On the focusing of the ocean swell images produced by a regular and by an interferometric SAR

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Abstract. Focusing of a monochromatic ocean wave image by either a regular or an interferometric SAR (INSAR) is studied. The model developed simulates the images of both SAR and INSAR for an arbitrary focus setting. The effects of the various operational parameters, i.e., integration and scene coherence times, ocean wave length, direction and amplitude, etc., on the focus sensitivity of both SAR and INSAR is examined. A dimensionless focusing parameter, $\alpha$, which depends on the platform velocity and on the azimuthal component of the phase velocity of the imaged ocean wave is introduced. It is shown that $\alpha$ is the major parameter which determines the image contrast of the regular SAR output, as well as that of the magnitude and the phase components of INSAR. The focus sensitivity, however, is also a function of the imaged ocean wave parameters. The effects of the varying SAR integration time and of the scene coherence time on the contrast of the resulting image is also examined.

1. Introduction

While the basic principles of SAR imagery of the ocean surface are now well understood and summarized in the consensus paper by Hasselmann et al. (1985), the issue of the focusing of the SAR ocean images still remains to a certain degree controversial. For stationary targets, matched filter operation is set to the SAR platform velocity, $V_0$, which gives the maximum output. When solid moving targets are imaged by SAR, the optimal speed of the matched filter operation, $W$, differs from $V_0$ by the azimuth (along track) component of the target velocity. This operation, known as refocusing, takes into account the movement of the target and thus reduces the image smear and improves SAR azimuthal resolution. The difference between the actual speed of the matched filter and the platform velocity, $\Delta V = W - V_0$, is the focus setting.

The fact that contrast images of ocean waves with wave length sufficiently large compared to SAR resolution, which move in the azimuth (along track) direction, can be enhanced by varying the speed in the matched filter operation in the SAR processing, has been noticed long ago (Shuchman and Shemdin 1983, Jain and Shemdin 1983). Improvement of the signal to clutter (S/C) ratio in the SAR imagery of ocean swell by defocusing was actually observed for an airborne SAR only, while for much faster space platforms no such effect could be obtained. The optimal amount of the focus setting usually appears to be of the order of magnitude of the azimuthal component of the phase velocity of the ocean swell. The swell phase velocity represents the noncoherent effect of the ocean wave pattern displacement. This effect does not contribute to the Doppler shift which is caused by the movement
of the scatterers at the ocean surface due to superposition of ocean currents and orbital velocity induced by the swell, and the phase velocity of the short resonant Bragg waves.

The part of the Doppler shift which is associated with the swell orbital velocities vary periodically in time as well as in space. This periodic variation of the Doppler shift in the azimuthal direction constitutes the major mechanism by which ocean waves are imaged by SAR, the so-called velocity bunching (Swift and Wilson 1979, Alpers and Rufenach 1979). Brüning et al. (1991) maintain that any relation between the optimum focus setting and the swell phase velocity is purely accidental. They put an accent on the temporal variability of the orbital velocity and show that the variation of the image contrast with the focus setting is strongly dependent on the radial component of the surface acceleration due to the orbital motion. Brüning et al. (1991) present some Monte Carlo simulations based on their theory. These simulations were carried out with parameters encountered during the Tower Ocean Wave and Radar Dependence (TOWARD) experiment (Shemdin 1988), and demonstrate that the velocity bunching theory yields optimum focus setting consistent with the experiment. Note that for a given ocean wavelength and propagation direction, the orbital acceleration is proportional to the wave amplitude. The interpretation of the focusing effect given by Brüning et al. (1991) thus seems to emphasize the optimum focus setting dependence on the imaged wave height.

On the other hand, there exist a number of other SAR imaging models, which account for the velocity bunching effect, but differ from the velocity bunching theory mainly by considering the temporal variation of the time-dependent ocean surface reflectivity (Lyzen 1988, Kasilingam and Shemdin 1988, Raney and Vachon 1988). The predictions of all these theories were compared by Kasilingam and Shemdin (1990) with the velocity bunching theory and were shown to be similar for short integration times. These models predict optimum focus setting at about half of the azimuthal component of the swell phase velocity $C_s = C / \cos \phi$, where $C$ is the phase velocity magnitude and the angle $\phi$ determines the swell propagation direction relative to the platform flight path. More accurately, the matched filter velocity $W$ which yields maximum image contrast is determined by

$$W^2 = V(V - C_s),$$

(1)

For $C_s \ll V$, (1) results in the optimum focus setting $\Delta V = C_s / 2$. The physical reasons for that were clarified recently by Kasilingam et al. (1991). Note that in contrast to the conjecture of Brüning et al. this result is wave amplitude independent.

