Observation of optical backflow

YANIV ELIEZER,† THOMAS ZACHARIAS,† and ALON BAHABAD*

Department of Physical Electronics, School of Electrical Engineering, Fleischman Faculty of Engineering, and Center of Light-Matter Interaction, Tel-Aviv University, Tel-Aviv 69978, Israel

*Corresponding author: alonb@eng.tau.ac.il

Received 1 July 2019; revised 5 December 2019; accepted 10 December 2019 (Doc. ID 371494); published 15 January 2020

Quantum backflow is a counterintuitive phenomenon in which a forward-propagating quantum particle propagates locally backwards. The actual counter-propagation property associated with this delicate interference phenomenon has not been observed to date in any field of physics, to the best of our knowledge. Here, we report the observation of an analog optical effect, namely, transverse optical backflow where a beam of light propagating to a specific transverse direction is measured locally to propagate in the opposite direction. This observation is relevant to any physical system supporting coherent waves.

1. INTRODUCTION

Quantum backflow (also known as retro-propagation) is a surprising phenomenon, first pointed out in 1969 [1–3] by Allcock in the context of the time-of-arrival problem in quantum theory. Allcock found that a local quantum probability current may become negative even for positive momenta quantum states, and thus cannot be a valid measure for the time of arrival. Further advances with regards to the time-of-arrival problem were made by Muga et al. [4–6]. The phenomenon was studied in detail in 1994 by Bracken and Melloy [7] who found a limit on the total amount of backflow. This led them to introduce a new dimensionless quantum number whose value has been reproduced more accurately in subsequent years [8,9]. Recently, there has been a renewed interest in backflow with various studies reintroducing and exploring various aspects of the phenomenon [10,11].

A recent development in the field explored the relation between backflow and superoscillation [12]. Superoscillation is the phenomenon in which a band-limited signal locally oscillates faster than its fastest Fourier component. The phenomenon appeared first in the microwave community [13–15] and later was revived by Aharonov et al. in 1988 [16] while establishing the theory of quantum weak measurements. Berry and Popescu further developed the mathematical theory of the phenomenon and proposed to use it to realize far-field sub-wavelength optical focusing [17,18]. This suggestion was verified experimentally shortly afterward [19]. Various experimental works involving superoscillations, which appeared in the last two decades, include super-resolution microscopy [20–22], optical beam shaping [23–25], nano-focusing of light [26], particle trapping [27], electron beam shaping [28], nonlinear optical frequency conversion [29], as well as optical temporal super-resolution and sub-Fourier focusing [30,31]. A complementary phenomenon to superoscillation, termed suboscillation, where a lower-bound-limited signal oscillates locally slower than its lowest Fourier component, was discovered recently [32].

Berry [12] analyzed the evolution of backflow regions in the interference of quantum wavepackets. He demonstrated that these regions are dependent on the overall momenta distribution of the wavepackets and showed that the superposition of many waves creates wider and stronger backflow regions when the Fourier components are highly correlated. Moreover, he also found this phenomenon to be extremely vulnerable, since the evolution in space–time causes the destruction of the delicate phase relations critical for backflow. In 2013, Palmero et al. [33] proposed an experiment to detect quantum backflow by applying a Bragg pulse to a Bose–Einstein condensate. A recent work investigated the effect of reflection and transmission processes on backflow [34]. Importantly, the first experimental observation of the local momentum associated with backflow near optical superoscillatory foci was reported [35]. Still, to the best of our knowledge, no experimental observation of any actual backward movement associated with backflow in any wave system has been reported to date.

Here, we report on the observation of counter-propagation due to optical backflow. We construct a light beam, based on a spectrally shifted suboscillatory function [32], made of the superposition of modes having negative transverse momentum relative to a chosen axis of propagation. While the expectation value of the transverse momentum of the beam is negative (i.e., the beam travels at a negative angle relative to the chosen axis of propagation), in certain locations, the local value of the transverse momentum is positive. Isolating these regions with a slit causes the local transverse momentum to be “projected” onto the expectation value of the beam’s transverse momentum, realizing a beam propagating at an overall positive angle relative to the propagation axis. The equivalence of the paraxial optical wave equation to the Schrodinger equation (where the propagation coordinate plays the role of time) [36] ensures that our experiment reported below constitutes a reliable simulator for the quantum phenomenon.
2. THEORY

