Experimental realization of structured super-oscillatory pulses

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Abstract: We demonstrate experimentally a generic method for the synthesis of optical femtosecond pulses based on Gaussian, Airy and Hermite-Gauss functions, which are transformed to exhibit fringes with tunable width. The width of the fringes is set in some cases to be much narrower than the inverse of the spectral bandwidth. Such pulses might be useful for ultrafast spectroscopy, coherent control and nonlinear optics.

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References and links

1. Introduction

Manipulating the amplitude and phase of optical pulses is important for numerous applications such as spectroscopy, metrology [1], optical communications [2], coherent control [3] and microscopy [4].

In recent years there is a growing interest in modifying the electric field of optical radiation in the form of super oscillating functions [5]. Super-oscillation is an interference phenomenon in which several Fourier modes interfere together to create local oscillations which are faster than the highest Fourier modes in the superposition. Such signals are known since the 1950’s in the context of Antenna theory [6], but the interest in these functions was revived in the 1980’s due to their relevance to quantum weak measurements [7]. Following a suggestion for the relevance of the phenomenon to optical super-resolution [5] many works followed, utilizing super oscillations in imaging [8–11], nonlinear optics [12], beam manipulation [13–16], electron microscopy [17] and particle manipulation [18]. Most of the works regarding optical super oscillations utilized the spatial degree of freedom, relating to the creation of super oscillating beams.

In the time domain, we would like first to mention two previous works which demonstrated temporal sub-Fourier focusing of optical pulses without employing super-oscillations. The first one, in 2005, applied a cosine amplitude and a Gaussian phase modulation, to generate a prominent central lobe which is about 30% narrower than the associated Fourier limit width [19] (which is proportional to the inverse of the spectral bandwidth). A year later another experimental work [20] used a Gaussian spectral phase to break the Fourier limit by 20%. It was first suggested to use super-oscillation in the time domain to deliver fast local oscillations through an absorbing resonance of a dielectric material [21]. A theoretical design for the generation of a temporal super oscillatory pulse was numerically demonstrated in 2016 [22]. Recently the first experimental demonstration of utilizing super oscillating pulses for realizing temporal super resolution was carried out [23]. The super oscillating function used in that work, although allowed for flexibility in setting the local rate of the superoscillation, relied on a very specific functional form based on superposition of frequency beats.

In this work we demonstrate numerically and experimentally a simpler, more generic and more flexible method for creating a variety of structured super oscillating pulses having arbitrarily short features (within technical limitations discussed below) which can be tuned in a controlled manner and are based on known functional forms such as Gaussian, Airy and Hermite-Gauss pulses. The modulation we employ is a simple \(\pi\)-phase shift over a finite bandwidth with its
width being a control parameter. It is well known for many years that a $\pi$-phase step can turn a transform limited Gaussian pulse to an Hermite-Gauss pulse [24], which is wider but contain faster oscillations. However, for an arbitrary pulse such a transformation would not yield faster oscillations (as a trivial case consider that such a modulation would turn an Hermite-Gauss pulse back into a Gaussian pulse). In contrary to that, the method we present behaves in a completely different manner, as it allows to create, within technical limits, arbitrarily fast modulations in any functional form. Following the wide range of uses optical super oscillations has found in the spatial domain, the current method might be relevant to various applications utilizing ultrashort pulses.

2. Theory

Consider the case in which an ultrafast pulse whose spectral envelope given with $F(\omega) = A(\omega) \exp (i\phi(\omega))$ centered around a carrier frequency $\omega_0$, is phase modulated by the transformation $\phi(\omega) \rightarrow \phi(\omega) + \phi_M(\omega)$ with:

$$\phi_M(\omega) = \begin{cases} \pi, & |\omega - \omega_0| < \omega_\pi \\ 0, & else \end{cases},$$

(1)

where $\omega_\pi$ is half the $\pi$ phase-shift bandwidth. The time domain realization of the phase modulated spectrum $F(\omega) \exp (i\phi_M(\omega))$ results, for some values of $\omega_\pi$, in a narrow feature on the envelope of the pulse [12] [25]. By tuning $\omega_\pi$ the feature will become arbitrarily narrow and would super-oscillate with the cost of lower intensity. A completely equivalent amplitude-only form of a spectral distribution $F(\omega)$ equipped with the $\pi$ phase-shift $\phi_M(\omega)$ is the following:

$$F(\omega) \exp (i\phi_M(\omega)) = F(\omega) \left[ 1 - \frac{2}{\pi} \phi_M(\omega) \right].$$

