An experimental and numerical study of the spatial evolution of unidirectional nonlinear water-wave groups

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Spatial evolution of nonlinear narrow-spectrum deep-water wave groups is studied experimentally in a wave tank. The experimental results are compared with the computations based on the unidirectional Zakharov equation and the Dysthe model. The very good agreement between the computational results based on both models with the experiments prompted an attempt to perform simulations for a wider initial spectral width, that formally violate the assumptions adopted in the derivation of the Dysthe model. The accuracy of the results based on the Dysthe model is checked against the solutions of the Zakharov equation, which is free of restrictions on the spectral width. Conclusions regarding the domain of validity of the Dysthe model are drawn.

1. INTRODUCTION

Frequency spectra of ocean waves are often quite narrow, and the waves therefore exhibit notable groupiness. As the wave groups propagate towards the shore in deep and intermediate-depth water, they are transformed by combined action of numerous factors, including dissipation and energy input due to wind. Of particular interest, however, is wave group envelope transformation as a result of the action of energy-conserving factors, like nonlinear interactions and dispersion.

The cubic Schrödinger equation (CSE)\(^1\)\(^-\)\(^2\) was the first model describing evolution of nonlinear gravity wave groups. This equation in its various versions (see, e.g., Mei\(^3\)) describes the slow temporal or spatial variation of group envelopes and is valid to the third order in the nonlinearity parameter \(\varepsilon = k_0 a_0\). This parameter represents the maximum wave steepness in the group, \(a_0\) being the maximum wave amplitude and \(k_0\) the carrier wave number. The equation is derived assuming that the spectrum of the wave field is sufficiently narrow, \(O(\varepsilon)\). The cubic Schrödinger equation was applied by Shemer \textit{et al.}\(^4\) to simulate evolution of wave groups generated in a laboratory tank. Comparison of the numerical results based on this model with the measurements demonstrated that certain features of the evolution of the group envelope along the tank could be predicted with a reasonable accuracy by the CSE. However, this equation is incapable to reproduce the salient feature of the group envelope shape observed in the experiments with waves of moderately high maximum amplitude, namely the asymmetry between the front and the tail of the envelope shape.

Zakharov\(^1\) obtained the CSE from a more general equation of the third order in the wave amplitude that takes into account four-wave (quartet) near-resonant nonlinear interactions. A somewhat different formulation of the Zakharov equation was presented by Krasitskii.\(^5\) The Zakharov equation describes the temporal evolution of a wave field in Fourier space and has no restrictions on spectral bandwidth. The equation was successfully applied to study the Benjamin–Feir instability of gravity waves.\(^6\)\(^-\)\(^7\) Results on the instability domains of gravity-capillary waves obtained using an appropriate modification of the Zakharov equation were recently supported by experiments in a laboratory wave tank.\(^8\) It should be stressed that application of the Zakharov equation requires a proper discretization. Such a discretization reduces an integro-differential equation to a set of coupled nonlinear ordinary differential equations. The problems related to this discretization procedure were discussed in Rasmussen and Stiassnie.\(^9\)

In order to perform quantitative comparison of the predictions based on the Zakharov equation with the experiments in a wave tank, a modification of the governing equation is required to describe the spatial (as opposed to temporal) evolution of the wave field. This task has been carried out in Shemer \textit{et al.}\(^10\) where the newly derived set of discrete equations was solved numerically. The numerical simulations compared favorably with the experimental results. In particular, the observed in the experiments increasingly asymmetric as they evolve along the tank shapes of group envelopes, as well as the corresponding asymmetry in the spectral shapes, were faithfully reproduced in the numerical simulations. However, in order to attain these results, in some occasions it was required to include in the wave frequency spectrum considered quite a large number of free modes, \(N\) (spectra including up to \(N = 39\) modes were taken into account in this paper). The number of equations is equal to the total number of free modes, while the number of nonlinear terms in the resulting set of ODEs is growing roughly proportionally to \(N^3\) (Jiao\(^11\)).

