STEEP TRANSIENT WAVES IN TANKS – EXPERIMENTS AND SIMULATIONS

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ABSTRACT

Very steep waves constitute an essentially nonlinear and complicated phenomenon. Inter-related experimental and theoretical efforts are thus required to gain a better understanding of their generation and propagation mechanisms. A nonlinear focusing process in which a single unidirectional steep wave emerges from an initially wide amplitude- and frequency-modulated wave group at a predicted position in the laboratory wave tank is studied both theoretically and experimentally. The spatial version of the Zakharov equation was applied in the numerical simulations. Experiments were carried out in the 330 m long Large Wave Channel (GWK) in Hannover, Germany and in the 18 m long Tel-Aviv University (TAU) wave tank. Quantitative comparison between the experimental and the corresponding numerical results is carried out. Good agreement is obtained between experiments and computations.

INTRODUCTION

Generation of very steep waves in wave tanks enables experimental study of the wave damage potential and is thus of great importance. Excitation of a single steep wave at a prescribed location in a laboratory wave tank of constant depth is also often required for model testing in coastal and ocean engineering. It is well known that such waves can be generated by focusing a large number of waves at a given location and instant. Dispersive properties of deep or intermediate-depth surface gravity waves can be utilized for this purpose. Since longer gravity waves propagate faster, a wave group generated at the wave maker in which wave length increases from front to tail may be designed to focus the wave energy at a desired location. Such a wave sequence can be seen as a group that is modulated both in amplitude and in frequency. One-dimensional theory describing spatial and temporal focusing of various harmonics of dispersive gravity waves based on the linear Schrödinger equation was presented by Pelinovsky & Kharif (2000). They suggested such a focusing as a possible mechanism for generation of extremely steep singular waves. However, the experiments of Brown & Jensen (2001) demonstrated that nonlinear effects are essential in the evolution of those waves. An extensive review of field observations of those waves, as well as of the relevant theoretical, numerical and experimental studies was recently presented by Kharif and Pelinovsky (2003).

The essentially nonlinear behavior of wave groups with high maximum wave steepness has been demonstrated in a number of studies. Attempts were made to describe the propagation of deep or intermediate depth gravity water-wave groups with a relatively narrow initial spectrum by a cubic Schrödinger equation (CSE). Shemer et al. (1998) demonstrated that while CSE is adequate for description of the global properties of the group envelope evolution, it is incapable to capture more subtle features such as the emerging front-tail asymmetry observed in experiments. For the weakly-dispersive wave groups in shallow water, application of the Korteweg – deVries equation provided results that were in a very good agreement with the experiments (Kit et al. 2000). In the case of stronger dispersion in deeper water, models that are more advanced than the CSE are required, since due to nonlinear interactions, considerable widening of the initially narrow spectrum can occur. The modified Schrödinger equation
(Dysthe 1979) is a higher (4th) order extension of the CSE. Application of this model indeed provided good agreement with experiments on narrow-band wave groups (Shemer et al. 2002). An alternative theoretical model that is free of band-width constraints is the Zakharov (1968) equation. Unidirectional spatial version of this equation was derived in Shemer et al. (2001) and applied successfully to describe the evolution of nonlinear wave groups in the tank. Kit & Shemer (2002) showed the relation between the spatial versions of the Dysthe and the Zakharov equations.

An attempt to check the limits of applicability of the Dysthe equation to describe evolution of wave groups with wider spectrum has been carried out by Shemer et al. (2002). Numerical solutions of the wave group evolution problem were carried out using both Dysthe and Zakharov equations. The obtained results demonstrated that while the Dysthe model performed in a satisfactory fashion for not too wide spectra, it failed for wave groups with initially very wide spectra.