Consider the following backflow function built using a spectrally shifted suboscillatory function \( f_{\text{sub}}(\xi) \) in the coordinate \( \xi \):

\[
 f_{\text{BF}}(\xi) = f_{\text{sub}}(\xi) \exp(i L \xi) = \frac{\exp(i L \xi)}{[\cos(k_0 \xi) + i a \sin(k_0 \xi)]^N},
\]

where \( a \in [0 < \Re a \leq 1], \ N \in \{ N > 0 \}, \ k_0 \) is the fundamental spatial frequency of the suboscillatory function, and \( \exp(i L \xi) \) acts as a spectral shifting function. The Fourier transform of \( f_{\text{BF}}(\xi) \) is given with

\[
 F_{\text{BF}}(k) = 2\pi \sum_{m=-\infty}^{+\infty} C_m(a) \cdot \delta(k - L - mk_0),
\]

where \( k \) denotes the spatial frequency, \( C_m(a) \) are Fourier coefficients, and \( L \) is the spectral shift parameter. By applying the local frequency operator on Eq. (1), we get

\[
 k_{\text{local}}(\xi) = \frac{\partial \ln[f_{\text{BF}}(\xi)]}{\partial \xi} = L - \frac{Nak_0}{\cos^2(k_0 \xi) + a^2 \sin^2(k_0 \xi)},
\]

It is obvious that \( k_{\text{local}}(\xi) \in [L - \frac{Nk_0}{2}, L - Nk_0a] \). The global spectrum of \( f_{\text{sub}}(\xi) \) is a negatively valued single-sided spectrum with the highest harmonic at \( -Nk_0 \) [32] (which sets it as the slowest frequency component). For the function \( f_{\text{BF}}(\xi) \), the \( \exp(i L \xi) \) term shifts the highest available spectral frequency to \( M = -Nk_0a + L \). This implies that for a proper selection of the parameters \( L, N, k_0, \) and \( a \), it is possible to obtain a single-sided spectrum composed of only negative components \( M < 0 \) while having a local positive frequency \( (L - Nk_0a) > 0 \), which is the hallmark of the backflow phenomenon. Consider, for example, the Fourier coefficients of Eq. (1) for \( N = 3 \), which are calculated by complex integration to be

\[
 C_m(a) = \begin{cases} \frac{\sin(\frac{(a+1)\pi}{2})}{\sin(\frac{\pi}{2})} \cdot \frac{\sin^2(\frac{(a-1)\pi}{2})}{2}, & m \in \{ \text{odd} < 0 \}, \\ 0, & \text{otherwise}. \end{cases}
\]

Together with \( k_0 = 1 \) and \( L = 2 \), the spectrum in Eq. (2) is completely single sided, having only negative components, with the highest harmonic at \( M = -1 \). The local frequency, however, as defined by Eq. (3), is positive in the regions close to \( \xi = 2\pi n \) \( (n \in \mathbb{N}) \) for \( a < 2/3 \).

The backflow function in Eq. (1) is periodic and extends from \( -\infty \) to \( \infty \) and so cannot be used in an experiment. We therefore derive a bounded signal by convolving the Fourier transform in Eq. (2) with a narrow Gaussian spectral distribution. The resulting spectrum of this finite backflow function is given with

\[
 F_{\text{FBF}}(k) = \sum_{m=-p}^{-N} C_m(a) \exp\left(\frac{-(k - L - mk_0)^2}{2\sigma_0^2}\right),
\]

where \( C_m(a) \) are the coefficients defined in Eq. (4), \( L \) is the spectral shift parameter, \( k_0 \) is the fundamental spatial frequency, and \( \sigma_0^2 \) is the spectral variance of each harmonic. Since \( C_m(a) \) decay rapidly, we synthesize our function using a finite number of negative harmonics, where \( (-P) \) and \( (-N) \) represent the indices corresponding to the lowest and highest harmonics, respectively. A sufficiently narrow spectral variance is chosen such that theoretically only a negligible fraction \((3.82 \times 10^{-12})\) of the energy of the highest mode \( M \) is positive (practically ensuring a one-sided spectrum). Experimentally (see below), we realize our beam using discrete sampling of the theoretical smooth function for which there is no energy at all associated with the Gaussian envelopes at positive transverse frequency values. The resulting spectrum is shown in Fig. 1 (left) for several values of the parameter \( a \). The

![Fig. 1.](image-url)
in inverse Fourier transform of this spectrum, which corresponds to the finite backflow function $f_{BF}(\xi)$, is shown in Fig. 1 (center). Figure 1 (right) shows the local momentum calculated by Eq. (5) for each case. It can be seen that for the case of $a = 0.4$, while the entire spectrum is negative [Fig. 1(c) (left)], positive local momentum regions are clearly evident [Fig. 1(c) (right)].