It can thus be concluded that the relatively simple CSE model does not provide a satisfactory description of the wave group evolution in an experimental wave tank, while the ap-
plication of the Zakharov model requires considerable computer resources. An intermediate path was suggested by Dysthe.\textsuperscript{12} By taking the perturbation expansion to the next order, Dysthe extended the envelope equation to the fourth order in $\varepsilon$, while retaining the spectral width constraint as in the cubic Schrödinger equation, i.e., $O(\varepsilon)$. This modified Schrödinger (Dysthe) model is derived for deep-water conditions and actually consists of two coupled partial differential equations, describing the evolution of the wave envelope and of the velocity potential of the induced current. Stiassnie\textsuperscript{13} demonstrated that the Dysthe model could be derived from the Zakharov equation assuming narrow frequency spectrum approximation and retaining terms of the fourth order in $\varepsilon$. By a proper selection of the dimensionless scaled variables, Lo and Mei\textsuperscript{14} transformed the Dysthe model to describe the spatial evolution of the group envelope and demonstrated that the solutions indeed exhibit the front-tail asymmetry. Kit and Shemer\textsuperscript{15} applied the approach of Stiassnie\textsuperscript{13} to derive spatial versions of the Dysthe evolution model separately for the surface elevation amplitude and the velocity potential amplitude from the generalized spatial version of the Zakharov equation.

Trulsen and Dysthe\textsuperscript{16} developed a model in which the spectrum bandwidth restriction is relaxed to $O(\varepsilon^{1/2})$ while retaining the same accuracy in nonlinearity. They demonstrated that the application of their model for determination of the instability domains of steep Stokes waves yields a better agreement with the full nonlinear solution by McLean.\textsuperscript{17}

The spatial formulation of the Dysthe model by Kit and Shemer\textsuperscript{15} is applied here to study the evolution of nonlinear wave groups in a laboratory tank. The results of these simulations are compared with the experiments, as well as with the computations based on the CSE and the Zakharov models. Furthermore, the limits of applicability of the unidirectional Dysthe model are investigated by expanding the width of the spectral domain and comparing the computational results with the Zakharov model predictions.

II. GOVERNING EQUATIONS

Surface elevation for a modulated two-dimensional Stokes wave can be presented at the leading order as

$$\zeta = \Re(a(x,t)e^{i(k_0x-\omega_0 t)})$$

where $a(x,t)$ is the slowly varying in time and space complex amplitude of the carrier wave with the frequency $\omega_0$ and the wave number $k_0=\omega_0^2/g$. Dimensionless scaled variables are introduced as

$$a=a_0 A, \quad \phi=\omega_0 a_0^2 \Phi,$$

$$\varepsilon \omega_0 \left(\frac{2k_0}{\omega_0} x - t\right) = \tau, \quad \varepsilon^2 k_0 x = X, \quad \varepsilon k_0 z = Z.$$

Kit and Shemer\textsuperscript{15} derived two versions of the normalized coupled systems of the Dysthe equations to describe the spatial evolution of the group envelope. One version was obtained for the amplitude of the velocity potential at the free surface $A(x,t)$, while a slightly different equation was derived for the amplitude of the surface elevation $A(x,t)$. Note that the system of equations for $A(x,t)$ obtained by Kit and Shemer is identical to that used by Lo and Mei.\textsuperscript{14} To carry out comparison with the measured variation of the surface elevation, the system of equation for $A(x,t)$ and for the potential of the induced mean current $\phi$ is solved here. This system (with the subscript omitted) has the following form:

$$\frac{\partial A}{\partial X} + i A \frac{\partial^2 A}{\partial \tau^2} + i |A|^2 A + 8 \varepsilon |A|^2 \frac{\partial A}{\partial \tau} + 2 \varepsilon |A|^2 A^* \frac{\partial A^*}{\partial \tau} + 4 i \varepsilon A \frac{\partial \Phi}{\partial \tau} \bigg|_{Z=0} = 0,$$

$$4 \frac{\partial^2 \Phi}{\partial \tau^2} + \frac{\partial^2 \Phi}{\partial Z^2} = 0, \quad -\varepsilon k_0 h < Z < 0,$$

with $\Phi$ satisfying the following boundary conditions:

$$\frac{\partial \Phi}{\partial Z} \bigg|_{Z=0} = \frac{\partial |A|^2}{\partial \tau}, \quad \frac{\partial \Phi}{\partial Z} \bigg|_{Z=-\varepsilon k_0 h} = 0. \quad (5)$$

Equations (3)–(5) are valid for deep water, i.e., $kh = O(k_0 a_0)^{-1} \gg 1$.