The focusing is more effective when the number of free wave harmonics generated at the wavemaker is large. Excitation of a single wave with extreme amplitude thus requires wide spectrum of the initial wave group that is generated at the wavemaker. Extremely steep (freak) wave therefore can be seen as wave groups with a very narrow envelope and correspondingly wide spectrum. In the current study we perform an experimental investigation of propagation of unidirectional steep wave groups with wide spectrum in two wave tanks that differ in size by an order of magnitude, i.e. in the 18 m long Tel-Aviv University (TAU) wave tank, and in the 330 m long Large Wave Channel (GWK) in Hannover, Germany. The experiments are accompanied by numerical simulations based on modification of the spatial version of the Zakharov equation derived by Shemer et al. (2001) is used:

\[ \frac{dB_j}{dx} = \sum_{\omega_j+\omega_0=\omega_m+\omega_n} \alpha_{j,m,n} B_j^* B_m B_n e^{-i(k_j+k_m-k_n-k_0)x} \]

where \( \alpha_{j,m,n} \) denotes complex conjugate and the interaction coefficient \( \alpha_{j,m,n} \) is given by

\[ \alpha_{j,m,n} = V(k(\omega_j),k(\omega_m),k(\omega_n),c_{g,j}) \]

Each one of equations (3), written for the \( j \)-th free spectral component \( B_j = B(\omega_j) \) of the surface elevation spectrum, describes its slow evolution along the tank due to the quartet (Class I) nonlinear interactions with all spectral harmonics. In (4), the values of \( V \) represent the interaction coefficient in the temporal Zakharov equation as given by Krasitskii (1994), and \( c_{g,j} \) is the group velocity of the \( j \)-th spectral component. Equations (3) and (4) accurately describe the spatial evolution of the wave field in inviscid fluid of constant (infinite or finite) depth. The expression (4), however, is valid only as long as the quartet nonlinear interactions considered occur among components that are

\[ \omega_0 = 2\pi T_0 \]
\[ \zeta_0 \]
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relatively close. When the spectrum considered is wide, this limitation can be removed by modifying (4) for the interaction coefficient. The modified expression is

$$\alpha_{ij,mn} = \frac{V(k, \omega_i) k(\omega_j, k(\omega_h))}{2a(k)} \left[ k(\omega_j) - \kappa \right] \left[ \omega - \alpha(k) \right]$$

(4a)

where \(\kappa = k(\omega_m) + k(\omega_h) - k(\omega_i)\), \(\chi = \omega_m + \omega_h - \omega_i\).

The dependent variables \(B(\omega_j, x)\) in (3) are related to the complex ‘amplitudes’ \(b(\omega_j, x)\) composed of the temporal Fourier transforms of the surface elevation \(\zeta(\omega_j, x)\) and of the velocity potential at the free surface \(\phi_x(\omega_j, x)\):

$$b(\omega_j, x) = \left( \frac{g}{2\omega_j} \right)^{1/2} \zeta(\omega_j, x) + \left( \frac{\omega_j}{2g} \right)^{1/2} \phi_x(\omega_j, x)$$

(5)

The “amplitudes” \(b\) consist of free and the bound waves:

$$b(\omega_j, x) = [\varepsilon B(\omega_j, x) + \varepsilon^2 B'(\omega_j, x) + \varepsilon^3 B''(\omega_j, x)] \exp(ikx)$$

(6)

The second order \(B'\) and third order \(B''\) bound components can be computed at each location once the free wave solution \(B(x)\) is known. The phase velocity of these components depends on the parent free waves and can not be determined using (2). The corresponding formulae, as well as the kernels necessary for their computations are given in Krasitskii (1994) and in Stiassnie and Shemer (1984, 1987). Inversion of (5) allows computing the Fourier components of the surface elevation \(\zeta(\omega, x)\). Inverse Fourier transform then yields the temporal variation for the surface elevation \(\zeta(x, t)\).

In this paper, the spatial Zakharov equation (3) is used with the modified interaction coefficient (4a). The spectrum corresponding to (1) is integrated from the designed focusing location \(x_0\) backwards up to the wavemaker at \(x = 0\). The waveforms derived from the computed spectra serve as a basis for computing the wavemaker driving signals. These computations take into the account the theoretical wavemaker transfer function for a given wavemaker shape (piston in Hanover and paddle in TAU) with corrections that account for the actual wavemaker response.