(2)

It is clear from Eq. (2) that the $\pi$ phase modulation creates a negative offset of the spectral amplitude over the modulated spectral region. In order to understand why this $\pi$ phase shift generates a narrow temporal feature, let us consider a simple case based on modulating a Gaussian pulse as shown in Fig. 1. Starting with the envelope (its spectrum is centered around frequency zero [see Fig. 1(a)], and subtracting from it the part dictated by the modulation [Fig. 1(b)] leads to a fast oscillation in the middle of the envelope in the time domain [Fig. 1(c)]. Finally, we shift the spectrum to the optical frequency $\omega_0$ [Fig. 1(d)].

We emphasize that the method we outline here is flexible and can be used to generate many types of super oscillating pulses based on functions other than the Gaussian function. To demonstrate this we now consider three different functions. In the first case $F_G(\omega)$ is a flat phase Gaussian around the carrier frequency $\omega_0$ [see Fig. 2(a), Theory]:

$$F_G(\omega) = \exp \left( -\frac{(\omega - \omega_0)^2}{\sigma_G^2} \right),$$

(3)

where $\sigma_G$ defines the spectral width. Applying the phase modulation of Eq. (1) results with a new and narrow central lobe in the time domain. For a relatively small range of $\omega_\pi$ the central lobe is stronger compared with its side lobes and narrower in comparison with the initial Gaussian envelope [see Fig. 2(b), Theory]. Increasing further $\omega_\pi$, the central lobe becomes narrower, yet weaker in intensity with respect to the side lobes [see Fig. 2(c)-(d), Theory].

In the second case $F(\omega)$ is a $HG_{10}$ Hermite-Gauss function around the carrier frequency $\omega_0$:

$$F_{HG}(\omega) = H_{10} \left( \frac{\sqrt{2} (\omega - \omega_0)}{\sigma_H} \right) \exp \left( -\frac{(\omega - \omega_0)^2}{\sigma_H^2} \right),$$

(4)
Fig. 1. $\pi$ phase modulated Gaussian pulse. (a) Zero-centered Gaussian spectral distribution (left) and its corresponding Gaussian pulse envelope in time (right). (b) Subtracted low frequencies band (left), corresponding to a subtracted wide function in time (right). (c) Spectral distribution after subtraction of the low frequencies band (left) and the corresponding super oscillatory function in time (right). Note that much of the function is now shifted below zero due to the negative low frequencies band (purple areas) leading to the generation of a superoscillatory region (red area). (d) Spectral shifting the spectrum to frequency $\omega_0$ (left) modulates the envelope in the time domain with a carrier frequency (right). Red dashed lines show the absolute value of the envelope in the time domain.
Fig. 2. Gaussian pulse with different π phase modulation widths $\Delta \lambda_{\pi,G}$ - theory (left) vs. measurement (right) (a) No π phase modulation. (b) $\Delta \lambda_{\pi,G} = 5.5 \pm 1 nm$. (c) $\Delta \lambda_{\pi,G} = 7.4 \pm 1nm$. (d) $\Delta \lambda_{\pi,G} = 8.1 \pm 1nm$. (e) $\Delta \lambda_{\pi,G} = 25.2 \pm 1nm$. In the frequency domain - amplitude is shown with a continuous blue line and phase with a dotted red line.

$\sigma_H$ defines the spectral width, $H_1$ is the first order Hermite polynomial) which is an $HG_{10}$ Hermite-Gauss function in the time-domain as well. Applying the π phase modulation in a relatively narrow spectral range results in a pair of inner lobes which are narrower in comparison with the initial Hermite-Gauss profile and the appearance of weaker, wider, outer lobes [see Fig. 3(b), Theory]. As in the previous case of the Gaussian pulse, these inner lobes become narrower but also weaker as $\omega_{\pi}$ increases. The pulse shaping scheme shown here can be extended by using higher order Hermite-Gauss functions to generate pulses with more than two super-oscillatory lobes [18].

