Double Fano resonance in a plasmonic double grating structure

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Abstract: It is shown theoretically and numerically that a simple gratings-based plasmonic structure can support a nearly-degenerate double Fano resonance which can lead to a relatively narrow spectral line shape. The double-resonance spectral location and line-shape are controllable by either adjusting the periodicity and unit-cell of the gratings or by adjusting the angle of incidence of the incoming radiation.

References and links
1. Introduction

Asymmetric resonances are currently the subject of considerable research efforts in photonic and plasmonic nanostructures [1–3]. Historically, asymmetric profiles were found in rare gas spectra [4] and were explained by Fano [5] by taking into account the electron energy dependence on the interaction between a discrete autoionized state and the continuum. A similar resonance was discovered by Feshbach when studying nuclear reactions [6]. In both cases the unique line shape is the result of interference between two pathways - one involving direct scattering to a continuum and the other a transition to the continuum through a metastable discrete bound state. The universality and ubiquitous nature of Fano-Feshbach profiles suggests that a generalization of the formalism to include several discrete states or several continua might be relevant to different multi-resonance systems. Indeed, Fano’s original work [5] considered such generalizations, while similar analysis under the Fano or Feshbach models followed [7–9]. Such treatments can easily be simplified to the case of two discrete state coupled to the same continuum [4, 10, 11].

This last case where two bound states are coupled to the same continuum is particularly interesting because it can result in an extremely narrow line-shape, which is important for applications such as sensing and slow light. In some such cases the generated line-shape is identical to the one associated with the phenomenon of Electromagnetically Induced Transparency (EIT) [12]. To date, most nanophotonic structure supporting a double Fano-Feshbach resonance (double FFR) are made of resonant structures associated with local oscillations (that is - nano-antenna based structures) [13–20], including a super-cell grating structure based on Fabri-Perot resonances, which was experimentally demonstrated in the radio-frequency regime [21].

In this work, we suggest a simple plasmonic structure, based on propagating Surface Plasmon Polaritons (SPPs), to generate a double FFR line-shape. The proposed structure is designed as an asymmetric Insulator-Metal-Insulator (IMI) configuration, with a periodic grating on each metal-insulator interface. Each grating couples radiation modes to either the SPP mode at the top or at the bottom metal-insulator interface (see Fig. 1). The asymmetric design guarantees the existence of two distinct SPP modes with different momentum at the energy degenerate case,
Fig. 1. (a) A simple scheme for generating a double Fano-Feshbach resonance (double FFR). An asymmetric IMI structure, whose metallic layer is shown in the inset, is characterized by two dispersion curves (continuous and dashed lines) for the two metal-dielectric interfaces of the structure. For nearly energy-degenerate double FFR line shape, two gratings are etched into the interfaces allowing mode coupling of two nearly degenerate radiation modes (whose dispersion line is denoted with a dot-dash line) to corresponding plasmonic modes at the two interfaces. The gratings have periodicities which are inversely proportional to the momentum mismatch denoted with a continuous and a dashed double-arrow lines. (b) The structure geometry. The double gratings geometry is characterized with periodicities \( \Lambda_1, \Lambda_2 \), duty cycles \( F_1, F_2 \) and thicknesses \( h_1, h_2 \). An intermediate metallic layer of thickness \( h_3 \) between the gratings serves as a backbone. Indices of refraction of the two dielectric materials are \( n_1 \) and \( n_2 \). The structure is designed for incoming (outgoing) radiation at an angle \( \theta_1 (\theta_2) \).

coupled to the same continuum. When the two coupled SPP modes are non degenerate, this configuration ensures the formation of a double FFR. For radiation impinging at a given angle, the specific line-shape is dependent upon both the gratings’ periods and the unit-cell configuration (depth and duty cycle of the corrugations). Thus, the form of the double FRR line-shape is easily tunable, allowing our proposed structure to provide for high-performance plasmonic devices. If the periods are such that the bounded SPP modes are nearly energy-degenerate, the line-shape can posses a narrow feature, whose narrowness is limited by the losses in the metal and the existence of other decay channels.

