ON GENERATION OF UNIDIRECTIONAL SINGLE
STEEP WAVES IN TANKS

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Abstract. Very steep waves constitute an essentially nonlinear and complicated
phenomenon. Inter-related experimental and theoretical efforts are thus required to
gain a better understanding of their generation and propagation mechanisms. A
nonlinear focusing process in which a single unidirectional steep wave emerges
from an initially wide amplitude- and frequency-modulated wave group at a
predicted position in the laboratory wave tank is studied both theoretically and
experimentally. The spatial version of the Zakharov equation was applied in the
numerical simulations. Experiments were carried out in the 330 m long Large
Wave Channel in Hanover and in the 18 m long Tel-Aviv University wave tank.
Quantitative comparison between the experimental and the corresponding
numerical results is carried out. Good agreement is obtained between experiments
and computations.

Key-words: Nonlinear water waves, rogue waves, freak waves, Zakharov equation,
spatial evolution, bound (locked) waves

1 Introduction

Generation of very steep waves in wave tanks enables experimental study of the
wave damage potential and is thus of great importance. Excitation of a single steep
wave at a prescribed location in a laboratory wave tank of constant depth is also
often required for model testing in coastal and ocean engineering. It is well known
that such waves can be generated by focusing a large number of waves at a given
location and instant. Dispersive properties of deep or intermediate-depth surface
gravity waves can be utilized for this purpose. Since longer gravity waves propagate
faster, a wave group generated at the wave maker in which wave length increases
from front to tail may be designed to focus the wave energy at a desired location.
Such a wave sequence can be seen as a group that is modulated both in amplitude
and in frequency. One-dimensional theory describing spatial and temporal focusing
of various harmonics of dispersive gravity waves based on the linear Schrödinger
equation was presented by Pelinovsky & Kharif (2000). They suggested such a
focusing as a possible mechanism for generation of extremely steep singular waves.
However, the experiments of Brown & Jensen (2001) demonstrated that nonlinear
effects are essential in the evolution of those waves. An extensive review of field
observations of those waves, as well as of the relevant theoretical, numerical and
experimental studies was recently presented by Kharif and Pelinovsky (2003)

The essentially nonlinear behavior of wave groups with high maximum wave
steepness has been demonstrated in a number of studies. Attempts were made to
describe the propagation of deep or intermediate depth gravity water-wave groups
with a relatively narrow initial spectrum by a cubic Schrödinger equation (CSE).
Shemer et al. (1998) demonstrated that while CSE is adequate for description of the
global properties of the group envelope evolution, it is incapable to capture more
subtle features such as the emerging front-tail asymmetry observed in
experiments. For the weakly-dispersive wave groups in shallow water, application of the Korteweg – deVries equation provided results that were in very good agreement with the experiments (Kit et al. 2000). In the case of stronger dispersion in deeper water, models that are more advanced than the CSE are required, since due to nonlinear interactions, considerable widening of the initially narrow spectrum can occur. The modified Schrödinger equation (Dysthe 1979) is a higher (4th) order extension of the CSE. Application of this model indeed provided good agreement with experiments on narrow-band wave groups (Shemer et al. 2002). An alternative theoretical model that is free of band-width constraints is the Zakharov (1968) equation. Unidirectional spatial version of this equation was derived in Shemer et al. (2001) and applied successfully to describe the evolution of nonlinear wave groups in the tank. Kit & Shemer (2002) showed the relation between the spatial versions of the Dysthe and the Zakharov equations.

An attempt to check the limits of applicability of the Dysthe equation to describe evolution of wave groups with wider spectrum has been carried out by Shemer et al. (2002). Numerical solutions of the wave group evolution problem were carried out using both Dysthe and Zakharov equations. The obtained results demonstrated that while the Dysthe model performed in a satisfactory fashion for not too wide spectra, it failed for wave groups with initially very wide spectra.

