Measurements of the dissipation coefficient at the wavemaker in the process of generation of the resonant standing waves in a tank

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Abstract. The complex dissipation at the wavemaker used for direct excitation of nonlinear standing waves in the vicinity of the cut-off frequency, is measured experimentally. The results indicate that the absolute value of this dissipation coefficient exceeds significantly the estimate based on purely viscous dissipation. This is attributed to the turbulent effects resulting from the vortex shedding at the discontinuities of the wavemaker surface. It is shown that incorporation of this dissipation in the boundary condition at the wavemaker as a nonlinear term in a form generally used to describe dissipation in a turbulent boundary layer, is more appropriate allowing to remove hysteresis in the value of the wavemaker dissipation coefficient. Such hysteresis is observed when a linear dissipation model is applied.

1 Introduction

In recent years, the problem of both directly and parametrically excited nonlinear standing cross-waves in a rectangular tank has obtained considerable attention (Kit et al. 1987; Li and Chen 1987; Kit and Shemer 1988, Shemer and Kit 1988). The interest in these waves stems from the fact, that they reveal in a relatively simple system a rich variety of nonlinear phenomena, including envelope soliton generation and propagation, regular or chaotic modulation on slow time and space scales, bifurcating solutions, hysteresis, etc. It was realized in some of the studies of the nonlinear resonant standing waves that dissipation at the wavemaker can play a crucial role in the overall pattern of the long-time evolution of such waves. The physical reason for this lies in the fact that under the conditions prevailing in certain experimental facilities which are capable of generating directly standing resonant waves, the rate of work by the wavemaker, the dissipation at the wavemaker and the energy flux from the wavemaker to the potential wave field may all be of the same order of magnitude. Kit et al. (1987) and Shemer and Kit (1988) therefore arrived at a conclusion that the boundary condition at the wavemaker in the theoretical model has to be modified so as to include the appropriate complex dissipation term. Both the absolute value and the argument of the dissipation coefficient were chosen by trying to fit the numerical results obtained from the theoretical model to the experimental observations. This fit appeared necessary since the experimental data on the dissipation coefficients at the wavemaker in particular, and at the solid walls in a wave tank in general, is very limited. As a matter of fact, to our best knowledge, the most recent direct measurements of dissipation coefficients in deep tanks were performed about 30 years ago (Case and Parkinson 1957; Keulegan 1959). The agreement between these experimental results and the damping rates calculated theoretically when only viscous effects were taken into account depended strongly on the properties of fluids and on the particular materials of which the solid walls are made. The necessity to take into account the capillary effects has been therefore realized since then.

The theoretical attempts to incorporate free-end edge conditions into the model describing the damping effects were made by Miles (1967), Mei and Liu (1973) and recently by Whittaker (1987). The results of some of these calculations show that the actual damping rate at smooth surface depends critically on the surface tension coefficient of the fluid and on the wetting angle. These properties, in turn, appear to be a strong function of contamination of the fluid surface by surface-active agents. The actual coefficient of dissipation even on the smooth wall can therefore only be determined by experiment.

In the case of the wavemaker which is used to excite directly standing cross-waves (sometimes called also sloshing waves) in a tank, dissipation defies description yet to a greater extent. In addition to all complicating factors which are present in the attempts to solve theoretically the problem of dissipation at a smooth wall, the wavemaker surface necessary to excite such waves always has discontinuities (Barnard et al. 1977; Kit et al. 1987). Due to these discontinuities, vortex shedding from the sharp edges of the wavemaker has to be taken into account. Shemer and Kit (1988) have suggested that dissipation by this vortex shedding is the main mechanism of energy losses in the vicinity of the wavemaker. Such vortex shedding may be one of the reasons for appearance of the turbulent boundary layer at the wavemaker. Breaking of quite steep resonant waves which are
generated in these experiments is an additional reason for the origin of turbulence in the vicinity of the wavemaker. The comparison of the experimental and the numerical results of Shemer and Kit (1988) indicated indeed that due to the resulting turbulent effects, the boundary layer at the wavemaker may become relatively thick and thus the dissipation there becomes dominant compared to relatively weak viscous damping in the narrow Stokes layer at the side walls of the channel.

In order to substantiate this assumption, in the present work we measure experimentally the coefficient of dissipation at the wavemaker in the process of direct generation of nonlinear sloshing waves in the tank. The obtained values are compared with the earlier estimates based on the numerical solutions.