3. EXPERIMENT

In our experiment (see Fig. 2), a reflective phase-only spatial light modulator (SLM, Holoeye Pluto) was used to realize finite backflow functions, set according to Eq. (5) for different values of the tuning parameter $a = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, with $P = 21$, $N = 3$, and $L = 0.733 \text{ mm}^{-1}$. The fundamental frequency and each harmonic spectral variance were set to $k_0 = 0.391 \text{ mm}^{-1}$ and $\sigma_0 = 0.0611 \text{ mm}^{-1}$, respectively. The resulting complex-valued spectral functions were encoded into a set of phase-only masks using a known phase-encoding technique [37]. The masks were equipped with a phase-only blazed grating (with period $\Lambda = 64 \text{ mm}$) whose first-diffraction-order optical axis is the propagation direction associated with the transverse frequency $k = 0$. The intensity of the encoded masks corresponding to the values of $a = 0.4, a = 0.7$, and $a = 1$ appear in Fig. 3 (left).

In the experiment, the laser beam (Quantum Ventus 532 Solo) was expanded and collimated before the SLM, reflected off it (embedded with the phase masks), and Fourier transformed using a 50 cm focal lens into the first focal plane where the finite backflow functions are realized. The intensity patterns of the beams in this plane were measured using a CMOS camera (Ophir Spiricon SP620U) and are presented in Fig. 3 (center). Next, the camera was removed from the first focal plane and replaced by a movable 100 mm wide slit mounted on top of a 25 mm long stepper motor linear stage (Newport MFA-PPD) that was set to move in steps of 5 mm along $\xi$. The spatially filtered beam in the first focal plane was Fourier transformed again by a 50 cm focal lens into the second focal plane. Multiple intensity images were taken at the second focal plane for different positions of the slit. The right column in Fig. 3 shows the beam intensity in the second focal plane averaged over the $y$ axis for different positions of the slit. Note that the application of an optical Fourier transform twice using two consecutive $2f$ systems results in a coordinate inversion. The curves in Fig. 3 (right) represent the expectation value of the beam’s position and hence describe the measured deflection of the beam in the second focal plane, relative to the propagation axis. These expectation values were calculated directly from the captured beam images (red continuous line), compared to an analytical expression based on the infinite backflow beam after it was slit-filtered (dashed blue line), and to a numerical calculation based on two consecutive optical Fourier transforms of the SLM patterns (dotted green line). All results show a very high level of agreement. Notice that we start in the SLM plane with a momentum representation of the backflow function and end back at a plane representing momentum. Thus, the beam deflection represents expectation values of the transverse momentum. Note that the exact location of the propagation (optical) axis at the second focal plane, associated with $k = 0$, was calibrated beforehand by applying a single slit mask to the center of the SLM and measuring the expectation value of the image at that plane.

A comparison of the expectation curve in the cases of $a = 1$ (a), $a = 0.7$ (b), and $a = 0.4$ (c) indicates that, as expected, the degree of deflection due to backflow increases as the value of $a$ decreases (and the beam does not contain backflow for $a = 1$). Note that the $a = 0.4$ case achieves the maximal backflow value, and the beam’s deflection completely crosses the propagation axis for certain positions of the slit, thus materializing local positive momentum.

It is also clear that a larger backflow entails lower intensity in the beam, which is a defining characteristic of super- and suboscillating functions [38–42]. We further define a quantitative backflow measure, $\mu_{BF}$, as the maximum ratio (over slit positions) of the power detected over negative values of the camera axis (Fig. 3, right) to the total detected power. The calculated values of this backflow measure are $\mu_{BF} = 0.34, 0.48, 0.54$ for $a = 1, 0.7, 0.4$, respectively. When $\mu_{BF} > 0.5$, the expectation value of the beam position crosses the propagation axis, and there is backflow.

In a sense, the filtering by the slit applied on the backflow beam realizes a nonlinear “projection” operation of a local property (local transverse momentum, which is not an eigenvalue of the momentum operator $\hat{k} \rightarrow -i\partial_\xi$) to a global property (eigenvalue of the momentum operator), thus allowing to observe the backflow as an actual deflection of the beam. More formally, our beams comprise a superposition of plane waves (eigenvectors of the momentum operator), and without the slit, measurement of the momentum amounts to a selection of one of these plane waves. With the slit, we first select the beam at a specific position and then measure the momentum. This no longer yields one of the momentum eigenvalues of the original beam, as it was changed by going through the slit, and the result can be a value outside the spectrum (set of all eigenvalues) of the original beam. This is related to the formalism of weak measurements [12,43]: the local momentum is considered as the result of a weak measurement, giving rise to a “weak value” observable $A_{\text{weak}}$ through the operation of the momentum operator $\hat{k}$ on a preselected state, which, in our case, the backflow beam $\psi$ (whose representation in momentum space is $|k\rangle\langle\psi| = F_{BF}(k)$ and in coordinate space $|\xi\rangle\langle\psi| = f_{BF}(\xi)$), and after post selecting with a coordinate state $|\xi\rangle$ (describing the position of the slit) [12],