The focusing is more effective when the number of free wave harmonics generated at the wavemaker is large. Excitation of single wave with extreme amplitude thus requires wide spectrum of the initial wave group generated at the wavemaker. Extremely steep (freak) wave therefore can be seen as wave groups with very narrow envelope and correspondingly wide spectrum. In the current study we perform an experimental investigation of propagation of steep wave groups with wide spectrum in two wave tanks that differ in size by an order of magnitude, i.e. in the 18 m long Tel-Aviv University (TAU) wave tank, and in the 330 m long Large Wave Channel (GWK) in Hanover, Germany. The experiments are accompanied by numerical simulations based on modification of the spatial version of the Zakharov equation. Some preliminary results of this study were presented in Goulitski et al. (2004) for measurements carried out in the TAU wave tank, and in Shemer et al. (2005) for the experiments performed in the Hanover experimental facility.

2 Theoretical background

The purpose of the present study is to obtain at a prescribed distance from the wavemaker, \( x = x_0 \), a steep unidirectional wave group with a narrow, Gaussian-shaped envelope with the surface elevation variation in time, \( \zeta(t) \), given by

\[
\zeta(t) = \zeta_0 \exp\left(-t/mT_0^2\right)^2 \cos(\omega_0 t)
\]  

(1)

where \( \omega_0 = 2\pi/T_0 \) is the carrier wave frequency, \( \zeta_0 \) is the maximum wave amplitude in the group, and the parameter \( m \) defines the width of the group. The small parameter representing the magnitude of nonlinearity \( \varepsilon \) is the maximum wave steepness \( \varepsilon = \zeta_0 k_0 \). The wave number \( k \) is related to the frequency \( \omega \) by the finite depth dispersion relation

\[
\omega^2 = k g \tanh(kh).
\]  

(2)
g being the acceleration due to gravity. The parameter \( m \) determines the width of the group; higher values of \( m \) correspond to wider groups and narrower spectra. The spectrum of the surface elevation given by (1) is also Gaussian.

The wave field at earlier locations, \( x < x_0 \) is obtained from the computed complex surface elevation frequency spectrum at this location. To this end, the unidirectional discretized spatial version of the Zakharov equation derived by Shemer et al. (2001) can be used:

\[
\sum_{m,l,j} \frac{dB_j(x)}{dx} = \sum_{m,l,j} \alpha_{j,l,m,n} B^*_l B_{m,n} e^{-(i(k_j + k_m - k_n)x)}
\]

where \( \cdot \) denotes complex conjugate and the interaction coefficient \( \alpha_{j,l,m,n} \) is given by

\[
\alpha_{j,l,m,n} = V(k(\omega_j), k(\omega_l), k(\omega_m), k(\omega_n)) / c_{g,j}
\]

In (4), the values of \( V \) represent the quartet interaction coefficient in the temporal Zakharov equation as given by Krasitskii (1994), and \( c_{g,j} \) is the group velocity of the \( j \)-th spectral component. Equations (3) and (4) accurately describe the slow evolution along the tank of each free spectral component \( B_j = B(\omega_j) \) of the surface elevation spectrum in inviscid fluid of constant (infinite or finite) depth, as long as the quartet nonlinear interactions considered occur among components that are relatively close. When the spectrum considered is wide, this limitation can be removed by modifying (4) for the interaction coefficient. The modified expression is

\[
\alpha_{j,l,m,n} = V(\kappa, k(\omega_l), k(\omega_m), k(\omega_n)) \frac{k(\omega_j) - \kappa}{\chi - \omega(\kappa)}
\]

where \( \kappa = k(\omega_m) + k(\omega_n) - k(\omega_l) \), \( \chi = \omega_m + \omega_n - \omega_l \)

The dependent variables \( B(\omega_j, x) \) in (3) are related to the generalized complex ‘amplitudes’ \( b(\omega_j, x) \) composed of the Fourier transforms of the surface elevation \( \zeta(\omega_j, x) \) and of the velocity potential at the free surface \( \hat{\phi}^s(\omega_j, x) \):

\[
b(\omega_j, x) = \left( \frac{g}{2\omega_j} \right)^{\frac{1}{2}} \zeta(\omega_j, x) + i \left( \frac{\omega_j}{2g} \right)^{\frac{1}{2}} \hat{\phi}^s(\omega_j, x)
\]