2 Experimental procedure

Experiments were carried out in a rectangular wave tank which is 18 m long, 1.2 m wide and filled to a mean water depth of 0.6 m. The side walls are made of large glass plates and are therefore quite smooth. Waves were generated by a four-segment paddle-type wavemaker. Each segment could be operated independently. This modular structure of the wavemaker caused certain discontinuities in the wavemaker surface. In order to generate standing waves of the second mode, the two inner segments were operated with 180° phase shift relative to the two outer paddles. Such mode of operation which can not be eliminated in practice when generating sloshing waves in a tank, results in even stronger discontinuity between the wavemaker sections moving out of phase. Note, however, that the surface of each paddle taken separately was quite smooth, in contrast to the wavemaker structure in earlier experiments reported in Kit et al. (1987). The close view of the wavemaker in our experimental facility was presented in Fig. 1 of Shemer and Kit (1988). Instantaneous surface elevation was measured by conductance-type wave gauges. Up to four gauges were used simultaneously. The gauges were placed on a bar which was parallel to the channel walls and could be moved along the tank. All measurements reported in the present work were performed at the center line of the tank.

Since the subject of the present investigation is a resonant phenomenon, extremely stable forcing frequency and possibility of fine tuning were necessary for the experiment. This was achieved by operating the wavemaker using a computer-generated sinusoidal signal. The computer was also used in order to sample the instantaneous surface elevation from the wave gauges and the instantaneous position of each wavemaker segment using the output from the appropriate position potentiometers. All this information, together with the computer-generated forcing signal which served as a phase reference, was sampled and recorded by a computer. In the present investigation the sampling duration was usually about 150 consecutive wave periods. Ensemble averaged quantities were calculated from the recorded information. The amplitudes and the phases of all measured quantities were therefore obtained by averaging over a large ensemble, which contributed to a significant reduction of the experimental error. A more complete description of the experimental facility, wave gauge calibration procedure and of the data acquisition and processing was given in Shemer et al. (1987) and Kit et al. (1987).

3 Theoretical background

Following Kit et al. (1987) and Shemer and Kit (1988), we consider a semi-infinite rectangular tank of width $b$ and water depth $h$, with the wavemaker at its end $x = 0$. All variables are rendered dimensionless using $h$ as a length scale and $(b/g)^{1/2}$ as a time scale. Deep water approximation holds for our experimental conditions. For a general case of $n$-th mode, the wave number $k_n$ and the cut-off frequency $\omega_n$ are given by

$$k_n = n \pi; \quad \omega_n = \sqrt{\epsilon n}.$$  

(1)

In the present work all experiments were performed for $n = 2$ (second sloshing mode, with the dimensional wave length equal to the tank width $b$). The small parameter of the problem $\epsilon$ is proportional to the dimensionless stroke of the wavemaker at the mean surface level, $s$. The relation between $\epsilon$ and the wavemaker stroke $s$ for any mode of excitation and for an arbitrary wavemaker shape is presented in the Appendix (A1). For a particular case of the second mode and our wavemaker geometry,

$$\epsilon = 0.0205 \, s.$$  

(2)

This small parameter is used to scale the velocity potential and the time and space variables. The scaling relations, the definition of the detuning coefficient $\delta$, as well as the nonlinear Schrödinger equation, which governs the evolution of the scaled dimensionless velocity potential $C$, as a function of the scaled time $T$ and space $X$, variables are presented in the Appendix (A4). The boundary condition at the wavemaker has the following form:

$$\frac{\partial C}{\partial X} = -i \delta \frac{\partial C}{\partial T} \quad \text{at} \quad X = 0$$  

(3)

where

$$\delta = \frac{\alpha_s \cdot \exp i \phi_s}{\sqrt{2 \pi \epsilon}}$$  

(4)

is the effective complex dissipation coefficient at the wavemaker. Note that the value of $\delta = \pi/2 - \phi_s$ represents the angle between the wall shear stress at the wavemaker and the velocity in the potential flow outside the wavemaker dissipative layer (Kit and Shemer 1989). Absolute value of $\delta$ for the 2nd mode has the following form:

$$\alpha_s = \frac{\delta}{h^{1/2}}$$  

(5)
where $\delta$ is the characteristic thickness of the dissipative layer. For purely viscous dissipation, $\delta$ is the thickness of the Stokes layer given by

$$\delta = \sqrt{2} v/\omega,$$  \hspace{1cm} (6)

while the argument of $\phi_2$, $\phi_2 = \pi/4$. As was mentioned above, the value of $\alpha_3$ given by (5) with the Stokes layer thickness calculated from (6) appears to underestimate substantially the actual damping rate. In the present work both the absolute value $\alpha_3$ and the phase angle $\phi_3$ of this dissipation coefficient are therefore determined experimentally from (3) by measuring the complex values of $\partial C/\partial X$ and $C$. Note that the only assumption made in derivation of (3) is the linear dependence between the wall shear stress at the wavemaker and the potential velocity at the outer edge of the dissipative layer. While this assumption obviously holds in the case of purely viscous dissipation, its validity may be questionable when vortex shedding and turbulent effects at the wavemaker become important, and thus may only be established by experiment.