**EXPERIMENTAL FACILITIES AND PROCEDURE**

The TAU wave tank is 18 m long, 1.2 m wide and has the water depth of 0.6 m. A paddle-type wavemaker hinged near the floor is located at one end of the tank. The instantaneous surface elevation is measured simultaneously by four resistance-type wave gauges made of blackened platinum wire 0.4 mm in diameter for better sensitivity. The probes are mounted on a bar parallel to the side walls of the tank and fixed to a carriage which can be moved along the tank. Focusing location in different experiments varied from 5 m to 10 m from the wavemaker.

The Hanover tank has a length of 330 m, width of 5 m and maximum depth of 7 m. Water depth in the present experiments was set to be 5 m. At the end of the wave tank there is a sand beach starting at the distance of 270 m with slope of 30°. The piston-type wavemaker is equipped with the reflected wave energy absorption system. The focusing location in all Hanover experiments was set at 120 m from the wavemaker. The instantaneous water height is measured using 25 wave gauges of resistance type, which are placed along the tank wall; higher concentration of the wave gauges is in the region of expected focusing of the wave group.

The Gaussian energy spectrum of (1) has a shape with the relative width at the energy level of \(1/2\) of the spectrum maximum that depends on the value of the parameter \(m\) in (1) and is given by

$$\Delta \omega / \omega_0 = 1 / m \pi \sqrt{1 / 2 \ln 2}$$

(7)

The value of the group width parameter in all experiments was selected to be \(m = 0.6\), so that (7) yields the relative spectrum width \(\Delta \omega / \omega_0 = 0.312\), which is beyond the domain of applicability of the narrow spectrum assumption of the cubic Schrödinger and Dysthe models, as demonstrated by Shemer et al. (2002).

In the Hanover experiments, the carrier wave period adopted in (1) is \(T_0 = 2.8\ s\), corresponding to the wave number \(k_0 = 0.52\ \text{m}^{-1}\) (carrier wave length \(\lambda_0 = 12.1\ m\)), so that \(k_0 h = 2.59\) and thus deep-water dispersion relation is only approximately satisfied. Therefore, in all expressions for the interaction coefficients finite depth versions were used. The focusing location in Hanover experiments is located at the distance of about 10 carrier wave lengths from the wavemaker.

TAU experiments were carried out with three carrier wave periods, \(T_0 = 0.85\ s\), \(k_0 = 5.59\ \text{m}^{-1}\) (\(\lambda_0 = 1.125\ m\)); \(k_0 h = 3.35\), corresponding to the intermediate depth conditions, \(T_0\)
\[ =0.6 \text{ s}, \quad k_0=11.2 \text{ m}^{-1} (\lambda_0=0.562 \text{ m}); \quad k_0=8.21 \text{ m}^{-1} (\lambda_0=0.765 \text{ m}); \quad k_0=4.93, \text{ the last two corresponding to deep water conditions satisfied for most harmonics in the spectrum. The driving amplitudes considered here are selected so that in most cases at the focusing location, the resulting carrier wave has the maximum amplitudes } \zeta_0 \text{ corresponding to the steepness } \varepsilon=k_0 \zeta_0 = 0.3. \]

For each set of the carrier wave period \( T_0 \), and the focusing location \( x_0 \), the solution of the system of \( N \) ordinary differential equations (3), \( N \) being the total number of wave harmonics considered, was obtained for all locations along the tank. The number of free wave harmonics considered is \( N = 120 \). The wavemaker-driving signal was adjusted to get as good as possible agreement between the calculated and the measured wave field at a location close to the wavemaker, but beyond the range of existence of evanescent modes (see, e.g. Dean and Dalrymple 1991).

**RESULTS**

![Figure 1. Calculated (a) and measured (b) in the GWK surface elevation within the group at different distances \( x \) from the wavemaker \( (T_0=2.8\text{ s}, \varepsilon=0.3) \).](image)

![Figure 2. Frequency spectra of the surface elevation variation with time along the GWK for \( x_0=120 \text{ m} \) and \( \varepsilon=0.3 \): a) computed; b) measured](image)

A representative selection of the findings accumulated in this study is discussed in this Section. Results obtained in Hanover are shown first. The computed and the measured temporal variations of the surface elevation at different locations along the tank are presented in Figs. 1a and 1b, respectively. The selected value of \( m = 0.6 \) in (1) yields a narrow wave group with a single steep wave at the focusing location. Closer to the wavemaker the group becomes notably wider, and the maximum wave amplitudes decrease accordingly. Modulation of the amplitude and the frequency within the group is clearly seen. The experimental results presented in Fig. 1b demonstrate good agreement with the computations, although the wave shape measured at the focusing location is not exactly symmetric.