The “amplitudes” \( b \) consist of a sum of free and the bound waves:

\[
b(\omega_j, x) = [\varepsilon B(\omega_j, x_2) + \varepsilon^2 B'(\omega_j, x, x_2) + \varepsilon^3 B''(\omega_j, x, x_2)] \exp(ikx)
\]

The higher order bound components \( B' \) and \( B'' \) can be computed at each location once the free wave solution \( B(\omega_j, x) \) is known. The phase velocity of these components depends on the parent free waves and can not be determined using (2). The corresponding formulae, as well as the kernels necessary for their computations

\[
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\]
are given in Krasitskii (1994) and in Stiassnie and Shemer (1984, 1987). In (6), the scaled slow coordinate $x_2 = \varepsilon^2 x$. Inversion of (5) allows computing the Fourier components of the surface elevation $\tilde{\zeta}(\omega, x)$. Inverse Fourier transform then yields the temporal variation for the surface elevation $\zeta(x,t)$.

In this paper, the spatial Zakharov equation (3) is used with the modified interaction coefficient (4a). The spectrum corresponding to (1) is integrated from the planned focusing location $x_0$ backwards up to the wavemaker at $x = 0$. The waveforms derived from the computed spectra serve as a basis for computations of the wavemaker driving signals that take into the account the theoretical wavemaker transfer function for a given wavemaker shape (piston in Hanover and paddle in TAU) with corrections that account for the actual wavemaker response.

3 Experimental facilities and procedure

The TAU wave tank is 18m long, 1.2m wide and has the water depth of 0.6m. A paddle-type wavemaker hinged near the floor is located at one end of the tank. The instantaneous surface elevation is measured simultaneously by four resistance-type wave gauges made of blackened platinum for better sensitivity. The probes are mounted on a bar parallel to the side walls of the tank and fixed to a carriage which can be moved along the tank. Focusing location in different experiments varied from 5 m to 10 m from the wavemaker. The Hanover tank has a length of 330 m, width of 5 m and depth of 7 m. Water depth in the present experiments was set to be 5 m. At the end of the wave tank there is a sand beach starting at the distance of 270 m with slope of 30°. The piston-type wavemaker is equipped with the reflected wave energy absorption system. The focusing location in all Hanover experiments was set at 120 m from the wavemaker. The instantaneous water height is measured using 25 wave gauges of resistance type, which are placed along the tank wall; higher concentration of the wave gauges is in the region of expected focusing of the wave group.

The Gaussian energy spectrum of (1) has a shape with the relative width at the energy level of $1/2$ of the spectrum maximum that depends on the value of the parameter $m$ in (1) and is given by

$$\Delta \omega / \omega_0 = \frac{1}{m\pi} \sqrt{\frac{1}{2} \ln 2}$$

(7)

The value of the group width parameter in all experiments was selected to be $m=0.6$, so that (7) yields the relative spectrum width $\Delta \omega / \omega_0 = 0.312$, which is beyond the domain of applicability of the narrow spectrum assumption of the cubic Schrödinger and Dysthe models.

In the Hanover experiments, the carrier wave period adopted in (1) is $T_0 = 2.8$ s, corresponding to the wavenumber $k = 0.52$ m$^{-1}$, so that $kHz = 2.59$ and thus deep-water dispersion relation is only approximately satisfied. Therefore, in all expressions for the interaction coefficients finite depth versions were used. The focusing location in Hanover experiments is located at the distance of about 10 carrier wave lengths from the wavemaker.
TAU experiments were carried out with two carried wave periods, \( T_0 = 0.85 \) s, \( k_0 = 0.056 \text{ cm}^{-1}; \) \( k_0h = 3.35, \) corresponding to the intermediate depth conditions, and \( T_0 = 0.60 \) s, \( k_0 = 0.112 \text{ cm}^{-1}; k_0h = 6.71, \) with deep water conditions satisfied even for the low harmonics in the spectrum. The driving amplitudes in the cases considered here are selected so that at the focusing location, the resulting carrier wave has the maximum wave amplitudes \( \zeta_0 \) corresponding to the steepness \( \varepsilon = k_0\zeta_0 = 0.3. \)