4 Experimental results and discussion

The experimental results of Shemer and Kit (1988) indicate that two qualitatively different sloshing wave regimes are possible in the present experimental facility. The first regime is a strongly modulated one, and is observed at higher frequencies of forcing, which correspond to the values of the detuning coefficient $\lambda$ above approximately $-1.2$. The details of the time and space evolution process in this regime were studied in detail by Shemer and Kit (1988). At lower forcing frequencies, steady standing wave field is observed in the tank. In the present investigation it was decided to focus the effort on this steady regime, since more accurate measurements of the dissipation coefficient can be performed when the additional complication of time dependence is removed.

Measurements were carried out at two amplitudes of forcing, $c = 0.43 \cdot 10^{-4}$ and $c = 0.59 \cdot 10^{-4}$. At each amplitude, the forcing frequency was varied monotonously with small increments in the values of the detuning coefficient $\lambda$. The results obtained when the forcing frequency was gradually increased are compared with the data obtained at identical forcing conditions, but attained by reducing the frequency. Two different wave steady patterns were observed in the tank in a wide range of the values of the detuning parameter $\lambda$. An example of a hysteresis loop obtained in the experiments is shown in Fig. 1a for the scaled according to (A3) amplitude of the standing wave in the vicinity of the wavemaker and in Fig. 1b for the phase angle of the surface elevation (relative to the displacement of the central module of the wavemaker) at the forcing amplitude $c = 0.59 \cdot 10^{-4}$. At other forcing amplitude the dependence of both the amplitude and the phase of the surface elevation on $\lambda$ were similar to those presented in Fig. 1. These results provided additional details to the earlier measurements of Barnard et al. (1977). Note that in their report different phase definition was employed. $\lambda_0$ in Fig. 1 denotes the transitional value of the detuning coefficient $\lambda$ at which the "jump" from the low to the high wave amplitude at the wavemaker occurs, while $\lambda_T$ denotes the value of $\lambda$ corresponding to the reverse transition. Note that the hysteresis loop is observed between the values of the detuning coefficient $\lambda$ corresponding to the two maximum possible values of the wave amplitude at the wavemaker, one obtained at $\lambda_0$ when the forcing frequency is gradually increased, while the second, which is also the absolute maximum, is obtained at $\lambda_T$.

The distributions of the scaled amplitudes and the phase angles of the surface elevation along the tank at identical forcing conditions ($c = 0.59 \cdot 10^{-4}$, $\lambda = -2.80$), but with dif-
Different "history", are given in Fig. 2. The distributions labeled by I were attained by gradual increase in the forcing frequency from some initial low value, while label II refers to the regime which is observed by reducing the forcing frequency from the initial value corresponding to the detuning coefficient $\lambda$ exceeding $\lambda_c$. The phase angles of the surface elevation are essentially constant along the tank in both wave regimes. Note that in all experiments with the wavemaker operation started in still water (zero initial condition), the wave pattern corresponding to the regime I was obtained eventually in the tank, after the transient effects vanished.