2. Results

In order to produce simultaneously two different SPP discrete states we propose an asymmetric IMI structure. For the simulations carried in this work we specifically chose silver (Ag) surrounded on its upper side by air and on its bottom side by sapphire (Al\(_2\)O\(_3\)) [22]. To couple these two SPP bounded states to a single continuum of incoming/outgoing radiation the two metal-dielectric interfaces were periodically corrugated with a period calculated using the usual SPP excitation condition [23]:

\[
\frac{2\pi n_1 \sin(\theta_1)}{\lambda} + m \frac{2\pi}{\Lambda_i} = \pm \frac{2\pi}{\lambda} \sqrt{\epsilon_m \epsilon_i} \quad i = 1, 2 \quad m = 0, \pm 1, \pm 2, \ldots
\]

with the incoming light coming from side \( i = 1 \). Here \( i = 1 (2) \) relates to the top (bottom) dielectric-metal interface, \( \lambda \) is the optical wavelength in vacuum, \( n_1 \) is the top dielectric material index of refraction, \( \theta_1 \) is the incident angle (see Fig. 1(b)), \( \epsilon_m \) is the metal permittivity, \( \epsilon_i \) are the dielectric materials permittivities and \( \Lambda_i \) are the period of the corrugations, matching the radiation
with the SPP excitation at interfaces $i = 1, 2$. We chose the following parameters for our proposed device: incident angle $\theta_1 = 15^\circ$, diffraction order $m = -1$ ($m = 1$) and negative (positive) sign of the right-hand-side in Eq. 1 for the upper (bottom) corrugation profile, and $\lambda = 800[\text{nm}]$ as the designated degenerate wavelength. This last parameter is the wavelength at which at the given incidence angle both SPP modes would be excited. This also indicates the resonance frequencies of the bound states of the two plasmons [23, 24] excluding inherent frequency shifts associated with Fano-type line shapes [5, 24]. For an incoming wave formally described with a temporal positive phasor of type $e^{i\omega t}$, at this wavelength, the materials permittivities are given by: $\epsilon_m = -24 - 1.85i$, $\epsilon_2 = 3.0276$ and $\epsilon_1 = 1$ [22]. For these values the corrugations periodicities on each interface are found to be $\Lambda_1 = 626[\text{nm}]$ and $\Lambda_2 = 506[\text{nm}]$. In our simulations we chose slightly different values of $630[\text{nm}]$ and $500[\text{nm}]$ to accommodate an integer number of periods of both gratings into an overall structure length of $63[\mu\text{m}]$. Optimization based on numerical simulations (all simulations were carried using the commercial COMSOL multiphysics software package) yielded the corrugation thickness on the air side to be $h_1 = 50[\text{nm}]$ and on the sapphire side $h_2 = 40[\text{nm}]$ and their duty cycle to be $F_1 = 0.878$ and $F_2 = 0.94$ respectively. For structural integrity, an additional unbroken metal layer separates between both corrugations. The thickness of this layer was chosen to be $h_3 = 10[\text{nm}]$ in order to minimize the total power losses inside the metal but still be thick enough to allow for possible fabrication. We note that the metal layer is thin enough to enable coupling the incoming radiation from the top side using the bottom corrugation.

Spectral characterization of the field transmission is carried with the parameter $|S21| = \sqrt{\frac{\text{Transmitted Power}}{\text{Incident Power}}}$ for a TM incident polarization field. At first we considered two simpler structures: one structure having a single corrugation on its upper side while the second structure having the corrugation on its lower side (see Fig. 2(a-b)). As one can see, in both cases, the interference between the direct scattering to the continuum indirect channel through the single bound discrete SPP state results in a standard asymmetric FFR line-shape [25–30]. (We note that the small resonance at around $660\text{nm}$ in Fig. 2(b) is due to a third order diffraction of the incoming light from the top grating to the bottom interface. It is irrelevant to the discussion at hand.) Both FFR line shapes in Fig. 2(a-b) can be matched to the usual form [5, 7, 10, 31–34]:

$$T(\kappa) = |S_{21}|^2 \propto \frac{(\kappa + q_r)^2 + q_i^2}{1 + \kappa^2}$$

(2)

where $\kappa = \frac{\omega - \omega_R}{\Gamma}$ is the reduced energy, $q_r$ - describes the degree of the asymmetry of the line shape, $q_i$ - describes the intrinsic losses, $\Gamma$ is the spectral line-width, and $\omega_R$ is the (shifted) resonance frequency [5, 32]. We note that the original derivation by Fano used a real asymmetry parameter $q$. This parameter is extended to a complex number $q = q_r + iq_i$ to account for losses through its imaginary part [33].