The computed and the measured spectra for the experimental parameters of Fig. 1 are presented in Fig. 2 at various locations along the tank. The variation of the...
spectral shape along the tank is evident and indicates that wave evolution is essentially nonlinear even at this relatively low amplitude of forcing. The agreement between experiments and computations is quite satisfactory and both the numerical simulations and the measurements exhibit similar features. The spectral shapes shown in Fig. 2b indicate that the spectrum becomes wider with the distance from the wavemaker and at the prescribed distance \( x_0 = 120 \) m approaches the Gaussian shape assumed in the numerical simulations. The peak frequency at \( x = 50 \) m is shifted to the right relative to the carrier frequency \( f_0 = 1/T_0 \). Note that the peak values within the group appear to be somewhat different in those figures. The low frequency part of the spectrum remains unaffected during the evolution process. It should be stressed that the computed surface elevation is obtained here by taking into account free modes only, while in the experiments the effect of the bound waves can be significant.

Careful analysis of the extensive data sets accumulated in Hanover and TAU experiments clearly indicate that in addition to accounting for the contribution of the bound waves to the generalized “amplitudes” \( b \), see (6), the effect of viscous dissipation has to be considered. Since the dissipation in the boundary layers at the tank walls and bottom is relatively weak, it is sufficient to account for the wave energy loss along the tank by adding to the r.h.s. of each one of the equations (3) a linear term \(-i\gamma_j B_j\). The dissipation coefficient for each harmonics, \( \gamma_j \), is calculated following Kit and Shemer (1989).

The substantially smaller dimensions of the TAU tank as compared to the Hanover facility make it possible to perform numerous experiments and to attain a better agreement of the computed and the actually obtained waveforms near the wavemaker. The detailed comparison of the theoretical predictions and experiments carried out in sequel is based therefore on the TAU-derived results.

Wave group evolution along the TAU tank is illustrated in Figs. 3 and 4 for the carrier wave period \( T_0 = 0.7 \) s and the maximum wave steepness at the focusing location, \( x_0 = 9 \) m, of \( \varepsilon = 0.3 \). The variation of the wave group shape is demonstrated in Fig. 3. Both experimentally measured and computed variations of the surface elevation within the group are given. Computations are performed taking into account viscous dissipation along the tank. Note that in the computational results at all locations, contributions of the 2nd order bound waves are also included. Those contributions are calculated from the free wave field that is computed at each location using (3). The effect of bound waves is manifested in Fig. 3 in sharper crests and flatter troughs, this effect being more pronounced at the focusing location.
The group shapes at equal distances before and after focusing in Fig. 3 represent a nearly mirror image, the minor differences resulting from dissipation. The contribution of bound waves is clearly seen in the spectra of Fig. 4, where the computed spectra are presented with and without the contribution due to bound waves. Note that at the focusing location, bound waves modify considerably the spectral shape not only at very low and very high frequencies, but in the range of free wave frequencies as well. The spectra at equal distances from the focusing in Fig. 4 are nearly identical.

In absence of dissipation, the energy flux along the tank should be conserved. In the linear approximation, the total energy flux $P(x)$ at any location along the tank can be computed from the known free wave amplitudes $a_j = a(\omega_j, x)$ as

$$P(x) = \sum_{j=1}^{N} c_{g,j} a_j^2(x) \quad (8)$$

To assess the effect of dissipation, the variation of the linear part of the energy flux was computed using (8) from the numerical simulations carried out with and without the dissipations term. In experiments, contribution of free waves only was computed from the whole measured wave field using an iterative procedure. This allowed estimating the energy flux from the experimental results also separately for free waves only. The computed and measured results representing two sets of experimental conditions the TAU tank are given in Figs. 5 and 6.
The results of Fig. 5 for the carrier wave period of 0.6 s (carrier wave length $\lambda_0 = 0.56$ cm) and the designed focusing distance from the wavemaker $x_0 = 6$ m, i.e. about 10 carrier wave lengths, similar to conditions in Hanover, clearly show that the non-linear contribution to the total wave field energy is essential mainly in the vicinity of the focusing location. The effect of dissipation is also visible, and the computations with dissipation accounted for are in good agreement with the experimental results.