Instantaneous measurements by two probes make it possible to obtain both the complex derivative $dC/dX$ and complex amplitude of the velocity potential $C$ and calculate the absolute value and the argument of $dC/dX$ from (3). The distance between the probes for the measurement of the derivative was $\Delta x = 15$ cm, while the distance between the first probe and the wavemaker was $x_1 = 5$ cm. For the higher forcing amplitude employed in the present experiments, $\varepsilon = 0.59 \cdot 10^{-4}$, these distances correspond to the normalized dimensionless $\Delta X = 0.095$ and $X_1 = 0.03$. As can be seen from (A-4), the dimensionless distances at other forcing amplitudes are of the same order of magnitude. Numerical results of Shemer and Kit (1988) reveal that these values of $\Delta X$ and $X_1$ can be considered as sufficiently small. On the other hand, the spacing between the probes was adequate to eliminate their mutual interference.
The variation of the absolute value of the wavemaker dissipation coefficient \( a_2 \) with \( \lambda \) (Fig. 3), calculated according to (3) and (4) indicates that the hysteretic behavior of the surface elevation is accompanied by the corresponding hysteresis in the dissipation coefficient \( a_2 \). The dissipation coefficient at the wavemaker \( a_2 \) in the case of purely viscous dissipation is independent of \( \lambda \). The absolute values of \( a_2 \) according to (5) are 0.024 for higher forcing amplitude \( (\varepsilon = 0.59 \cdot 10^{-4}) \) and 0.028 for weaker forcing \( (\varepsilon = 0.43 \cdot 10^{-4}) \). The results presented in Fig. 3 reveal that the experimentally obtained values are higher at least by order of magnitude. The average value of \( a_2 \) obtained in the present experiments is quite close to \( a_2 = 0.56 \), which was chosen in the numerical solutions of Shemer and Kit (1988) as giving the best agreement between the modulation patterns obtained numerically and those observed in their experiments. The results presented in Fig. 3 are very similar for all forcing amplitudes. When the wave regime I exists in the tank, the value of \( a_2 \) remains almost constant and is around 0.25. It increases gradually to about 0.5 - 0.6 when the detuning coefficient \( \lambda \) approaches the transitional value \( \lambda_t \). Comparison with Fig. 2a indicates that the dependence of \( a_2 \) on \( \lambda \) in this regime resembles strongly that of \( |C| \). The transition from the wave regime I to the wave regime II is always accompanied by a corresponding jump in the values of the wavemaker dissipation coefficients. At both forcing amplitudes the transitional value of the detuning coefficient \( \lambda_t \) was close to \( -2.5 \).

In the regime II, which corresponds to higher wave amplitudes at the wavemaker, the value of \( a_2 \) is close to unity for both values of \( \varepsilon \) employed in this investigation. This value remains nearly independent of \( \lambda \). The absolute value of the wavemaker dissipation coefficient \( a_2 \) at this amplitude decreases gradually with decreasing \( \lambda \) to about 0.8 prior the back transition to regime I. The value of the detuning coefficient where the transition from regime II to regime I is observed is about \( \lambda^*_t = -3.3 \) at \( \varepsilon = 0.59 \cdot 10^{-4} \) and \( \lambda^*_t = -4.27 \) at \( \varepsilon = 0.43 \cdot 10^{-4} \). In both cases therefore the hysteresis region has a substantial width in the order of unity in terms of \( \lambda \).

The arguments of \( a_2 \) shown in Fig. 4 also exhibit certain hysteresis. The values of \( \phi_2 \) appear to be more sensitive to the absolute dimensional wave amplitude than \( a_2 \). In the regime II the arguments of \( a_2 \) are practically independent of \( \lambda \) and are close to 30° for both \( \varepsilon \). This value is practically identical to \( \phi_2 = \tan^{-1}(0.5) \) chosen by Shemer and Kit (1988) as the best fit at higher frequencies of forcing, where the modulated wave pattern is observed. The present results therefore indicate that their guess was reasonably close to the actual values prevailing in the experimental facility at the values of the detuning parameter \( a_2 \), when only regime II is possible in the tank. In regime I, however, there appears to be notable decrease in \( \phi_2 \) with increasing forcing frequency.

It seems reasonable to assume that a correct model which takes into account all physical processes which contribute to the dissipation at the wavemaker should be independent of the wave field in the tank. The present results indicate, however, that not only the Eq. (5) with the Stokes layer thickness \( \delta \) defined by (6) does not provide the realistic value of the damping coefficient, but also the linear dependence between the wall shear stress at the wavemaker and the potential flow velocity, which is the major assumption in the derivation of the boundary condition (3), does not hold in present experiments. As was mentioned above, higher values of \( |C| \) (at increasing frequency) correspond to lower values of \( a_2 \). Such dependence of the dissipation coefficient on the wave amplitude suggests that the assumption that the wall shear stress \( \tau_w \) at the wavemaker is proportional to the square of the potential velocity, as it is usually the case in turbulent boundary layers, may provide a more realistic description of the physics of dissipation mechanism at the wavemaker. In order to check this assumption, we adopt the following relation between the shear stress and \( C \): \( \tau_w \propto C |C| \). As follows from Shemer and Kit (1988), this assumption

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**Fig. 4a and b.** The arguments of the dissipation coefficient at the wavemaker for the conditions identical to those of Fig. 3.
5. Conclusions

Measurements of directly excited resonant standing waves in a rectangular tank were performed at various forcing amplitudes. The forcing frequencies were chosen to be in a range where steady wave patterns were obtained. Accurate control of the experimental conditions is necessary in order to obtain meaningful results.