The simulated focusing curves of various imaging theories were carefully compared by Hayt et al. (1990) with TOWARD images obtained at different focus settings. Good agreement was found between the TOWARD data and the simulation curves based on those theories. The reason for similarity between the focusing curves predicted by the velocity bunching theory of Brüning et al. (1991) and those of time-dependent theories by Lyzen (1988) and Kasilingam and Shemdin (1988) was examined by Plant (1992). It was demonstrated in this study that all SAR imaging theories under consideration are reconciled and can be seen as a single consistent theory. They are identical for short integration times and diverge for longer times of integration. The question of the wave height dependence of the optimum focus setting was not addressed directly by Plant (1992). Kasilingam (1991), however, maintains that the optimum focus setting which is proportional to the orbital acceleration is consistent with (1). One of the goals of the present study is to resolve this apparent contradiction.
The other aspect addressed in the present study is the focusing sensitivity of an interferometric SAR (INSAR). The along-track INSAR was first employed by Goldstein and Zebker (1987) and Goldstein et al. (1989) for studying of slowly varying in space and time ocean currents. The instrument represents a modification of a conventional SAR and consists of two antennas separated spatially by distance $B$ along the platform flight path. In the present study a case will be considered when the first antenna only serves as a transmitter. Due to the spatial separation between the two INSAR antennas, the images of an identical scene obtained by each antenna are separated by time interval $\Delta t = B/2V$, $V$ being the platform velocity. The complex SAR images obtained by each antenna are combined interferometrically into a single complex image $I(X = Vt, y)$, where $y$ is the imaged point coordinate in the range direction. The modulus of the resulting complex map, $|I(X = Vt, y)|$, is similar to a regular SAR image, while the argument (phase) component of the image is directly related to the radial (line-of-sight) component of the scatterers velocity $U_r$ at the ocean surface:

$$\arg(I(X = Vt)) = 2k U_r(X, y) B/V$$

(2)

Generally speaking, the radial velocity $U_r$ results from the superposition of the phase velocity of the resonant Bragg waves, orbital velocity of the water elements at the surface due to gravity waves, and surface currents in the ocean (Marom et al. 1990). The two first contributions to $U_r$ do not depend on spatial coordinates (on the scale of ocean wave length) and thus can be seen as DC component, while the variation of $U_r$ due to the ocean swell can be seen as the AC component. This fact prompted an attempt by Marom et al. (1990, 1991) to apply INSAR to ocean wave measurements. It was shown in these studies that the direct imaging mechanism of INSAR contributes to a better quality of ocean imagery as compared to a regular SAR. Moreover, it appears that for the prevailing experimental conditions, the assumption that the direct relation (2) between the phase component of the complex INSAR image and the surface velocity still remains valid for the spatially and temporally varying ocean wave field. It was thus possible to measure using INSAR not only the lengths and the directions of the ocean waves, but also wave heights and directional wave energy spectra (Marom et al. 1991). The model describing the INSAR imagery of a monochromatic ocean wave field was developed by Shemer and Kit (1991). This model was further generalized by Shemer (1993) to include real aperture (tilt and hydrodynamic) modulation effects. It was demonstrated in these studies that for waves that are not too steep, so that the velocity bunching coefficient is not very high, the application of (2) is justified and INSAR can thus serve as a quantitative tool for measuring ocean wave heights. The phase component of the INSAR image also appears to be virtually insensitive to the real aperture radar modulation effects. In contrast to the complicated mechanism of the regular SAR imagery of the ocean surface, the phase component of the complex INSAR output is mainly affected by the surface scatterers velocity field, thus providing a more direct imaging mechanism. The conventional SAR imaging mechanisms are, however, also of importance in the INSAR imagery of the ocean surface. The velocity bunching mechanism contributes to the distortion of the image, while the effects of the real aperture modulation are relatively minor, in particular for waves which do not travel in close to the range direction.

The present study is based on the theoretical INSAR model by Shemer and Kit (1991), which is modified to incorporate the focusing adjustment in the matched filter. It should be stressed here that the analysis of a monochromatic ocean wave
considered in this model is particularly justified for the studies of focus sensitivity. This stems from the fact that focusing adjustment is tuned to a certain component in the wave spectrum, with a purpose to enhance the image contrast of the selected ocean wave, usually corresponding to the peak in the two-dimensional wave spectrum, while suppressing all other spectral components. The model suggested in the present study is exact in the sense that it fully accounts for the spatial and temporal variability of the orbital velocity during the integration period. The model computations examine the focus sensitivity of SAR as well as that of INSAR as a function of both the ocean swell phase velocity and the imaged wave height effect. These computations thus contribute to resolution of the existing controversy on this issue.

For the conventional SAR, the wave height cannot be found directly from the image, and thus the focus sensitivity is determined solely by the image contrast. Contrary to that, the phase component of INSAR is quantitatively related to the ocean wave height. The focusing sensitivity of INSAR can therefore be defined either by considering the image contrast, as it is the case for SAR, or, by comparing the accuracy of the resulting estimate of the swell amplitude for different focus settings. The INSAR simulations performed in this study show that the optimum focus setting is not necessarily identical for these alternative definitions of the focus sensitivity of the INSAR phase component.

2. Mathematical model

Consider constant range distance $R_0$ and thus constant range coordinate $y$. The fore antenna arrives at the point $X = Vt$ at the instant $t_1 = t - \Delta t$, while the aft antenna arrives at the same location at $t_2 = t + \Delta t$. The time delay is determined by $\Delta t = B/2V$. Note that the case with $B = 0$, and thus $\Delta t = 0$, corresponds to the regular SAR. For surface scatterers velocity with the radial (line-of-sight) component $U_r(x, t)$, $x$ being the azimuth coordinate of the scatterer, the backscattered signal is Doppler-shifted by

$$\Delta \omega(x, t) = 2k U_r(x, t). \quad (3)$$

where $k = |k|$ is the modulus of the radar wave vector. Adopting for convenience a Gaussian antenna pattern corresponding to the integration time $T_0$, the antenna outputs $s_1$ and $s_2$ were given by Shemer and Kit (1991) as