For each set of the carrier wave period \( T_0, \) and the focusing location \( x_0, \) the solution of the system of \( N \) ODEs (3), \( N \) being the total number of wave harmonics considered, was obtained for distances from the wavemaker up to \( x_0 \) and beyond. The number of free wave harmonics considered is \( N = 120. \) The wavemaker-driving signal was adjusted to get as good as possible agreement between the calculated and the measured wave field at a location close to the wavemaker, but beyond the range of existence of evanescent modes (see, e.g. Dean and Dalrymple 1991).

4 Results

A representative selection of the accumulated in this study results is discussed in this Section. Results obtained in Hanover are shown first. The computed and the measured temporal variations of the surface elevation at different locations along the tank are presented in Figs. 1a and 1b, respectively. The selected value of \( m =0.6 \) in (1) yields a narrow wave group with a single steep wave at the focusing location. Closer to the wavemaker the group becomes notably wider, and the maximum wave amplitudes decrease accordingly. Modulation of the amplitude and the frequency within the group is clearly seen. The experimental results presented in Fig. 1b demonstrate good agreement with the computations, although the wave shape measured at the focusing location is not exactly symmetric.

![Figure 1](image-url)  
Figure 1. Calculated and measured in the Hanover wave tank surface elevation within the group at different distances \( x \) from the wavemaker (\( T_0=2.8s, \varepsilon=0.3 \)).

The computed and the measured spectra for the experimental parameters of Fig. 1 are presented in Fig. 2 at various locations along the tank. The variation of the spectral shape along the tank is evident and indicates that wave evolution is essentially nonlinear even at this relatively low amplitude of forcing. The agreement between experiments and computations is quite satisfactory and both the numerical simulations and the measurements exhibit similar features. The spectral shapes shown in Fig. 2b indicate that the spectrum becomes wider with the distance from the wavemaker and at the prescribed distance (\( x_0 = 120 \) m) approaches the Gaussian shape assumed in the numerical simulations. The peak
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frequency at \( x = 50 \) m is shifted to the right relative to the carrier frequency \( f_0 = 1/T_0 \). Note that the peak values within the group appear to be somewhat different in those figures. The low frequency part of the spectrum remains unaffected during the evolution process. It should be stressed that the computed surface elevation is obtained here by taking into account free modes only, while in the experiments the effect of the bound waves can be significant.

![Figure 2](image)

Figure 2. Frequency spectra of the surface elevation variation with time along the Hanover tank for \( x_0 = 120 \) m and \( \zeta_0 k_0 = 0.3 \): a) computed; b) measured

Careful analysis of the extensive data sets accumulated in Hanover and TAU experiments clearly indicate that in addition to accounting for the contribution of the bound waves to the generalized “amplitudes” \( b \), see (6), the effect of viscous dissipation has to be considered. Since the dissipation in the boundary layers at the tank walls and bottom is relatively weak, it is sufficient to account for the wave energy loss along the tank by adding a linear term, \(-i\gamma j B_\ast\), to the r.h.s. of (3). The dissipation coefficient \( \gamma \) is calculated following Kit and Shemer (1989).

The substantially smaller dimensions of the TAU tank as compared to the Hanover facility make it possible to perform numerous experiments and to attain a better agreement of the computed and the actually obtained waveforms near the wavemaker. The detailed comparison of the theoretical predictions and experiments carried out in sequel is based therefore on the TAU-derived results.