The term I on the left-hand side of (3) represents the slow spatial evolution of the envelope $A(X,\tau)$, and the terms I–III comprise the cubic Schrödinger equation for deep water. All other terms are of the fourth order in $\varepsilon$. The IVth and the Vth terms include an odd derivative in $\tau$, which directly contributes to the asymmetry in $A$. The VIth term includes the first order derivative of the induced current in $\tau$ at the still water level, and its computation requires the solution of a coupled system of Eqs. (3)–(5). It can be seen from (5) that for an initially symmetric envelope, velocity potential is antisymmetric in $\tau$, thus the VIth term in (3) retains the asymmetry of the envelope.

The modified discretized spatial version of the Zakharov equation was presented Shemer et al.\textsuperscript{10} It describes the evolution of the complex amplitude $B_j=B(\omega_j)$ of each free component in the spectrum that comprises $N$ components, due to four-wave interaction in a unidirectional space domain:
\[
\frac{i \cdot c_s}{\partial x} \frac{\partial B_j(x)}{\partial t} = \sum_{i,m,n} T(\omega_j, \omega_i, \omega_m, \omega_n) \\
\times B_m B_n b(\omega_j + \omega_i - \omega_m - \omega_n) \\
\times e^{-i(k_j + k_i - k_m - k_n)x},
\] (6)

where \(c_s\) is the group velocity and asterisk denotes complex conjugate. In (6), only those four-wave interactions among waves are considered that satisfy the conditions of near resonance

\[
\omega_j + \omega_i - \omega_m - \omega_n = 0,
\] (7a)

\[
|k_j + k_i - k_m - k_n| = O(\varepsilon^2).
\] (7b)

A relatively compact expression for the kernel \(T\) valid for deep water and exact resonance conditions, i.e., the left-hand side of (7b) equals zero, was given by Zakharov\(^{18}\) and used in this study. This expression is sufficiently accurate at the fourth order considered here. The surface elevation variation with \(t\) and \(x\) is then calculated from the obtained spatial variation of the complex amplitudes \(B_j(x), j = 1, ..., N:\)

\[
\zeta(x,t) = \frac{1}{\pi} \text{Re} \left( \sum_{j=1}^{N} \frac{\omega_j}{2g} B_j(x) e^{i(k_j x - \omega_j t)} \right).
\] (8)

### III. DESCRIPTION OF THE EXPERIMENTS

Experiments were carried out in a laboratory wave tank that is 18 m long, 1.2 m wide and 0.6 m deep. The tank is equipped with a computer-driven paddle-type wavemaker. Instantaneous surface elevation is measured at various locations along the tank by a number of resistance-type wave gauges. Detailed description of the experimental facility and the data acquisition procedure was given in Shemer et al.\(^{4,10}\)

Two different shapes of signals to drive the wavemaker are used in this study. The first driving signal was adjusted so that the initial surface elevation envelope at the wavemaker has a Gaussian shape

\[
\zeta(t) = \zeta_0 \exp \left(-t/mT_0^2\right) \cos(\omega_0 t), \quad -16T_0 < t < 16T_0.
\] (9)

The selected carrier wave period \(T_0 = 2\pi/\omega_0 = 0.7\) s, corresponding to the wave number \(k_0 = 8.22\) m\(^{-1}\), so that deep-water dispersion relation is satisfied. The forcing amplitude \(\zeta_0\) was chosen so that the maximum wave amplitude in the group \(a_0 = 2.92\) cm, corresponding to the maximum wave steepness \(\varepsilon = k_0 a_0 = 0.24\). The energy spectrum of (9) also has a Gaussian shape with the relative width at the energy level of \(\frac{1}{2}\) of the spectrum maximum given by

\[
\Delta \omega = \frac{1}{m \pi} \sqrt{2 \ln 2}.
\] (10)

The value of the parameter \(m\) in the experiments was selected to be \(m = 4.0\), so that the relative spectrum width \(\Delta \omega/\omega_0 = 0.047 < \varepsilon\), thus satisfying the narrow spectrum assumption of the cubic Schrödinger and Dysthe models.