$$s_{1,2}(t) = \int_{-\infty}^{\infty} r_{1,2}(x, t \pm \Delta t) \exp \left( -\frac{2( x - Vt)^2}{V^2 T_0^2} \right) \exp \left( -\frac{ik(x - Vt)^2}{R_0} \right)$$

$$\times \exp \left( -2ik \int_0^{t \pm \Delta t} U_r(x, \eta) d\eta \right) dx, \quad (4)$$

where $r_{1,2}(x, t)$ are the corresponding radar reflectivities. The last integral in (4) represents the accumulated phase due to the contribution of the time-dependent Doppler shift given by (3). The complex SAR outputs $i_{1,2}(t)$ for each antenna are obtained by convolving $s_{1,2}(x, t)$, respectively, with the matched filter set at an arbitrary velocity $W$:

$$h(t) = \exp \left( ik W^2 t^2 / R_0 \right). \quad (5)$$

The resulting complex INSAR output is given by
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\[ I(t) = \langle i_j(t) \tilde{r}_j^2(t) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle r_1(x', t' - \Delta t) r_2^*(x'', t'' + \Delta t) \rangle \times \exp \left[ -\frac{2(x' - Vt')^2}{\nu^2 T_0^2} \right] \exp \left[ -\frac{2(x'' - Vt'')^2}{\nu^2 T_0^2} \right] \exp \left[ \frac{ik(x' - Vt')^2}{R_0} \right] \times \exp \left[ \frac{ik(x'' - Vt'')^2}{R_0} \right] \exp \left[ -2ik \int_{0}^{r-\frac{\tau}{2}} U_r(x', \eta) d\eta \right] \exp \left[ 2ik \int_{0}^{r+\frac{\tau}{2}} U_r(x'', \eta) d\eta \right] \times \exp \left[ ik W^2 (t - t')^2 / R_0 \right] \exp \left[ -ik W^2 (t - t'')^2 / R_0 \right] dx' dx'' dr' dr'', \] (6)

where \( \langle \ldots \rangle \) denotes ensemble averaging (cf. Hasselmann et al. 1985). For spatially white reflecting surface, the cross-covariance of the radar reflectivity is assumed to have the generally accepted form:

\[ \langle r_1(x', t' - \Delta t) r_2^*(x'', t'' + \Delta t) \rangle = \sigma(x', T) \delta(x'' - x') \exp \left[ -4(\tau + \Delta \tau)^2 / \eta^2 \right] \] (7)

see Hasselmann et al. (1985) and Shemer and Kit (1991).

In (7) new temporal variables are introduced

\[ T = (t' + t'') / 2; \quad \tau = (t'' - t') / 2. \] (8)

The real aperture radar (RAR) modulation effects are not considered in the present study, so that \( \sigma(x', T) = \sigma_0 \). The finite coherence of the ocean surface due to random surface movement at subresolution scale is represented in (7) by the scene coherence time \( \tau_S \) (see, e.g., Lyzenga 1988, Shemer and Marom 1993). The \( \delta \)-function in (7) allows replacing the spatial variables \( x' \) and \( x'' \) by a single variable \( x \).

For a monochromatic swell with the wave vector \( \kappa = (\kappa_x, \kappa_y) \) and wave height \( H \), propagating at the angle \( \phi \) relative to the platform direction, so that \( \tan \phi = \kappa_y / \kappa_x \), the radial velocity component at \( x=(x,y) \) is given by

\[ U_r(x, t) = \frac{H}{2} \Omega (\cos^2 \theta + \sin^2 \theta \sin^2 \phi)^{1/2} \cos (\kappa_x x + \kappa_y y - \Omega t) \] (9)

where \( \theta \) is the radar incidence angle. Thus, the amplitude of the radial velocity is

\[ \hat{U}_r = \frac{H}{2} \Omega (\cos^2 \theta + \sin^2 \theta \sin^2 \phi)^{1/2} \] (9a)

The radial frequency of the swell \( \Omega \) and the wave vector \( \kappa \) are related by the deep water dispersion relation \( \Omega^2 = g |\kappa| \).

We now take a closer look at those terms in (6) which include the radial component of the surface velocity \( U_r \). Invoking (7) to (9) allows rewriting those terms in the form

\[ \exp \left[ -2ik \int_{r-\frac{\tau}{2}}^{r+\frac{\tau}{2}} U_r(x, \eta) d\eta \right] = \exp \left[ -4ik \hat{U}_r \cos (k x - \Omega T) \sin (\Omega (\tau + \Delta \tau)) \right] \] (10)

Expression (10) is exact for a monochromatic wave. Assuming that \( \Omega \tau \ll 1 \), i.e., that the integration time \( T_0 \) is much shorter than the wave period, and that the time shift \( \Delta \tau \ll T_0 \), it is possible to represent the sine in (10) by its Taylor expansion

\[ \sin \Omega (\tau + \Delta \tau) = \Omega (\tau + \Delta \tau) + O(\Omega^3 \tau^3) \] (11)

Note that the expansion (11) is equivalent to the expression obtained in the Appendix of Kasilingam and Shemdin (1988), which was restricted to the linear in \( \tau \).
term in the Taylor expansion. Brünig et al. (1991) attribute the focus setting dependence on the orbital acceleration to the quadratic in \( \tau \) term. It is however obvious from (11) that this quadratic in \( \tau \) term actually vanishes for a monochromatic ocean wave. The expressions (10) and (11) are thus accurate to the second order in accounting for the temporal variability of the surface scatterers velocity during the integration time, as is the expression of Brünig et al. (1991). It is also important to stress here that it can easily be shown that the expansion (11) is valid for a monochromatic ocean wave with an arbitrary initial phase, and not necessarily the one assumed in (9).