When designing a grating structure to support a Fano resonance the parameters of the grating can be mapped to the parameters of the line-shape [35]: the resonance of the line-shape $\omega_R$ is obviously dictated by the period of the grating while the asymmetry parameter $q_r$ is dictated by the unit-cell configuration of the grating. The reason for this is that (generally speaking) the asymmetry parameter describes the relative coupling strength of the incoming radiation to the SPP state and to the scattered radiation state, and these couplings are determined by the shape of the grating’s unit-cell.

The matching of the line shapes parameters that fit Eq. (2) for the lossless case, were extracted from each of the two single FFR cases, shown in Fig. 2(a-b), by first solving three equations involving the three unknown parameters $\omega_R$, $\Gamma$ and $q_r$. The first equation, relates the maximum location of Eq. (2) to $\omega_R$, the second one relates the minimum location of Eq. (2) to $q_r$ and third one the relates the location of one half of the maximal value of Eq. (2) to $\Gamma$.  

The parameters resulting from solving the three equations are then used as an initial guess for a subsequent least-square curve-fitting optimization process to match the simulated line-shapes to the form given with Eq. (2). This optimization results in the parameters given in Table 1 for the two separate IMI structures; one having the corrugation only on its top side and the other having the corrugation only on its bottom side. In addition, for normalizing the amplitude of the analytic line shapes to the numerical results, the maximal value of each single FFR line-shape was normalized to the maximal value of the corresponding peak in the numerically simulated single FFR line-shape.

Table 1. FFR Profile Parameters For The Lossless Model Of Two Different Structures Having A Single Grating

<table>
<thead>
<tr>
<th>i</th>
<th>Γ[PHz]</th>
<th>ω_R[PHz]</th>
<th>q_r</th>
<th>λ_R[nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>2.29</td>
<td>-4.25</td>
<td>823</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
<td>2.47</td>
<td>10.34</td>
<td>763</td>
</tr>
</tbody>
</table>

These parameters are given by least-squares optimization fitting the data of the numerical simulations to Eq. (2); i = 1(2) is for the structure with top (bottom) corrugation.

For a double FFR there are two discrete bound modes which are coupled to the same continuum. Now when radiation is transmitted through a device supporting such a configuration it can be described as the contribution of three terms interfering together: a direct scattering path and two indirect paths through the bound states. It is straightforward to derive the line-shape in this case through an implicit analysis given in Fano’s original work [5]. The double FFR line shape is described with two asymmetry parameters [7, 11]:

\[
T(ω) \propto \left( 1 + \frac{q_1 Γ_1}{ω - ω_R_1} + \frac{q_2 Γ_2}{ω - ω_R_2} \right)^2 + \left( \frac{q_1 Γ_1}{ω - ω_R_1} + \frac{q_2 Γ_2}{ω - ω_R_2} \right)^2
\]

we note that for \( q_1 = q_2 = 0 \) this line shape is identical to the line-shape associated with EIT [36].

For our proposed device, we now combine two separated corrugations - each on one of the sides of the metal layer, so each corrugation is coupling the radiation mode to a different SPP mode. The transmission indeed results in a double FFR line-shape (see the continuous line in Fig. 2(c)). The simulated line-shape (still without losses) is now fitted to Eq. (3) using again least squares optimization where the initial guess for the parameters were taken as the single FFR line shape parameters given in Table 1. This procedure results in the parameters given in Table 2.

Table 2. Double FFR Profile Parameters For The Lossless Model For The Double-Grating Structure

<table>
<thead>
<tr>
<th>i</th>
<th>Γ[PHz]</th>
<th>ω_R[PHz]</th>
<th>q_r</th>
<th>λ_R[nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>2.29</td>
<td>-11.6</td>
<td>823</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>2.46</td>
<td>12.3</td>
<td>766</td>
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</table>

These parameters are given by least-squares optimization fitting the data of the numerical simulations to Eq.(3); i = 1(2) stands for resonance associated with the top (bottom) corrugation.
Comparing between these two tables we see that the overall width (associated with $\Gamma$) of the two resonances as well as their central frequency ($\omega_R$) do not change significantly when two separate resonances are engineered together into the same device. However the asymmetry parameters $q_r$ value changes significantly. This might be due the possibility that the bottom (top) SPP is now better coupled to the impinging (transmitted) radiation as there is less metal between them, which enlarges the asymmetry parameter. However, we note that the actual relations between the line-shape parameters of the two separated systems to the system containing both resonances is not straightforward or trivial as now there can be interactions between the resonances (via direct coupling between the SPP modes or via indirect SPP-radiation-SPP interaction). For example, with a single Fano resonance better coupling leads to increased line-shift of the resonance frequency from the frequency of the bound state [5, 24]. Here with the two separate resonances combined into the same system, such a shift is not observed, despite the increased coupling.