In the second series of experiments the initial surface elevation at the wavemaker has the following shape:

\[
\zeta(t) = \zeta_0 \cos(\omega_0/2\pi t) \cos(\omega_0 t), \quad -10T_0 < t < 10T_0.
\] (11)

The carrier wave frequency and the maximum amplitude in the experiments with the initial shape (11) were identical to those with the shape given by (9). The spectrum of this

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**FIG. 1.** Effect of various terms in the Dysthe model for \(T_0 = 0.7\) s, \(\varepsilon = 0.24\), and the Gaussian initial shape (9) with \(m = 0.4\): (a) CSE; (b) Dysthe model without terms IV and V; (c) CSE+ terms IV and V; (d) full Dysthe model.
FIG. 2. Measured and computed variation of the instantaneous surface elevation at three locations along the tank for $T_0=0.7 \text{ s}$, $e=0.24$, and the Gaussian initial shape (9) with $m=4.0$.

FIG. 3. Measured and computed variation of the instantaneous surface elevation at three locations along the tank for $T_0=0.7 \text{ s}$, $e=0.24$, and the initial shape given by (11).
surface elevation shape is bi-modal, with two equal-height peaks at \( \omega = \omega_0 \pm \Delta \omega \), where \( \Delta \omega = \omega_0/20 \), satisfying again the narrow spectrum approximation requirement. The drive signals corresponding to both (9) and (11) were repeated periodically throughout the duration of the experiments.

IV. RESULTS

The effect of the fourth order terms on the solution of the Dysthe model is studied first. Equations (3)–(5) are solved using pseudo-spectral method and split-step Fourier method as described in Lo and Mei.\(^{14}\) Usually integration step \( \Delta x = 1 \) cm and 1024 points in the temporal domain were used in the numerical solution of the Dysthe model in this study. The numerical results obtained for \( \varepsilon = 0.24 \) and the initial condition given by (9) with \( m = 4 \), are presented in Fig. 1. The solution of the deep-water CSE model [terms I–III in (3)] is given in Fig. 1(a). At this high forcing amplitude, this model yields considerable energy focusing along the tank, while retaining the symmetric shape of the envelope. At the next stage, the effect of the induced current is also considered by addition of the term VI in (3), Fig. 1(b). In this case, the simultaneous solution of the coupled Eqs. (3)–(5) is required. This modification does not violate the symmetry of the solution, but leads to the widening of the envelope shape as compared to the CSE solution in Fig. 1(a). Further, in Fig. 1(c) only the nonlinear terms IV and V in (3) are considered, while the term VI which represents effect of the current potential is disregarded. As expected, addition of terms IV and V to the CSE model results in the visible asymmetry of envelope shapes. Finally, the solution of the full Dysthe model is presented in Fig. 1(d). It thus can be concluded that terms IV and V cause the dramatic change in the initially symmetric envelope shape as observed in the experiments.\(^4,10,14\)

Comparison of the numerical simulations of the temporal and spatial variation of the surface elevation performed according to both models with the experiments is carried out in Figs. 2 and 3. The spatial Zakharov equation (6) is solved by using the modified Runge–Kutta method following the procedure described in Shemer et al.\(^{10}\) In both figures, the results are shown at three locations which were arbitrarily selected close to the wavemaker, in the middle of the tank and far away from the wavemaker. The experimental results are band-pass filtered to eliminate the contribution of bound waves.

Results for the initial condition (9) with \( m = 4.0 \) and the maximum wave steepness \( \varepsilon = 0.24 \) are given in Fig. 2. The total number of free modes considered in the solution of the Zakharov equation (6), \( N = 60 \), although much less modes are actually required to obtain satisfactory results. Close to
the wavemaker, the group envelope still retains its Gaussian shape, but farther away strong distortion of the initially symmetric shape appears, and the characteristic triangular shape of the envelope emerges. Both theoretical models faithfully describe the experimental oscillations, including the fine features of the observed wave shapes. The bi-modal forcing case is presented in Fig. 3. The Zakharov equation here was solved with $N = 20$ free modes. Here too, impressive resemblance between the experiments and the numerical simulations is obtained.