The next stages of the derivation follow closely those given in Shemer and Kit (1991). Substituting (7)–(11) into (6) allows integrating over \( \tau \). Introduction of the new variables

\[
\zeta = \frac{V}{C_x} (x - C_x T + y \cos \phi) \quad (12a)
\]

\[
\zeta_0 = x - V T \quad (12b)
\]

makes possible additional integration over \( \zeta_0 \). The following parameters appear as a result of these operations. The effective integration time \( T' \) is defined by

\[
\frac{2}{T'} = \frac{1}{\tau_0^2} + \frac{2}{T_0^2}. \quad (13)
\]

It is now convenient to define the focusing coefficient \( z \) as

\[
z = \frac{W^2}{V(V - C_x)}. \quad (14)
\]

Note that the value of \( z = 1 \) corresponds to the optimal focus setting given by (1) according to Kasilingam and Shemdin (1991) and Lyzenga (1988), while \( z = V/(V - C_x) \) represents SAR processing where no focusing correction is introduced.

The INSAR output (6) can be presented by

\[
I(X = V t) = \frac{\pi}{2} \frac{C_x}{V - C_x} \rho_0 \exp \left\{ \left( \frac{\Delta t'}{\tau_0} \right)^2 - \left( \frac{1 - (1 - z)^2 \rho_0^2}{\rho^2} \right) - 2 \right\} I_1, \quad (15)
\]

where the integral \( I_1 \) is given by

\[
I_1 = \int_{-\infty}^{\infty} \sigma_0 \exp \left\{ 4i k U_s(\zeta) \Delta t \right\} \exp \left\{ \left( \frac{f^2(\zeta)}{\rho^2} \right) - \frac{2i T' \Delta f(\zeta)}{\tau_0^2} \left[ 1 - \left( \frac{\rho_0/(1 - z)}{\rho} \right)^2 \right] \right\} \times \exp \left\{ \frac{2i T' \Delta f(\zeta)}{\tau_0^2 \rho} \left[ 1 - \left( \frac{\rho_0/(1 - z)}{\rho} \right)^2 \right] \right\} d\zeta, \quad (16)
\]

In these expressions, the azimuth SAR resolution \( \rho \) is given by

\[
\rho = \frac{R_0}{k V T'} \quad (17)
\]

and the velocity bunching coefficient is defined as

\[
h = \frac{\hat{U}_s R_0}{\rho V} \quad (18)
\]
The degraded azimuthal resolution is given by

\[ \rho_0 = \rho \left[ 1 + 4(1-\alpha)^2 \frac{V^2 T_0^2}{\rho^2} \right]^{1/2}. \tag{19} \]

It is clear from (19) that for SAR processing without the focusing correction, the azimuthal resolution is degraded as a result of the image smear due to the effect of the wave pattern movement with the phase velocity \( C \). The optimum azimuthal resolution of SAR/INSAR, which is determined by the effective integration time \( T' \) and is given by (17), is obtained for \( \alpha = \frac{1}{2} \). Any value of the focusing coefficient different from unity causes the degradation of the azimuthal resolution. In (16), the function \( f(\xi) \) is defined as

\[ f(\xi) = \xi - \frac{\hat{U}_r R_0}{V} \cos \left[ 2\pi \left( \frac{\xi \cos \phi}{\lambda x} - \frac{V - C_x X}{V \rho} \right) \right] \tag{20a} \]

and

\[ U_r(\xi) = \hat{U}_r \cos \left[ 2\pi \left( \frac{\xi \cos \phi}{\lambda x} - \frac{V - C_x X}{V \rho} \right) \right]. \tag{20b} \]

It follows from the model equations that the value of the focusing coefficient \( \alpha = \frac{1}{2} \) also corresponds to the minimum in the decay of the INSAR image magnitude due to the time shift \( \Delta t \). It can be easily seen that for \( \tau_s \ll T_0 \), the decay due to \( \Delta t \) nearly vanishes, and the INSAR signal intensity is practically identical to that of a regular SAR.

Expressions (15), (16) and (20) can be used to compute the complex INSAR output as a function of the focusing parameter \( \alpha \) for any ocean wave length, height or direction.

3. Results

The fixed values of the range distance \( R_0 = 10 \text{ km} \), the incidence angle \( \theta = 45^\circ \) and the platform velocity \( V = 200 \text{ m s}^{-1} \) are assumed in all model computations presented here for an L-band SAR with radar wave length 0.25 m. Simulations are performed by an ocean swell with wave length \( \lambda = 120 \text{ m} \); in most cases the wave height is assumed to be \( H = 1.2 \text{ m} \). The selected wave parameters closely resemble those observed during the TOWARD experiment (Brüning et al. 1991). In the computations depicted in figure 1, the scene coherence time \( \tau_s = 0.141 \text{ s} \) and the SAR integration time \( T_0 = 2 \text{ s} \). INSAR simulations are performed for the time delay \( \Delta t = 0.05 \text{ s} \), which corresponds to the antenna spacing and platform velocity of the NASA–JPL interferometric SAR, see Goldstein et al. (1989) and Marom et al. (1990, 1991). The phase \( \beta \) of the imaged wave which accounts for the scanning distortion is defined as

\[ \beta = \frac{V - C_x}{V} \frac{2\pi X}{\lambda_x} \] \tag{21}
Figure 1(a).