![Figure 3](image)

Figure 3. Variation of the total energy of free spectral modes along the TAU tank. \( T_0 = 0.6 \) s, \( \zeta_0 k_0 = 0.3 \) and \( x_0 = 6 \) m.
Experiments in the TAU tank for the carrier wave period of 0.6 s (carrier wave length $\lambda_0 = 0.56$ cm) were designed for the focusing distance from the wavemaker $x_0 = 6$ m, i.e. about 10 carrier wave lengths, similar to conditions in Hanover. The results of Fig. 3 clearly show that the non-linear contribution to the total wave field energy is essential mainly in the vicinity of the focusing location. As a result of dissipation along the tank, the amplitude of the waves generated by the wavemaker should be somewhat higher than that computed for a purely Hamiltonian case.

Further experiments in TAU were carried out for a longer carrier wave with the period $T_0 = 0.85$ s and length $\lambda_0 = 1.12$ m. In this case, focusing occurred at $x_0 = 9$ m, about 8 carrier wave lengths from the wavemaker. Notable decay of wave energy along the tank is visible. Away from focusing, the sum of squared amplitudes of all free waves adequately represents the total wave field energy, and excellent agreement between measurements and computations indicates that dissipation is properly accounted for. Around the focusing locations contribution of energy contained in bound waves is essential.

The effect of bound waves on both surface elevation and frequency spectrum is further investigated in Fig. 5. Since the effect of bound waves is mostly visible for very steep waves, those waves are computed here at the focusing location. In Fig. 5a, the experimentally measured temporal variation of the surface elevation is compared with computations performed both with and without contribution of the 2nd order bound waves, denoted by $B'$ in (6). As expected, bound waves contribute to steeper crest and flatter trough of the wave, and result in a better agreement with the measured wave shape.

Comparison of the corresponding amplitude spectra in Fig. 5b demonstrates that when the contribution of bound waves is accounted for, the agreement of theoretical predictions with the experiments is improved drastically, in particular in the high frequency region. In this frequency domain, bound waves can be seen as the 2nd harmonic of the dominant free waves. Certain improvement of the agreement between experiment and computations is also obtained for lower frequencies. The remaining discrepancies between experiments and computations can be attributed to difficulties in exact reproduction of the computed wave forms by the wavemaker.
Figure 5. The effect of 2nd order bound waves, wave parameters as in Fig. 3.
a) Computed and measured surface elevation at the focusing location;
b) the corresponding amplitude spectra.

Figure 6. Maximum wave height variation along the tank, wave parameters as in
Fig. 4.
Wave height $H$ is defined as the difference between the consecutive minimum and maximum surface elevation. The evolution of the maximum wave height, $H_{\text{max}}$, within the group along the tank for $\omega_0 k_0 = 0.3$ is shown in Fig. 6. Very good agreement is observed, and the measured and computed rates of increase of the maximum wave height during the focusing process and the following decrease in the maximum wave height during defocusing for $x > x_0$ are practically identical.

5 Conclusions

The ability to excite focused steep waves at any desired location along the tank is demonstrated in two very different experimental facilities. Large number of wave harmonics is required to generate very steep wave at the focusing location. It is shown that the focusing process is accompanied by a notable change of the spectral shape and is thus essentially nonlinear. The modified unidirectional spatial discrete version of the Zakharov equation as given by (3) and (4a) is adequate to describe nonlinear evolution of steep wave groups with wide spectrum propagating in water of constant intermediate depth. To achieve not only qualitative but also quantitative agreement between the model predictions and the experiments, it is insufficient, however, to consider the nonlinear evolution process of the free wave components only. At least two additional effects have to be accounted for. First, dissipation along the tank is essential and can be adequately described by an additional linear term in (3) that represents the decay of amplitude of each spectral mode as a result of viscous boundary layers at the bottom and side walls of the tank. Secondly, effects related to the bound waves can not be neglected. These effects strongly depend of the wave steepness and become important mainly in the vicinity of focusing. Second-order bound are accounted for in the present study, and the appropriate corrections are introduced. The effect of the 3rd order bound waves will be investigated in future. With both dissipation and 2nd order bound waves accounted for, very good agreement between experiments and numerical simulations is achieved.

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References