Figure 2(c) clearly shows the difference between the double FFR line-shape which is constructed by applying the line-shape parameters of the two separate single FFR devices (dotted...
line) to the double FFR line shape (dashed line) that best fits the actual line-shape of the device (continuous line). When the intrinsic losses of the metal are included in the modelling, it changes the parameters of the line-shapes. Generally, the transmission of the field will be a little bit weaker and the width of the resonances would increase. The absolute value of the asymmetry parameter would also decrease (or be "damped" see Ref. [24, 32]). In addition the minima in the line-shape would increase. These can be clearly seen in the line shapes of the single FFR devices depicted in Fig. 3(a-b) (continuous line) when compared to the lossless case seen in Fig. 2(a-b). We used the same procedure as for the lossless-case to extract the single FFR line-shape parameters that best fit the simulated line-shapes for the single grating devices. The extracted parameters are given in Table 3. The single FFR line-shape with these parameters are shown in Fig. 3(a-b) (dashed line). When the two gratings are combined into the same device, apart from the asymmetry parameters, the parameters extracted to best-fit the double FFR line-shape (given in Table 4) are relatively similar to the parameters extracted for the two different single grating structures (given in Table 3), as was with the lossless models. Although the asymmetry parameters are still different, the differences are slightly less severe as was with the lossless case. This is reflected in the fact that the best-fit line-shape (Fig. 3(c) dashed line) and the line-shape with the parameters of the two single-grating devices (Fig. 3(c) dotted line) are quite similar. This similarity might be attributed to reduced interaction between the top and bottom SPP modes compared with the lossless case.

Table 3. FFR Profile Parameters For The Lossy Model Of Two Different Structures Having A Single Grating

<table>
<thead>
<tr>
<th>i</th>
<th>$\Gamma [\text{PHz}]$</th>
<th>$\omega_R [\text{PHz}]$</th>
<th>$q_r$</th>
<th>$q_i$</th>
<th>$\lambda_R [\text{nm}]$</th>
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<td>2.46</td>
<td>3.35</td>
<td>0</td>
<td>766</td>
</tr>
</tbody>
</table>

*The parameters are given by least-squares optimization fitting the data of the numerical simulations to Eq. (2); $i = 1(2)$ is for the structure with top (bottom) corrugation.*

Table 4. Double FFR Profile Parameters For The Lossy Model For The Double-Grating Structure

<table>
<thead>
<tr>
<th>i</th>
<th>$\Gamma [\text{PHz}]$</th>
<th>$\omega_R [\text{PHz}]$</th>
<th>$q_r$</th>
<th>$q_i$</th>
<th>$\lambda_R [\text{nm}]$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-1.33</td>
<td>9</td>
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<td>2.44</td>
<td>7.78</td>
<td>4.69</td>
<td>772</td>
</tr>
</tbody>
</table>

*These parameters are given by least-squares optimization fitting the data of the numerical simulations to Eq. (3); $i = 1(2)$ stands for resonance associated with the top (bottom) corrugation.*