$$k_{\text{local}}(\xi) = \text{Im} \left( \frac{\partial}{\partial \xi} \ln |\psi(\xi)| \right) = \frac{\text{Re} \left( \frac{\langle \xi | \hat{k} | \psi \rangle}{|\psi|} \right)}{|\psi|} = A_{\text{weak}}. \quad (6)$$

To examine the deflection’s sensitivity to the slit’s width, $W$, we have used a beam propagation simulation to calculate the beam’s deflection as a function of the slit’s position and width. Figure 4 shows the simulated beam image at the second focal plane,
Fig. 3. Experimental measurements. (Left) Generated SLM phase-only masks. Each line creates a propagating mode with a well-defined negative transverse momentum. The dotted line represents the center of the x axis, related to zero transverse momentum. (Center) Measured intensity distribution (in counts) in the first focal plane (backflow beam). The two dashed white lines represent the width of the slit. (Right) Measured beam image at the second focal plane, averaged over the y coordinate for each slit position. The dashed-dotted black line denotes the center of the propagation axis. Continuous red line: measured expectation value of the beam position (equal to momentum in the first focal plane). Dashed blue line: analytically calculated expectation value for a theoretical infinite periodic backflow beam after it is slit-filtered. Dotted green line: expectation value derived from Fourier transforming the SLM image and then Fourier-transforming again the slit-filtered image. (a) \( a = 1 \), (b) \( a = 0.7 \), and (c) \( a = 0.4 \).

Fig. 4. Intensity distribution for different slit widths. Simulated beam image at the second focal plane, averaged over the y coordinate for each slit position, for the case of \( a = 0.4 \) and for different values of the slit’s width: (left) \( W = 50 \, \mu\text{m} \), (center) \( 100 \, \mu\text{m} \), and (right) \( 200 \, \mu\text{m} \). The dotted green curve represents the expectation value of the beam’s position.

averaged over the y coordinate, for different values of the slit’s width: \( W = 50 \, \mu\text{m} \) (a), \( W = 100 \, \mu\text{m} \) (b), and \( W = 200 \, \mu\text{m} \) (c), all cases with the backflow parameter \( a = 0.4 \). The backflow measure \( \mu_{BF} \) is found to be 0.59, 0.54, 0.50 for \( W = 50, 100, 200 \, \mu\text{m} \), respectively. The green dotted curves represent the expectation value of the beam’s position along x. It is evident from the curves that the degree of backflow decreases as the slit width increases. Of course, reducing the slit width increases the variance in the position of the beam (its width), but the change in position of the center of mass of the beam (its expectation value) is due to the local momentum at the slit position.

4. CONCLUSION

We experimentally constructed, measured, and observed beam deflections associated with backflowing beams. We first designed a backflow beam based on the mathematical form of suboscillatory functions that were discovered recently. This allowed controlling the degree of backflow in the beam. Slit-filtering the generated beams allowed, counterintuitively, their deflection towards a direction opposite to that associated with the momentum states comprising the original beam. This effect is the result of a delicate interference phenomenon that, until now, hindered the observation of movement or deflection associated with backflow in any wave system (from quantum particles to optical waves to acoustic waves, etc.). Our results are also relevant to single photons, where each photon is in a backflow state comprising a superposition of different transverse momentum states. The backflow we demonstrated in this work is transverse optical backflow. It would be more challenging to demonstrate longitudinal optical backflow [12] along the axis of propagation of a light beam, which is more in line with the original concept of retro-propagation. In the future,
backflow and possible generalizations of the phenomenon might be used for unique applications. Controlling the far-field radiation patterns of light going through small apertures is an obvious direction. It can also be very interesting to use backflow for stand-off spectroscopy [44,45], where optical sensing of remote areas is based on information propagating backwards towards the source. It might also be relevant to nonlinear and ultrafast optics, especially if this phenomenon is manifested in the time domain, generating local negative frequencies.

Acknowledgment. We would like to thank Liran Hareli for fruitful discussions.

These authors contributed equally to this work.

REFERENCES

17. M. Berry, Quantum Coherence and Reality: In Celebration of the 60th Birthday of Yakir Aharonov (World Scientific, 1994).