Figure 1(b).
Figure 1(c).

Figure 1(d).
Figure 1. Simulations of INSAR and SAR outputs of an ocean wave with length $\lambda = 120$ m and height $H = 12$ m for various values of the focusing parameter $z$; $T_o = 2$ s, $\tau_s = 0.141$ s. (a) INSAR magnitude for $\cos \phi = 1.0$. (b) INSAR phase component for $\cos \phi = 1.0$. (c) INSAR magnitude for $\cos \phi = -1.0$. (d) INSAR phase component for $\cos \phi = -1.0$. (e) Regular SAR output for $\cos \phi = 1.0$. (f) Regular SAR output for $\cos \phi = -1.0$. 
Simulations of a regular SAR image of an identical wave system are given in figure 1(e) for $\cos \phi = 1.0$ and in figure 1(f) for $\cos \phi = -1.0$.

The results in figure 1 are presented for 4 values of the focusing coefficient $\alpha$. The computations were performed for a case when the matched filter is set at the platform velocity, $V = W$, which according to (14) corresponds to $\alpha = 1.073$ for $\cos \phi = 1$, and $\alpha = 0.936$ for $\cos \phi = -1$, for an optimum focusing according to the existing time-dependent models, $\alpha = 1$, as well as for two additional values around $\alpha = 1$, i.e., $\alpha = 0.085$ and $\alpha = 1.15$. The phase component of the complex INSAR output given in figures 1(h) and (d) is compared with the undistorted ideal INSAR phase output $4U, k \Delta \cos \beta$, which is directly proportional to the local instantaneous velocity. It appears that for both propagation directions, matched filter setting with $\alpha = 1$ indeed provides an image which has an enhanced contrast, with the apparent wave height exceeding the ideal INSAR phase value by about 5 per cent for both propagation directions. When no focus adjustment is applied, the output contrast is decreased notably, and the apparent wave height is below the exact value. The relative error for the matched filter velocity $W = V$ is higher than for the optimal focusing case, being about 12 per cent. The relatively high relative error in the INSAR estimate of the ocean wave height can be attributed to the nonlinear effects resulting from the high value of the velocity bunching coefficient, $b = 1.52$. When SAR processing is performed with focusing coefficient values which differ notably from unity, both the image contrast and the accuracy of the wave height estimate are considerably decreased.

Note that although the focusing correction is related to the phase velocity of the ocean swell, it only allows increasing the contrast of the signal, but does not affect the distortion of the image wave length as a result of the finite scanning velocity $V$. The apparent wave length in the image is longer than $\lambda$ for $\cos \phi > 0$, and shorter than $\lambda$ for $\cos \phi < 0$, the scanning distortion ratio in both cases being $(1 - C_2/V)$, in agreement with Tajirian (1988). The fact that the focusing correction is not related to the scanning distortion can be easily seen from (20). Note also that for all values of $\alpha$, the zero-crossing points in the INSAR phase component image coincide, while the asymmetry of this signal varies strongly with $\alpha$.

Contrary to the phase component, the absolute value of the INSAR signal, presented in figure 1(a) and (c), as well as the regular SAR images (figures 1(e) and (f)) remain symmetrical for all $\alpha$. Generally speaking, the behaviour of the INSAR signal magnitude is quite similar to the regular SAR output. For the wave parameters selected, the image contrast for INSAR is slightly better than that of SAR. For both SAR and INSAR, processing with $\alpha = 1$ yields image contrast which is higher than that obtained for the alternative values of the focusing coefficients. For both propagation directions, processing with $\alpha = 1.15$ results in a dip at the output maximum of the INSAR absolute value, while no such dip is observed for a regular SAR. This dip in figures 1(a), (c) can again be attributed to the quite high value of the velocity bunching coefficient for the selected ocean wave parameters, and thus essentially non-linear imaging.

The ocean swell propagation direction relative to that of the platform affects not only the apparent wave length in the images due to the scanning distortion, but also the symmetry of the phase component of INSAR, cf. figures 1(h) and (d). Variation of the wave propagation direction results in corresponding change in the effective ocean wave length in the azimuth direction, $\lambda_a = \lambda / \cos \phi$. The change of the sign only of the $\cos \phi$, however, does not affect the value of the velocity bunching coefficient,
as defined by (18), as well as that of other parameters which determine the SAR or INSAR output for a given value of $\beta$, see (16) and (20). In the absence of the real aperture radar modulation, the range of variation of both the phase and the magnitude components of the INSAR output, as well as that of a regular SAR, along the imaged wave, does not depend on whether the wave propagates towards the platform or with the platform. The simulation results are therefore presented in the following figures only for waves propagating with the platform, i.e., for $\cos \phi > 0$.

The SAR and INSAR simulations presented in figure 1, as well as those reported elsewhere (Shemer and Kit 1991, Shemer 1993) indicate that for INSAR antenna spacing $B$ which results in $\Delta t \ll \tau_s$, the absolute value of the INSAR image and the regular SAR output are qualitatively similar with minor quantitative deviations. Thus, for the sake of brevity the study of the effects of various parameters on the focusing sensitivity of the SAR and INSAR imagery of ocean wave will be restricted to INSAR simulations.