In the third and final case $F(\omega)$ has a flat amplitude and a cubic spectral phase around the carrier frequency $\omega_0$:

$$F_A(\omega) = \text{Rect}(\omega - \omega_0, \omega_R) \exp \left( i k_A (\omega - \omega_0)^3 \right),$$  \hspace{1cm} (5)

where $\text{Rect}(\omega, \omega_R)$ is 1 for $|\omega| \leq \omega_R/2$ and zero elsewhere, $\omega_R$ denotes the spectral width of the rectangle function and $k_A$ is an arbitrary cubic phase coefficient. In the time domain the envelope is in the form of a truncated Airy function [26]. The additional π phase modulation results in energy transfer from the originally primary Airy lobe to both sides - to the adjacent lobe originally present (i.e. the originally Airy secondary lobe) and to a newly formed lobe on the other side [14]. The originally secondary lobe, which is narrower than the originally primary lobe, gets relatively stronger [see Fig. 3(d), Theory]. The originally primary lobe clearly gets narrower as the width of the phase modulation increases and its intensity drops, exhibiting the hallmarks of super oscillations.
Fig. 3. HG\textsubscript{10} Hermite-Gauss and Airy pulses with different $\pi$ phase modulation widths $\Delta_\pi$ - theory (left) vs. measurement (right) (a) HG\textsubscript{10}. No $\pi$ phase modulation. (b) HG\textsubscript{10}, $\Delta_\pi_{HG} = 9.8 \pm 1nm$. (c) Airy, No $\pi$ phase modulation. (d) Airy, $\Delta_\pi_{A} = 5.5 \pm 1nm$. In the frequency domain - amplitude is shown with a continuous blue line and phase with a dotted red line.

3. Results

Numerical simulations

To verify that the applied transformation indeed can result with the emergence of super-oscillatory features, we use here a short time Fourier analysis [18, 21] of the original pulse fields vs. their $\pi$ modulated version. Figure 4 shows a short time Fourier analysis performed using Gaussian windows (shown with dotted red lines) centered at the location of the narrow lobes resulting from the $\pi$ phase modulation. The Gaussian, Hermite-Gaussian and Airy pulses are initially set with a bandwidth having Full Width Half at Maxima (FWHM) of 17nm, 38nm, 60nm respectively. The $\pi$ phase modulation widths applied for the corresponding cases are 7nm, 23nm and 22nm. The Gaussian windows are set to have FWHM values which are equal to the distance between the zeros of the narrowest lobe (two central lobes for the case of the Hermite-Gauss pulse) resulting from the $\pi$ phase modulation. The short time Fourier spectrum for all considered cases - initial pulses in the form of a Gaussian in Fig. 4(a), an Hermite-Gauss in Fig. 4(b) and an Airy function in Fig. 4(c) - clearly shows local higher frequency content due to the transformation which is the hallmark of superoscillation. Note that the increased content at high frequencies for the $\pi$ modulated functions is larger than accounted for by the Gaussian window acting on the original function. At this point we would like to remark that we applied our procedure for both symmetric pulses (Gaussian, Hermite-Gauss) and also for an asymmetric pulse (Airy). For symmetric pulses (which are symmetric both in the temporal and in the spectral domains) we naturally centered the $\pi$ phase modulation around the symmetry axis of the pulse spectrum. This guarantees that the generated pulse would still be symmetric. Shifting the location of the modulation would destroy the symmetry. As we verified numerically, we can still achieve super oscillating features with the modulation, but generally the optimal width of the modulation is dependent upon its location relative to the spectrum of the input pulse.
Fig. 4. Short-time Fourier transform analysis to verify the emergence of super-oscillatory features. (left) Original normalized Gaussian (a) Hermite-Gauss (b) and Airy (c) pulses in time domain (dot-dashed blue line), corresponding normalized $\pi$ modulated pulses (continuous black line) and the applied Gaussian window (dotted red line) used in the short time Fourier transform. (middle) Normalized pulse after the application of the Gaussian window. (right) Frequency domain representation of the Gaussian-cut functions. The bandwidth’s full width half maximum for all initial pulses were: $\text{FWHM}_G = 17 \text{nm}$ (for the Gaussian pulse), $\text{FWHM}_{HG} = 38 \text{nm}$ (for the Hermite-Gaussian pulse) and $\text{FWHM}_A = 60 \text{nm}$ (for the Airy pulse) while the $\pi$ phase modulation width applied for the different cases were: $\Delta \lambda_{\pi,G} = 7 \text{nm}$, $\Delta \lambda_{\pi,HG} = 23 \text{nm}$ and $\Delta \lambda_{\pi,A} = 22 \text{nm}$ correspondingly.