The agreement between the numerical results of the Dysthe and the Zakharov models is observed in Figs. 2 and 3 for the initial conditions with narrow spectrum. More extensive comparison of the numerical simulations based on the Zakharov equation with wave tank experiments has been carried out in Shemer et al.\textsuperscript{10} It thus has been demonstrated that the spatial Zakharov equation represents an adequate model to describe evolution of a nonlinear wave field along the tank. Since the Zakharov model is free of any limitations on the spectral width, it can be effectively used for any initial wave spectrum. In the narrow spectrum case, results of Figs. 2 and 3 clearly demonstrate that the Dysthe model is sufficient to emulate appropriately the details of the wave field evolution. Since the Dysthe model requires essentially lesser computer resources, it is thus tempting to check the limits of validity of this model for wider initial wave spectra.

Generation of narrow initial wave group envelopes (with correspondingly wide initial spectrum) represents not an easy task in a relatively short experimental wave tank, in part due to the presence of long waves in the spectrum. On the other hand, the validity of the Zakharov model for wave groups with narrow initial spectra was confirmed by extensive experiments. Since this model is free of any restrictions on the spectral width, the numerical solutions based on the Zakharov equation can serve as a basis for determining the domain of applicability of the Dysthe model. Computations based on both those models are carried out for the initial

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**FIG. 5.** Amplitude spectra of the surface elevation for the Gaussian initial shape, $e = 0.24$ and $m = 1.0$. 

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conditions that violate the formal limitations on the spectral width inherent to the Dysthe model.

Spectrum of desired width is obtained in the present study by varying the value of the parameter $m$ in the Gaussian envelope shape given by (9). Simulations are carried out for the carrier wave period $T_0 = 0.7$ s, maximum wave steepness $\varepsilon = 0.24$ and three values of the coefficient $m$, 1.0, 0.6, and 0.4. For $m = 1.0$, the effective relative spectral width given by $\Delta \omega/\omega_0$ is 0.19, just below the value of $\varepsilon$. For $m = 0.6$, $\Delta \omega/\omega_0 = 0.31$, somewhat higher that the maximum wave steepness. Finally, for $m = 0.4$, the relative initial spectrum width attains an extremely high value of 0.47, exceeding the maximum possible steepness of propagating deep gravity waves. Both variation of the surface elevation with time and the amplitude spectra are presented at three distances from the location of wave generation, at $x = 5$ m, $x = 10$ m, and $x = 20$ m.

Results for the surface elevation variation with the distance for $m = 1.0$ are presented in Fig. 4. Strong dispersion leads to dramatic variation of the group shape, which is obvious in both computations. The resemblance between the results of both models seems to indicate that the Dysthe model is adequate for these parameters even at relatively large distances. Additional insight into the nonlinear physics of the wave group transformation along the tank is obtained by analyzing the discrete amplitude spectra of the surface elevation. Smoothed lines representing those spectra at the three selected locations are compared with the initial spectral Gaussian shape in Fig. 5. The similarity between the two simulations is impressive, excluding the low frequency range. The Dysthe model correctly represents the gradual narrowing of the spectrum with the distance, with the corresponding growth of the peak value.

For $m = 0.6$, the Dysthe model-based simulated temporal variation of the surface elevation at different locations along the tank still compare favorably with the computations based on the Zakharov equations, although in this case the agreement becomes less impressive at larger distance of 20 m, Fig. 6. The groups retain their identity up to the distance of about $x = 10$ m, and farther away the faster moving longer waves penetrate to the slower moving shorter waves of the previous group. The corresponding amplitude spectra obtained from the Dysthe model retain definite similarity to those obtained using the Zakharov equation, especially at higher frequencies. As in Fig. 5 in both simulations the spectrum becomes narrower with the distance, although for $m = 0.6$ this effect is
less pronounced than for \( m = 1.0 \). The rate of variation of the spectrum in Fig. 7 is quite fast at the first stages of evolution, but at larger distances from the location of wave generation the spectral shape remains nearly constant. This can be attributed to the spreading of the wave energy over the computational domain visible in Fig. 6, which results in gradual linearization of the problem with increasing distance. At the lower end of the spectrum, however, the Dysthe model exhibits notable noise, in contrast to the smooth behavior of the spectrum obtained from the Zakharov equation at those frequencies.