In both SAR and INSAR imagery of the ocean surface, the information on the dominant wave systems is obtained by performing the Fourier analysis of the images. In order to estimate the performance of SAR or INSAR in the ocean surface imagery, by using computations based on the current theoretical model, the amplitudes of the dominant harmonic of the simulated images of a monochromatic ocean wave were calculated. The focusing curve for the given in each figure SAR and wave parameters represents the dependence of the resulting amplitudes on the focusing parameter $x$. Most focusing theories indicate that the width of the focusing curve decreases with the SAR integration time $T_0$ (Hayt et al. 1990). The effect of $T_0$ on the INSAR focusing curves is studied for the selected swell with $\lambda = 120$ m and $H = 1.2$ m for two propagating directions: with the platform, $\cos \phi = 1$, in figures 2(a) and (b), and in the close to the range direction, $\cos \phi = 0.33$, figures 2(c) and (d). The scene coherence time in all simulations summarized in figure 2 is $\tau_s = 0.141$ s.

The magnitude of the INSAR output depends quite strongly on the duration of integration $T_0$, similarly to the regular SAR behaviour. In order to facilitate the comparison of the model results for different values of $T_0$, the dominant harmonic amplitudes in all focusing curves presented are normalized by the corresponding averaged along the imaged ocean wave values. These mean INSAR output magnitudes which serve as normalization factors, can also be obtained as a DC component in the Fourier analysis. For the phase component of INSAR, it seems reasonable to prefer a different normalization procedure. The accuracy of the amplitude of the phase component of INSAR at various values of the focusing parameter $x$ can be best estimated by using the amplitude of the ideal INSAR phase component, $4Ux\Delta t$, as a normalization factor. The deviation of the normalized amplitude of the INSAR phase component from unity represents the error of the INSAR measurement of the ocean wave height.

For an azimuthally propagating wave, the width of the focusing curves decreases sharply with $T_0$ both for the magnitude (figure 2(a)) and the phase (figure 2(b)) of the INSAR output. This is in agreement with the generally accepted SAR focus sensitivity dependence on the integration time duration (Hayt et al. 1990). Note that for the selected wave length $\lambda = 120$ m, the value of the focusing coefficient $x$ calculated from (1) for the focus setting corresponding to the platform velocity, $W = V$, is $x = 1.073$ for $\cos \phi = 1.0$, and $x = 0.936$ for $\cos \phi = -1.0$. Since the
azimuthal resolution $\rho$ is only a weak function of $T_0$, see (13) and (17), the velocity bunching coefficient for the selected in figure 2(a) and (b) wave length and height varies only slightly from $b=1.41$ for $T_0=0.5$ s to $b=1.52$ for $T_0=5.0$ s. The imaging procedure for all simulations of figure 2 is thus moderately nonlinear.

The focusing curves are not quite symmetrical with respect to the location of the maximum. For short integration time, $T_0=0.5$ s, the INSAR output is only weakly dependent on the focusing coefficient. The maximum image contrast for this value of $T_0$ is attained for $x=0.98$ in figure 2(a) and for $x=1.04$ in figure 2(b). For longer integration times, the maximum of the focusing curves is attained at even closer to the unity values; at $T_0=1$ s, the location of the maximum is at $x=0.99$ in figure 2(a) and at $x=1.01$ in figure 2(b); the deviation of the maximum location from $x=1.0$ becomes negligible for $T_0=2$ s and $T_0=5$ s. The maximum attainable image contrast, in the vicinity of the optimum focus setting, is essentially independent of $T_0$ for both the magnitude and the phase components of INSAR. For the conditions of figure 2(b), the amplitude of the dominant harmonic of the phase component remains below the exact value for all focusing parameters. The error in determination of the wave amplitude is thus minimum at the location of the maximum image contrast. The both criteria, i.e., the maximum INSAR phase image contrast, and the minimum error in wave height determination, result here in the optimum focus setting at the same value of $x$.

When the swell propagates close to the range direction ($\cos \phi=0.33$ in figures 2(c) and (d)), the length of the imaged waves in the azimuthal direction is much higher, $\lambda_x=\lambda/\cos \phi=360$ m. The velocity bunching coefficient is also higher due to larger amplitude of the radial velocity component of the surface scatterers $U_r$, see (8a). The value of the velocity bunching coefficient now varies from $b=1.95$ for $T_0=0.5$ s, to $b=2.09$ for $T_0=5.0$ s. As in figures 2(a) and (b), the focus sensitivity in figures 2(c) and (d) increases with $T_0$, but the shape of the focusing curves, in particular for the INSAR magnitude, figure 2(c), changes notably as compared to that obtained for the azimuthally propagating wave. For shorter integration times, the maximum of the focusing curves in figure 2(c) is now shifted to the values of the focusing coefficient notably below unity: at $x=0.77$ for $T_0=0.5$ s, and at $x=0.91$ for $T_0=1.0$ s. At higher values of $T_0$, the location of the maximum approaches $x=1$: for $T_0=2$ s the INSAR magnitude image contrast is maximum at $x=0.98$, while at $T_0=5$ s it attains its limiting value $x=1.0$. Note that at $x=1.0$, the INSAR magnitude image contrast is identical for all integration times $T_0$.