Experimental results

In our experiment, we use a home-made Pulse Shaper [27] for synthesizing the pulses and a custom-made Frequency Resolved Optical Gating (FROG) [28] for characterizing the pulses. The FROG was built using a 50 : 50 beam splitter, a 50$\mu$m BBO Second Harmonic Generation crystal, a 0.1$\mu$m step linear motor stage, and an off-axis parabolic mirror having a reflected focal length of 4$''$. The pulse shaper was built using a pair of 35cm focal length cylindrical mirrors, and a pair of 1200 lines/mm holographic gratings. At the Fourier plane we used a 640 pixel, dual-mask Spatial Light Modulator (Jenoptik SLM-S640d). The laser source used in the experiments was Coherent Vitara-T Ti:Sapphire laser. Figure 5 depicts a detailed schematic of our experimental setup.

At the first stage of the experiment we have set the pulse shaper to produce a Gaussian spectrum with FWHM of 14.3 ± 1nm. Figure 2(a)(Measurement) shows the time domain FROG reconstruction for this case - a Gaussian envelope having FWHM of 58 ± 5 fs. (Note that for every measurement appearing in a graph we also show the expected theoretical form for comparison). Next we used the pulse shaper to apply the additional $\pi$ phase modulation profile to the Gaussian spectrum around the central wavelength of $A_0 = 778 \pm 5nm$. The additional phase modulation profile was set with several widths: $\Delta \lambda_{\pi,G} = \{5.5, 7.4, 8.1, 25.2\} \pm 1nm$. The temporal envelope was reconstructed for each case using the FROG apparatus [see Fig. 2(b)-(e)(Measurement)]. The modulation has resulted with the creation of a new central lobe in the time domain. The FWHM of the center lobe were measured to be $\Delta t = \{36.5, 33.8, 31.0, 132.7\} \pm 5 fs$ for the corresponding $\Delta \lambda_{\pi,G}$ above. For the case of $\Delta \lambda_{\pi,G} = \{5.5\} \pm 1nm$ the middle lobe is narrower in comparison to the original Gaussian envelope by approximately a factor of 1.6. As $\Delta \lambda_{\pi,G}$
Fig. 5. Experimental setup. The pulses emitted by an ultra-fast laser oscillator are shaped in a 4f Fourier domain pulse shaper. The shaped pulses amplitude and phase are retrieved through a measurement in a FROG apparatus. M=Mirror, CM=Cylindrical Mirror, G=Grating, BS=Beam Splitter, PM=Off-axis Parabolic Mirror, SHGC=Second-Harmonic-Generation Crystal, B=Beam Blocker.

increases, the middle lobe becomes increasingly narrower and less intense in comparison to the two outer lobes. Note that further increasing the phase modulation width would finally results in the elimination of the narrow central lobe [see Fig. 2(e)].