When an even wider spectrum is considered, \( m = 0.4 \), the wave energy spreads quite fast, so that already at \( x = 10 \) m the consecutive groups become indistinguishable, Fig. 8. In this case, 70 free modes were considered in the solution of the Zakharov equation (6). Although the initial spectrum in this case can by no means be considered as narrow, the temporal variation of the simulated using the Dysthe model surface elevation up to the distance of about 10 m is similar to the results of the Zakharov equation. At larger distance, \( x = 20 \) m, quite a chaotic temporal variation of \( \xi(t) \) is obtained by the Dysthe model, qualitatively (but not quantitatively) similar to the solution at this location of the Zakharov equation. Comparison of the corresponding amplitude spectra in Fig. 9 sheds additional light on the nonlinear wave transformation process. The solution of the Zakharov equation indicates that the energy spreading along the group is in this case even faster than that for \( m = 0.6 \), so that already at \( x = 5 \) m the wave spectrum attains its finite shape and no energy transfer occurs anymore among various wave components. This final spectrum shape, however, for \( \omega / \omega_0 > 0.8 \) is notably different from the initial Gaussian spectrum and exhibits a wider peak and a more narrow extent of the spectrum. The performance of the Dysthe model cannot be considered as adequate. It still reflects relatively well the behavior of the higher-frequency part of the spectrum, but the shape of the spectrum at energy-containing frequencies is
notably different from that obtained from the Zakharov equation. Moreover, the oscillations in the spectral amplitudes at the low-frequency end of the spectrum, which were already clearly visible in Fig. 7, become now much stronger, indicating that the Dysthe model fails for this wider initial spectrum.

V. CONCLUSIONS

A very good agreement between the experimental results and the simulations based on both Zakharov and Dysthe models for unidirectional spatial evolution of nonlinear wave groups with narrow initial spectrum was obtained. The validity of both those nonlinear theoretical models is confirmed. The application of the Dysthe model is advantageous since it requires substantially less computer resources. Moreover, the Dysthe model is relatively simple and the evolution, the linear dispersion and the nonlinear terms are clearly identified. The contribution of each term in the equation to wave group transformation along the tank was analyzed in Fig. 1. The complete Dysthe model involves coupling between the envelope and the potential of the induced current equations. It is shown that both the complete Dysthe coupled model and the truncated model in which the effect of the current is disregarded can give quite similar results and the current term does not affect the asymmetry of the simulated envelope forms.

The Zakharov equation that is free of any restriction on the spectral width was used in this study as a basis for determining the limits of the Dysthe model applicability for wider initial spectra. It is demonstrated that for the relative spectral width of the order of $\varepsilon$ the relatively simple deep-water Dysthe model yields both surface elevation variation and amplitude spectra that are in a reasonable agreement with the solution of the Zakharov equation. The accuracy of the Dysthe model reduces significantly when the initial spectral width is substantially higher than the maximum initial wave steepness.

This study illustrates that a better understanding of the complex problem of the nonlinear wave evolution can be achieved by analyzing both surface elevation history and amplitude spectra at various locations. The surface elevation history plots make apparent the effects of dispersion, while the spectra clearly show the contribution of nonlinearity.

The present simulations for wider initial spectra demonstrate that the wave energy from the high-frequency components in the spectrum is shifted in the course of unidirec-
tional evolution process towards lower frequencies, thus changing substantially the spectral shape. The frequency of the peak in an initially wide spectrum, though, shows a trend towards higher frequencies. No significant energy exchange is observed in the low-frequency range of the spectrum.

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15E. Kit and L. Shemer, “Spatial versions of the Zakharov and Dysthe...

