The requirement of a uniformly bounded convergence time for all initial conditions is important for modern control theory. In particular the requirement appears in design of hybrid or switched systems. It can happen in such systems that a dynamic system repeatedly changes its configuration, and the configuration only lasts for some bounded time with its lower estimation known in advance. The natural requirement in that case is to get the desired system behavior before the next structural change takes place. Therefore, the exact regulation is to be achieved in uniformly bounded time irrespectively of the initial state at the time of the structural change. The corresponding transient convergence is called uniformly exact convergence (Angulo, Moreno, Fridman, 2011, 2012).

The problem is further aggravated in the case when the system features significant uncertainty. Sliding mode (SM) control is considered to be effective in coping with heavy uncertainty conditions. The corresponding approach is based on the exact keeping of a properly chosen function (sliding variable) at zero by means of high (theoretically infinite) frequency of control switching. In the case when the sliding variable has the sense of a tracking error, the established sliding mode corresponds to exact output regulation.

While the traditional SM approach deals with sliding variables of the relative degree one, the higher-order SM (HOSM) methods are applicable for any relative degree \( r \), and the corresponding sliding mode is said to have the sliding order \( r \). The control strategy consists of the reaching phase, when the sliding variable is forced to vanish, and of the sliding mode itself, when it is kept at zero. Respectively the reaching phase can also be considered as the finite-time stabilization of an uncertain feedback-linearizable system of the dimension \( r \).

While the problem is relatively simple with the relative degree (dimension) 1, already with \( r = 2 \) the problem turns to be non-trivial. Meantime uniform exact convergence has only been obtained for observers (Angulo, Moreno, Fridman, 2011). Only convergence into some vicinity of second-order sliding-mode has been obtained for the problem of output regulation with \( r = 2 \) (Angulo, Moreno, Fridman, 2012). The goal of this paper is to develop a uniformly exact second-order sliding-mode controller effective for dynamic systems of the relative degree 2 under uncertainty conditions.

The system is described by the equation

\[
x' = a(t,x) + b(t,x)u, \quad \sigma = \sigma(t,x),
\]

where \( x \in \mathbb{R}^n \), \( a, b \) and \( \sigma \): \( \mathbb{R}^{n+1} \to \mathbb{R} \) are unknown smooth functions, \( u \in \mathbb{R} \) is the control, \( n \) might be also uncertain. The output \( \sigma \) has the sense of the output regulation error. The task is to get \( \sigma = 0 \).

All differential equations are understood in the Filippov sense, which allows discontinuous dynamics. The system relative degree \( r \) is assumed to be constant and known, which implies that

\[
\sigma^{(r)} = h(t,x) + g(t,x)u,
\]

where \( h(t,x) = \sigma^{(r)} \big|_{\sigma = 0}, \quad g(t,x) = \frac{\partial}{\partial u} \sigma^{(r)} \neq 0 \) are some unknown smooth functions. It is supposed that

\[
0 < K_m \leq g(t,x) \leq K_M, \quad |h(t,x)| \leq C
\]

for some \( K_m, K_M, C > 0 \). In the considered case the full state is assumed to be known.

The idea is to use the well-known twisting controller (Levant 1993) having the form
\[
    u = \begin{cases}
        -\alpha \text{ sign} \sigma \text{ with } \sigma \dot{\sigma} > 0, \\
        \frac{1}{\mu} \alpha \text{ sign} \sigma \text{ with } \sigma \dot{\sigma} > 0,
    \end{cases}
\]

where \( \mu > 1 \). The trajectory of the system in the plane \( \sigma, \dot{\sigma} \) revolves around the origin \( \sigma = \dot{\sigma} = 0 \) and "twisting" converges to it in finite time, performing the infinite number of rotations on the way.

The idea is to assign such value of \( \alpha \) at each rotation that the resulting sum of the rotation times would be bounded by a prescribed value. The corresponding modified controller obtained from (4) is naturally called accelerated twisting controller. The resulting convergence trajectory and the graph \( \sigma(t) \) are shown in Fig. 1.

![Graphs showing convergence to zero](image)

**Fig. 1:** Convergence of the output to identical zero.

**References.**