Contrary to the focusing curves of figure 2(c), the maximum of the dominant harmonic of the INSAR phase image is located much closer to $x=1.0$ for all integration times. The variation of the phase image contrast with the focusing parameter is extremely weak for short integration times, with the maximum attained at $x=1.05$ for $T_0=0.5$ s and at $x=1.01$ for $T_0=1.0$ s. As in figure 2(b), all focusing curves in figure 2(d) are located below unity, so that the location of maximum image contrast also corresponds to the minimum error in the INSAR estimate of the ocean wave amplitude. In the vicinity of the optimum focus setting, which is thus uniquely determined, this error is much less than 1 per cent for the conditions of figure 2(d) and is thus negligible.

The effect of the scene coherence time $\tau_s$ is studied in figure 3. The focusing curves are presented in this figure for the same length and height as in figure 2, for two propagating directions and for three values of $\tau_s$ in the range $0.1$ s $\leq \tau_s \leq 0.2$ s, which corresponds to the scene coherence time actually observed in the ocean (cf.,
Figure 2. INSAR focusing sensitivity as a function of SAR integration time. Wave conditions and $t_s$ as in figure 1. (a) INSAR magnitude for $\cos \phi = 1.0$. (b) INSAR phase component for $\cos \phi = 1.0$. (c) INSAR magnitude for $\cos \phi = 0.33$. (d) INSAR phase component for $\cos \phi = 0.33$. 
Figure 3. INSAR focusing sensitivity as a function of the scene coherence time $\tau_s$. Wave conditions and $\tau_s$ as in figure 1; SAR integration time $T_0 = 1$ s. (a) INSAR magnitude for $\cos \phi = 1.0$. (b) INSAR phase component for $\cos \phi = 1.0$. (c) INSAR magnitude for $\cos \phi = 0.33$. (d) INSAR phase component for $\cos \phi = 0.33$. 
e.g. Shemer and Marom 1993). Qualitatively, all curves in figure 3 look similar. The image contrast decreases with increasing $\tau_S$ for the absolute values as well as for the phase INSAR images. For an azimuthally propagating wave ($\cos \phi = 1-0$), the maximum of the focusing curve for the magnitude image, Figure 3(a) is located at $\alpha$ varying from 0.99 to 1.0, while for $\cos \phi = 0.33$, Figure 3(c), the INSAR output magnitude image contrast is notable below that obtained in figure 3(a) and the optimum focus setting corresponds to $\alpha \approx 0.9$.

The maximum amplitude of the INSAR output phase component (figures 3(b) and (d)) is obtained at $\alpha \approx 1.0$ for both propagation directions and for all values of the scene coherence time. For higher values of $\tau_S$, the relative phase is below unity for all $\alpha$, and thus the INSAR measurement gives the ocean wave height which is somewhat below the exact value. The relative error has a minimum at $\alpha \approx 1.0$; it is about 5 per cent in figure 3(b), and less than 1 per cent in figure 3(d). For short coherence time, $\tau_S = 0.1$ s, the value of the maximum relative phase exceeds unity by few percent. If the optimum focus setting for the INSAR phase image is seen as the value of $\alpha$ minimizing the error in determination of the wave amplitude, it then corresponds to $\alpha = 0.92$ or $\alpha = 1.1$ in figure 3(b), and to $\alpha = 0.83$ or $\alpha = 1.25$ in figure 3(d).

We now proceed to the study of wave amplitude effects on the INSAR output for various focus settings. The INSAR focusing curves are presented in figure 4 for a number of ocean wave heights. The maximum wave height considered for both $\cos \phi = 1-0$ (figures 4(a) and (b)) and $\cos \phi = 0.33$ (figures 4(c) and (d)) is $H = 1.5$ m. This wave height corresponds to the velocity bunching coefficients of $b = 1.87$ and $b = 2.57$, respectively. For these conditions, the spectra of both the absolute value and the phase components of the complex INSAR output have dominant harmonics which correspond to the imaged swell wave length. For higher ocean wave heights, the strongly nonlinear imaging mechanism results in a totally distorted output signal. Moreover, for wave heights $H > 2.5$ m for $\cos \phi = 1-0$, and $H > 1.8$ m for $\cos \phi = 0.33$, the amplitude of the ideal INSAR phase component exceeds the value of $\pi$, beyond which the INSAR imagery of waves ceases to be effective, see Shemer and Kit (1991).

It is clear from the examination of the curves of figure 4(a) ($\cos \phi = 1-0$) and figure 4(c) ($\cos \phi = 0.33$) that the focus sensitivity of the magnitude component of the INSAR output increases with the wave amplitude. For an azimuthally propagating wave, the value of the focusing coefficient $\alpha$ where the image contrast is maximum remains around unity and varies slightly with the wave height from $\alpha = 0.95$ for $H = 0.25$ m, to $\alpha = 1.01$ for $H = 1.5$ m. The situation is different in figure 4(c), where for low amplitude waves, the image contrast decreases monotonously with $\alpha$. The focusing sensitivity of the INSAR imagery of these waves is, however, quite weak. For a steeper wave, $H = 1.5$ m, the maximum of the focusing curve is attained at $\alpha = 0.95$, and the focusing sensitivity of the output becomes stronger.