Hysteresis was observed in the steady wave patterns in the tank. Both the amplitudes and the phase angles of the surface elevation in the region of hysteresis depend on the previous history of the wave field.

The complex value of the wavemaker dissipation coefficient which appears in the boundary condition of the nonlinear model describing the time and space evolution of the resonant standing waves in a rectangular tank was measured experimentally. Generally speaking, both the absolute value and the argument of the dissipation coefficient obtained in the present investigation are in agreement with the appropriate values adopted in Shemer and Kit (1988) chosen so as to fit the results of the numerical calculations to the experimentally observed long-time evolution patterns. The assumption of the linear dependence between the wall shear stress at the wavemaker and the potential velocity resulted in a similar hysteresis in the complex dissipation coefficient at the wavemaker \( \delta_C \).

The absolute values of \( \delta_C \) are by order of magnitude higher and its argument may be quite different from the \( \pi/4 \) value predicted from the purely viscous model.

The hysteresis in the \( \delta_C \) may be removed if the assumption of the main dissipation mechanism at the wavemaker by vortex shedding from the edges of the wavemaker segments and the resulting turbulence is adopted. This assumption is also justified by high wave steepness typically obtained in the present experiments, which causes wave breaking.

Appendix

Following Shemer and Kit (1988), the small parameter of the problem \( \varepsilon \) is defined as

\[
\varepsilon = \frac{2(\alpha \pi)^2}{\int_0^{\infty} \int_0^{\infty} \cos^2(\alpha \pi y) \exp(2 \alpha \pi z) dy dz}
\]

(A1)

where \( f(y,z) \) is the shape function of the wavemaker, so that the instantaneous coordinate of the wavemaker \( z(y,x,t) \) is given by

\[
z(y,x,t) = \frac{1}{2} s f(y,z) (e^{-\Im} + \Im)
\]

(A2)

where \( * \) denotes complex conjugate. For the present experimental conditions

\[
f(y,z) = \begin{cases} (1 + z/\bar{a}); & 0 < y < 1/4; 3/4 < \bar{a} < 1, \\ (1 + z/\bar{b}); & 1/4 < y < 3/4. \end{cases}
\]

(A3)

The slow length variable \( X \) and the slow time variable \( T \) for the 2nd mode are given by

\[
X = (2 \pi)^{1/3} \varepsilon^{1/2} s, \quad T = 2 \pi^2 s t.
\]

(A4)
The ratio of deviation of the forcing frequency $\omega$ from the resonant cut-off value for the $n$th mode $\omega_n$ to the forcing amplitude $\varepsilon$ is represented by the dimensionless detuning coefficient $\lambda$, defined by

$$\lambda = \frac{\omega - \omega_n}{\omega_n} \cdot \lambda.$$  \hspace{1cm} (A5)

The dimensional velocity potential $\phi$ is related to complex normalized potential amplitude $C$ by

$$\phi = \sqrt{\frac{gh}{\varepsilon}} \cos \frac{2\pi}{T} \exp [C (X, T) \cdot \exp (-i \omega t + \delta)].$$  \hspace{1cm} (A6)

The dimensional free surface elevation $\eta$ is related to the velocity potential $\phi$ through the linearized dynamic boundary condition:

$$\eta = -\frac{1}{g} \frac{\partial \phi}{\partial t}.$$  \hspace{1cm} (A7)

The phase angle of $\eta$ is therefore shifted by $\pi/2$ relative to the argument of $C$, and the following relation between the directly measurable amplitude of the surface elevation $\langle \eta \rangle$ and $|C|$ is obtained from (1), (A6) and (A7) for $n = 2$:

$$\langle \eta \rangle = 2 \sqrt{\frac{2 \pi \varepsilon h}{|C|}}.$$  \hspace{1cm} (A8)

As was shown by Kit et al. (1987), the time and space variation of the nonlinear wave field in the presence of dissipation along the tank is governed by the nonlinear Schrödinger equation:

$$\frac{\partial C}{\partial T} + \frac{\partial^2 C}{\partial X^2} + \lambda C + \dot{\Delta} C + 2 |C|^2 C = 0$$  \hspace{1cm} (A9)

where $\dot{\Delta}$ is the appropriate complex dissipation coefficient along the tank.

References


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