The double FFR line-shape can be easily modified by changing the periodicity of one of the gratings. This way one of the resonances can be "scanned" over the other resonance. This is demonstrated in a series of simulations (depicted in Fig. 4, where the period of the top corrugation is set in steps which moves the top SPP resonance from lower to higher wavelengths compared with the resonance of the bottom SPP. The vertical dashed lines in this figure denote the wavelengths at which Eq. (1) is satisfied for the different cases for both the top corrugation.
Fig. 3. Single FFR and double FFR resonances for the lossy case. Numerical simulations of the transmission profile (continuous line) and best-fit FFR line-shapes (dashed line) for: (a) a single FFR for a device with a bottom corrugation having a periodicity of $\Lambda_2 = 500\,[\text{nm}]$ (b) a single FFR for a device with a top corrugation having a periodicity of $\Lambda_1 = 630\,[\text{nm}]$ (c) a double FFR for a double-grating device with periodicities of $\Lambda_1 = 630\,[\text{nm}]$ ($\Lambda_2 = 500\,[\text{nm}]$) at the top (bottom) side. The dotted line represents Eq. (3) with the parameters that fits the two separate single FFR devices.
Fig. 4. Relative shift of resonances. Numerical simulations (continuous line) of the double FFR line-shape (including losses) when the location of top SPP resonance is scanned over the position of the bottom SPP resonance by changing the top corrugation periodicity to be: (a) $\Lambda = 510$ nm (b) $\Lambda = 530$ nm (c) $\Lambda = 560$ nm (d) $\Lambda = 580$ nm (e) $\Lambda = 600$ nm (f) $\Lambda = 620$ nm. The dashed line represents the simulated single FFR line shape for a structure containing only the top corrugation. The vertical red (blue) dashed line marks the wavelength at which Eq. (1) is satisfied for the different grating periods for the structure containing only the top (bottom) corrugation.
Fig. 5. The effect the incidence angle have on the double FFR line-shape for different incident angles for a structure with $\Lambda_1 = 630\,[\text{nm}]$ at the top side and $\Lambda_2 = 500\,[\text{nm}]$ at the bottom side (a) $\theta_{\text{inc}} = 14^\circ$  (b) $\theta_{\text{inc}} = 15^\circ$ (c) $\theta_{\text{inc}} = 16^\circ$ (d) $\theta_{\text{inc}} = 17^\circ$. The vertical red (blue) dashed line marks the wavelength at which Eq. (1) is satisfied for the different grating periods for the structure containing only the top (bottom) corrugation.

and associated SPP and the bottom corrugation and associated SPP. These wavelengths are not the resonant wavelengths as the actual resonant frequencies are shifted from these values due to the interplay with other channels in the system (such as direct scattering or interaction between the bound states) [5, 24]. The case of degeneracy where the actual resonant frequencies are the same $\omega_{R1} = \omega_{R2}$ is depicted in Fig. 4(d). As predicted by theory the double FFR profile is reduced in this case to a single FFR line-shape having a single zero. The interesting cases, application-wise are the nearly-degenerate cases shown in Fig. 4(e-f). These cases exhibit features which are narrower than the corresponding single Fano resonance given by a single-grating device (compare the narrower features shown with the continuous lines compared with the features given with the dashed lines). Changing the unit-cell configuration would also change the form of the line-shape as these modify the asymmetry parameters. However, we do not have a simple model which explains how the unit-cell configuration is mapped to specific values of the asymmetry parameters. Thus if one wishes to achieve a specific goal, such as minimizing the width of the prominent feature of the line-shape, numerical optimizations should be used.

Lastly, we simulated the dependence of the double FFR line-shape on the incident angle of the impinging radiation. The results are shown in Fig. 5. Obviously, modifying the incident angle changes the wavelengths which are coupled to SPP modes. As the two SPP dispersion curves are different, this change in the coupled wavelength would be different for each SPP. As a result, scanning the incident angle has a similar effect to a change in the periodicity of one of
the gratings, so one resonance can be scanned over its counterpart in the double FFR line-shape. As such, the spectral response of our proposed structure is tunable - it can be manipulated by modifying the angle of incidence of the incoming light.

3. Conclusions and discussion

In this work we suggested and numerically demonstrated a geometrically simple asymmetric IMI structure able to support a double FFR spectral line-shape. The overall line-shape is determined by the periodicities of the gratings at the metal-insulator interfaces of the structure, and also by their unit-cell configuration. The location of the resonances is related to the gratings periodicities as was shown by our simulations. The asymmetry parameters are determined mainly by the shape of the unit-cell of each grating. Here a straight forward mapping between these shapes to the asymmetry parameter does not exist, and so to get a desired exact line-shape one needs to use optimization procedures in the available parameters space. The fact that the double FFR line-shape generally exhibit sharper features compared to the single FFR line-shape, together with the possibilities of tuning its features, and the simplicity of the structure, should make it relevant for many possible applications, such as sensing, field enhancement and slow-light devices [2].

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