The situation is not very different when the phase component of the INSAR output is considered. For an azimuthally propagating wave shown in figure 4(b), the maximum image contrast is always attained at $\alpha \approx 1.0$, the exact location of the maximum shifting from $\alpha = 0.99$ for $H = 0.25$ m, to $\alpha = 1.02$ for $H = 1.5$ m. For $H = 1$ m, the relative error in wave amplitude determination at the optimum focus setting is essentially zero. For a steeper wave, INSAR phase image provides wave heights which are underestimated, while for $H < 1$ m, the INSAR-based measurements at close to the optimum focus setting will provide wave amplitude estimates
which can exceed the actual value by few per cent. Considerable relative error, exceeding say 10 per cent, however, is only obtained for waves of vanishing amplitude. When ocean wave propagation direction is close to radial, figure 4(d), the INSAR phase component is practically insensitive to the focus setting, and the INSAR estimate of the ocean wave amplitude appears to be quite accurate.

4. Discussion and conclusions

In the present study the INSAR model developed by Shemer and Kit (1991) was generalized to include the variable focus setting in the SAR processing. This modified model makes it possible to obtain simulations of SAR and INSAR images of a monochromatic ocean swell for any given operational conditions. Some examples of the resulting simulations for both regular and interferometric SAR were presented in figure 1. These simulations demonstrate that the output of a regular SAR is quite similar to the magnitude component of INSAR for the chosen radar parameters and ocean wave field. This observation allows restricting the presentation to the magnitude and the phase components of the interferometric SAR. The conclusions regarding the focus behaviour of the regular SAR can be drawn from the focusing curves which show the dependence of the contrast of the INSAR magnitude on the focusing parameter $\alpha$.

An important feature of the theoretical model is that it accounts for the temporal variability of the imaged surface and is accurate to the third order in the coherent time $\tau$, see (11), thus fully incorporating the effects of the surface acceleration. In the process of deriving the model equations, the dimensionless focusing coefficient $\alpha$ as defined by (15) appears in a natural way. It thus is advantageous to use $\alpha$ instead of the generally accepted focus setting $\Delta V = V - W$. The introduction of the focusing coefficient $\alpha$ into the model expressions describing the INSAR image of the ocean swell, (15), (16) and (20), makes it obvious that the value of $\alpha = 1.0$ is special in the sense that it results in the optimum azimuthal resolution, as well as in the minimum decay of the INSAR output magnitude due to the time shift $\Delta t$. The focus setting $\Delta V$ which corresponds to $\alpha = 1.0$ is also suggested by Kaslingam and Shemdin (1988), Lyzenga (1988) and Raney and Vachon (1988) as giving the maximum SAR image contrast.

The results of the present simulations, generally speaking, lead to a conjecture that the value of $\alpha = 1$ constitutes the optimal focus setting not only for a conventional SAR and the absolute value component of the complex INSAR output, but also when focusing the INSAR phase images is considered. It was shown that one can define the optimum value of the focus coefficient $\alpha$ with respect to the INSAR phase images based on two alternative considerations, i.e., the image contrast, or the accuracy of the wave height measurement. The maximum image intensity of the INSAR phase component is always obtained in close vicinity of $\alpha = 1$. At this focus setting, the wave amplitude determination by INSAR is also usually most accurate, with minimum relative error. There exist, however, conditions when the INSAR-based measurements with the focus setting corresponding to $\alpha = 1$, lead to overestimate of the actual wave height. This can occur, e.g., for very short scene coherence times $\tau_s$, figure 3(b), or for low amplitude waves propagating in the azimuth direction, figure 4(b). In such occasions, though, the measurement error obtained when the processing is performed with $\alpha = 1.0$ is still relatively insignificant. The conclusion can thus be made that even under these operational conditions, the value of $\alpha = 1.0$ can be recommended as the desirable focus setting.
Figure 4. INSAR focusing sensitivity as a function of the ocean wave height. Scene coherence time $\tau_S=0.141$ s, SAR integration time $T_0=1$ s, wave length $\lambda=120$ m.

(a) INSAR magnitude for $\cos \phi=1.0$. (b) INSAR phase component for $\cos \phi=1.0$.
(c) INSAR magnitude for $\cos \phi=0.33$. (d) INSAR phase component for $\cos \phi=0.33$. 

$H=2.0\text{m}$
$H=1.0\text{m}$
$H=0.5\text{m}$
The INSAR focus sensitivity was studied for various scene coherence times $\tau_S$, SAR integrations times $T_n$, as well as ocean wave directions and amplitudes. The focus sensitivity increases sharply with $T_n$, see figure 2, in agreement with the results obtained elsewhere (Kasilingam and Shemdin 1990, Hayt et al. 1990). The variation of the sea state, which manifests itself in the scene coherence time $\tau_S$, affects both the magnitude and the phase component of INSAR at all values of the focusing coefficient $\alpha$ in a similar way, figure 3.

While the present computations support in general the result of the so-called 'time-dependent' models that the optimum focus setting is determined by (1), it appears that the complicated nature of the SAR processing makes this result somewhat oversimplified. In particular, the focusing curves presented in this study show that the maximum image contrast is obtained at close to $\alpha=1.0$, but not necessarily at exactly this value. The deviation of the optimum focus setting from unity may sometimes be quite meaningful, especially for waves which have low amplitudes and significant wave length component in the azimuth direction, $\lambda_x$, see figure 4. The optimum focus setting for given conditions also may differ for the magnitude and for the phase components of INSAR, and is dependent, among the other parameters, on the imaged wave height. This wave height dependence of the optimum focus setting has been stressed in particular by the so-called velocity bumping model (Brüning et al. 1991).

References