At the second stage of the experiment we have set the pulse shaper to produce a $HG_{10}$ Hermite-Gauss profile. Figure 3(a)(Measurement) shows the time domain FROG reconstruction for this case - a $HG_{10}$ Hermite-Gauss shaped envelope. The FWHM of each of the main envelope’s lobes was measured to be $63.6 \pm 5\, f\sec$. Next we have modulated the Hermite-Gauss spectral shape with the additional $\pi$ phase modulation profile around $\lambda_0 = 793 \pm 5\, nm$ with several widths: $\Delta \lambda_{\pi,HG} = \{9.8, 13.9, 15.9\} \pm 1\, nm$ and reconstructed the corresponding temporal shape of the pulses. The case of $\Delta \lambda_{\pi,HG} = 9.8 \pm 1\,nm$ can be seen in Fig. 3(b)(Measurement). In all cases the modulation has resulted with the creation of a pair of new narrower inner lobes, i.e. in comparison to the original Hermite-Gauss pulse (with no additional phase modulation), which get narrower and less intense as the $\pi$ phase modulation gets wider. The FWHM of each of the inner lobes was measured to be $\Delta t = \{49.8, 44.5, 37.1\} \pm 5\, f\sec$ for the corresponding $\Delta \lambda_{\pi,HG}$ above. At the final stage of the experiment we have set the pulse shaper to produce a rectangular amplitude function whose width is $56 \pm 1\, nm$ accompanied with a cubic phase profile $\phi(\lambda) = c_0 (\lambda - \lambda_0)^3$, where $c_0 \equiv 10^{24} [1/m^3]$ and $\lambda_0 = 770 \pm 5\, nm$. Figure 3(c)(Measurement) shows the time domain FROG reconstruction for this case - an Airy function envelope. The FWHM of the main envelope’s lobe was measured to be $90.8 \pm 5\, f\sec$. Next we have modulated the Airy pulse with the $\pi$ phase modulation function around $\lambda_0 = 770 \pm 5\, nm$ using several widths:
\( \Delta \lambda_{\pi,A} = \{2.2, 5.5, 10.2\} \pm 1 \text{nm} \) and reconstructed the corresponding temporal shape of the pulses. The case of \( \Delta \lambda_{\pi,A} = 5.5 \pm 1 \text{nm} \) is shown in Fig. 3(d). As the \( \pi \) phase modulation width increased energy is transferred from the originally primary lobe to both the originally secondary lobe and a newly generated lobe. The originally primary lobe became narrower with following FWHM values: \( \Delta t = \{80.1, 33.6, 15.2\} \pm 5 \text{fs} \) corresponding to the set of \( \Delta \lambda_{\pi,A} \) given above. The FWHM of the originally secondary lobe were measured to be approximately \( \Delta t = 48 \pm 5 \text{fs} \) for all cases, while its intensity increased. Further increasing the of the phase modulation width eventually resulted in the transfer or loss of energy from the originally secondary lobe into the originally tertiary lobe.

The discrepancies between the experimental results and theoretical waveforms are due to technical limitations in our pulse-shaper, mainly the existence of residual spatio-temporal coupling [29]. We note that the spectral resolution of our pulse-shaper \( d\omega = 1.15 \times 10^{12} \text{ Rad}/\text{Sec} \) easily obeys the required Nyquist-Shannon sampling criterion \( 2\pi/d\omega > 2T \) as the longest pulse duration \( T \) in our experiment was below 1 picosecond. This means we have no distortions due to sampling-induced aliasing. Any system producing super-oscillating waveforms has technical limitations in its ability to create fast super-oscillations. As a rule of thumb - a super-oscillatory function would be destroyed when the amplitude noise on its constructing (Fourier) modes is larger than the amplitude at the super-oscillation, and when the phase noise for a spectrum of width \( \Omega \) is larger than \( \pi \Omega/\omega_{SO} \) where \( \omega_{SO} \) is the local frequency of the super-oscillation (please refer to Ref. [21] for details). We can interpret these rules as constraints on the required amplitude and phase resolution of the SLM in our setting. For example, the phase resolution should be better than \( \pi \Omega/\omega_{SO} \).

4. Discussion and conclusions

To conclude, we have demonstrated numerically and experimentally the synthesis of complex femtosecond pulses which are the result of applying a finite width spectral \( \pi \) phase modulation to known functional forms. For each of the cases we studied (i.e. Gaussian, Airy and Hermite-Gauss pulses) the modulation generates narrow temporal features, where for some cases these features present local frequencies which exceed the highest component in the available spectrum, realizing a super-oscillation. There are technical limits in any physical system for its ability to create fast super-oscillating features. These limits are mostly dependent on the available resolution and fidelity at which the phase and amplitude of the desired waveforms can be synthesized. A relevant analysis for this case, considering what phase and amplitude noise would destroy a given super-oscillating feature was given in reference [21]. We have shown that the width of the narrow temporal features is tunable and can easily be made faster by optimizing the spectral width of the \( \pi \) phase modulation (at the expense of lower amplitude). Such a manipulation, can be advantageous to other known techniques [23], in that it allows applying a very simple transformation to any functional form for achieving a super oscillating feature in time domain. The simplicity of the transformation allows to replace the SLM used in our experiment with a thin transparent \( \pi \) phase step plate when working with a regular Gaussian pulse. The resulting generation of temporally structured narrow features can be useful for many applications of femtosecond pulses such as coherent control and nonlinear optics.

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