} &\sigma_{out}^2 = \sigma_r^2 + \sigma_s^2 \Rightarrow \\ &\sigma_s^2 = \sigma_{in}^2, \\ &\sigma_r^2 = \frac{\alpha''_r d}{(2\pi)^2} + \frac{\left[\frac{\beta''_r d}{(2\pi)^2}\right]^2}{\sigma_{in}^2 + \frac{\alpha''_r d}{(2\pi)^2}}. \end{aligned} \tag{10}$$

From the above formula it is obvious that a change in the pulse width is only due to the resonant contribution, in the free propagating scenario the pulse retains its width as expected. To calculate the change in the square root of the pulse power we take the absolute value of the analytical approximation of A_{out} (given in [1,2]) in $t = t_d$:

$$\begin{aligned} &PF = |A_{out}(t_d)| = PF_s \cdot PF_r \Rightarrow \\ &PF_s = \frac{d_0}{d_0 + d}, \\ &PF_r = \frac{\sigma_{in}}{\sqrt[4]{(\sigma_{in}^2 + \frac{\alpha''_r d}{(2\pi)^2})^2 + (\frac{\beta''_r d}{(2\pi)^2})^2}} \\ &\quad \cdot \exp\left[-\alpha_{0r} \cdot d + \frac{1}{2} \frac{(\alpha'_r d)^2}{(2\pi\sigma_{in})^2 + \alpha''_r d}\right]. \end{aligned} \tag{11}$$

The free space power factor PF_s just scales linearly with the distance as expected, for the resonant part PF_r we obtain a more complex expression, which contain dominant exponential factors.

2.3. Approximation validity

In calculating the above analytical expressions one should bare in mind that the following approximations are made, first $\chi \cdot Q = \varepsilon_1 \ll 1$. Second: it is assumed that

$$\sum_{n=0}^{2} \frac{1}{n!} \cdot k^{(n)} \cdot \sigma_{in}^{-n} \gg \sum_{n=3}^{\infty} \frac{1}{n!} \cdot k^{(n)} \cdot \sigma_{in}^{-n}. \tag{12}$$

From Eq. (12) in which derivatives are taken to third order one derives the condition

$$1 \gg \varepsilon_2 = \frac{2Q}{\sigma_{\text{in}} f_{\text{r}}} \tag{13}$$

at resonance, in the case that Q is large. This means that the width of the Gaussian in the frequency domain should be much smaller than the width of the Lorentzian curve. Further more in [1,2] we state an additional condition for the analytical approximated transfer function integral to converge, the condition takes the following form for resonant frequencies

$$\sigma_{\text{in}}^2 > 8Q^3 \cdot \frac{\chi t_{\text{S}}}{4\pi f_{\text{r}}}. \tag{14}$$

2.4. Pulse with a carrier frequency at resonance

Bearing in mind the above conditions we can now calculate the various quantities of interest. For the resonant delay time we obtain the result

$$t_{\text{r}} = \frac{1}{2\pi}\beta_{\text{r}}' \cdot d = -t_{\text{S}}\chi Q^2. \tag{15}$$

The negative sign of t_{r} indicates that signal arrives faster than should be expected by speed of light propagation, hence it appears super-luminar. However, this is only an illusion, which is caused by the infinite extent of the Gaussian pulse. In fact the super Luminal Gaussian results from the tail of the original Gaussian as is explained in many papers and text books [10–12]. (The propagation of the pulse still has causal characteristics that can be described by the Sommerfeld forerunner [13]).

To evaluate the effect of the resonance on the pulse width we calculate the α, β derivatives in Eq. (10) at resonance, this can rewritten as

$$\sigma_{\text{out}}^2 = \frac{\chi t_{\text{S}}}{4\pi f_{\text{r}}}\left[\hat{\sigma}_{\text{in}}^2 - 8Q^3 + \frac{4Q^4}{\hat{\sigma}_{\text{in}}^2 - 8Q^3}\right], \quad \left(\hat{\sigma}_{\text{in}}^2 = \frac{4\pi f_{\text{r}}}{\chi t_{\text{s}}}\sigma_{\text{in}}^2\right). \tag{16}$$

The above formula shows clearly that the largest pulse expansion in the time domain (and hence the best coherence in the frequency domain) is achieved when $\hat{\sigma}_{\text{in}}^2$ is very close to $8Q^3$. Introducing the small parameter $\Delta^2 = \hat{\sigma}_{\text{in}}^2 - 8Q^3$ we obtain the pulse expansion coefficient

$$\text{ER}^2 = \frac{\sigma_{\text{out}}^2}{\sigma_{\text{in}}^2} \cong \frac{Q}{2\Delta^2}. \tag{17}$$

For minimal expansion and compression we utilize an equation from [1,2], which gives the width of the input signal needed to achieve maximum compression. It has the resonant value

$$\sigma_{\text{in}}^2 = \frac{d}{(2\pi)^2}(|\beta_{\text{r}}''| - \alpha_{\text{r}}'') = \frac{\chi t_{\text{S}} Q^2}{2\pi f_{\text{r}}}(1+4Q). \tag{18}$$

For which the minimal width of the output signal and compression ratio can be written as

$$\sigma_{\text{out}_{\min}}^2 = 2\frac{|\beta_{\text{r}}''|d}{(2\pi)^2} = \frac{\chi t_{\text{S}} Q^2}{\pi f_{\text{r}}}, \quad \text{CR}^2 = \frac{\sigma_{\text{out}_{\min}}^2}{\sigma_{\text{in}}^2} = \frac{2}{1+4Q}. \tag{19}$$

We see that whatever the effect: time delay, pulse expansion or compression the size of the effect depends on the quality factor of the Lorentzian.

3. Conclusions

In this work we suggest a method to 'tailor' the features of the medium in order to obtain some desired effects on the signal. Such effects as pulse compression, expansion or introducing a 'negative' time delay can be realized for a Lorentzian absorption line in which the pulse carrier frequency is equal to the resonance frequency of the absorption line. Unfortunately for such effects to be realized intense electro-magnetic wave radiation is needed in order to compensate the large absorption the signal has to suffer.

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