![Graph](image1)

*Figure 6. As in Fig. 5, for $T_0 = 0.85$ s, $\zeta_0k_0 = 0.3$ and $x_0 = 9$ m.*

Results for a longer carrier wave with the period $T_0=0.85$ s and length $\lambda_0=1.12$ m are presented in Fig. 6. In this case, focusing occurred at $x_0=9$ m, about 8 carrier wave lengths from the wavemaker. Notable decay of wave energy along the tank is visible. Away from focusing, linear contribution to the total energy flux $P$ calculated for free waves only adequately represents the total wave field energy flux, and excellent agreement between measurements and computations indicates that dissipation is properly accounted for in this case as well. Around the focusing locations contribution of bound waves to the total energy flux is essential.

The effect of bound waves on both surface elevation and frequency spectrum is investigated in more detail in Fig. 7. Since the effect of bound waves is mostly visible for very steep waves, those waves are computed here at the focusing location. In Fig. 7a, the experimentally measured temporal variation of the surface elevation is compared with computations performed both with and without contribution of the 2nd order bound waves, denoted by $B^e$ in (6). As expected, bound waves contribute to steeper crest and flatter trough of the wave, and result in a better agreement with the measured wave shape.

![Graph](image2)

*Figure 7. The effect of 2nd order bound waves, $T_0 = 0.6$ s $\zeta_0k_0 = 0.3$.
  a) Computed and measured surface elevation at the focusing location;
  b) the corresponding amplitude spectra.*

Comparison of the corresponding amplitude spectra in Fig. 7b demonstrates that when the contribution of bound waves is accounted for, the agreement of theoretical predictions with the experiments is improved drastically, in particular in the high frequency region. In this frequency domain, bound waves can be seen as the 2nd harmonic of the dominant free waves. Certain improvement of the agreement between experiment and computations is also obtained for lower frequencies. The remaining discrepancies between experiments and computations can be attributed to difficulties in exact reproduction of the computed wave forms by the wavemaker.
Wave height $H$ is defined as the difference between the consecutive minimum and maximum surface elevation. The evolution of the maximum wave height, $H_{\text{max}}$, within the group along the tank for $T_0 = 0.85$ s, $\omega \kappa_0 = 0.3$ and the focusing distance $x_0 = 9$ m is shown in Fig. 8. Very good agreement is observed, and the measured and computed rates of increase of the maximum wave height during the focusing process and the following decrease in the maximum wave height during defocusing for $x > x_0$ are practically identical.

CONCLUSIONS

The ability to excite focused steep waves at any desired location along the tank is demonstrated in two very different experimental facilities. Large number of wave harmonics is required to generate very steep wave at the focusing location. It is shown that the focusing process is accompanied by a notable change of the spectral shape and is thus essentially nonlinear. The modified unidirectional spatial discrete version of the Zakharov equation as given by (3) and (4a) is adequate to describe nonlinear evolution of steep wave groups with wide spectrum propagating in water of constant intermediate depth. To achieve not only qualitative but also quantitative agreement between the model predictions and the experiments, it is insufficient, however, to consider the nonlinear evolution process of the free wave components only. At least two additional effects have to be accounted for. First, dissipation along the tank is essential and can be adequately described by an additional linear term in (3) that represents the decay of amplitude of each spectral mode as a result of viscous boundary layers at the bottom and side walls of the tank. Secondly, effects related to the bound waves can not be neglected. These effects strongly depend of the wave steepness and become important mainly in the vicinity of focusing. Second-order bound waves are accounted for in the present study, and the appropriate corrections are introduced. The effect of the 3rd order bound waves will be investigated in future. With both dissipation and 2nd order bound waves accounted for, very good agreement between experiments and numerical simulations is achieved.

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REFERENCES


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