# Binary Dirty MAC with Common State Information


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***Abstract*—The general two-user memoryless multiple-access channel, with common channel state information known to the encoders, has no single-letter solution which explicitly characterizes its capacity region. In this paper a binary "dirty" multiple-access channel (MAC) with "common interference", when the interference sequence is known to both encoders, is considered. We determine its sum-capacity, which equals to the capacity when full-cooperation between transmitters is allowed, contrary to the Gaussian case. We further derive an achievable rate region for this channel, by adopting the "onion-peeling" strategies which achieve the capacity region of the "clean" binary MAC. We show that the gap between the capacity region of the clean MAC and the achievable rate region of dirty MAC stems from the loss of the point-to-point binary dirty channel relative to the corresponding clean channel.**


## I. Introduction

Consider the two-user memoryless state-dependent multiple-access channel (MAC) with transition and state probability distributions

$$p(y|x_1,x_2,s) \quad \text{and} \quad p(s)\,, \tag{1}$$

where $s \in \mathcal{S}$ is known non-causally at both encoders, but not to the decoder. The channel inputs are $x_1 \in \mathcal{X}_1$ and $x_2 \in \mathcal{X}_2$, and the channel output is $y \in \mathcal{Y}$. The memoryless property of the channel implies that

$$p(\boldsymbol{y}|\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{s}) = \prod_{i=1}^{n} p(y_i|x_{1i},x_{2i},s_i). \tag{2}$$

The capacity region of this channel is still not known in general, and remains an open problem. See, e.g., [1]. Interestingly, this model appears to be a bottleneck in many wireless networks, ad hoc networks and relay problems.

The MAC model in (1) generalizes the point-to-point channel with side information (SI) at the transmitter considered by Gel'fand and Pinsker [2]. Gel'fand and Pinsker proved a direct coding theorem using a random binning technique, an approach widely used in the analysis of multi-terminal source and channel coding problems [3]. They obtained a general capacity expression which is given in terms of an auxiliary random variable $U$:

$$C = \max_{p(u,x|s)} \left\{ H(U|S) - H(U|Y) \right\}\,, \tag{3}$$

where the maximization is over all joint distributions of the form $p(u,s,y,x) = p(s)p(u,x|s)p(y|x,s)$.

Using this result, Costa [4] showed that in the Gaussian additive channel with known interference, the capacity is equal to that of the AWGN channel, i.e., as if the interference $S$ were not present. Nevertheless, this does not carry on to the binary modulo-additive case ("binary dirty-paper channel"):

$$Y = X \oplus S \oplus Z\,,$$

where $X,S,Z \in \mathbb{Z}_2$ and $\oplus$ denotes addition mod $-\,2$ (XOR). The input constraint is $\frac{1}{n} w_H(\mathbf{x}) \le q$, where $0 \le q \le 1/2$ (We shall refer to this constraint as "power constraint"), $w_H(\cdot)$ denotes Hamming weight, and $n$ is the length of the codeword. The noise $Z \sim \text{Bernoulli}(\varepsilon)$ is independent of $S,X$ (w.l.o.g. we assume $\varepsilon \le 1/2$) ; the state information ("interference") $S \sim \text{Bernoulli}\,(1/2)$ is known non-causally to the encoder. The capacity of this ("dirty") channel is equal to

$$C^{\text{P2P}}_{\text{Dirty}} = \text{uce}\Big\{ \max\left\{ H_b(q) - H_b(\varepsilon), 0 \right\} \Big\}\,, \tag{4}$$

where $H_b(\cdot)$ denotes the binary entropy [3] and $\text{ucre}$ is the upper convex envelope operation with respect to $q$ ($\varepsilon$ is held constant). The capacity of the interference-free ("clean") channel (the binary symmetric channel with a Hamming input constraint), given by

$$C^{\text{P2P}}_{\text{Clean}} = H_b(q \circledast \varepsilon) - H_b(\varepsilon)\,, \tag{5}$$

is higher than that of the dirty binary channel (4), since $H_b(q \circledast \varepsilon) \ge H_b(q)$, where $\circledast$ denotes binary convolution, defined as

$$p_1 \circledast p_2 \triangleq p_1(1-p_2) + (1-p_1)p_2\,.$$

See [5], [6].

One approach to finding achievable rates for the MAC with common interference (1), is to extend the Gel'fand and Pinsker result [2] to the two-user case [1]. This extension leads to the following inner bound for the capacity region of (1) (see [1]):

$$\begin{aligned} \mathcal{R} \triangleq \text{cl conv} \Big\{ (R_1,R_2) : R_1 &\le I(U;Y|V) - I(U;S|V) \\ R_2 &\le I(V;Y|U) - I(V;S|U) \\ R_1 + R_2 &\le I(U,V;Y) - I(U,V;S) \Big\}\,, \end{aligned} \tag{6}$$

where $\text{cl}$ and $\text{conv}$ are the closure and convex hull , respectively, taken over all admissible auxiliary pairs $(U,V)$ satisfying:


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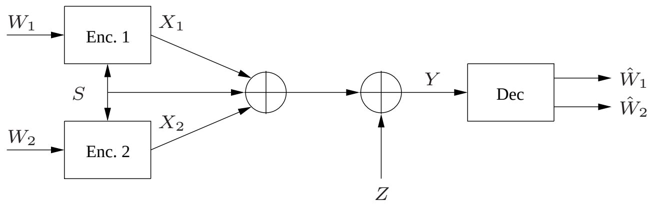


Fig. 1. Dirty MAC with common state information

$$(U, X_1) \leftrightarrow S \leftrightarrow (V, X_2)$$
$$(U, V) \leftrightarrow (X_1, X_2, S) \leftrightarrow Y.$$

Philosof et al. [7], [8] considered the "doubly-dirty MAC": a Gaussian additive MAC, with an additive interference which is composed of a sum of two independent Gaussian interferences, where each interference is known non-causally only to one of the encoders.

The capacity region of the Gaussian "dirty MAC", where the interference is known non-causally to both transmitters ("binary MAC with common interference"), was found by Gel'fand and Pinsker [9] (and rediscovered by Kim, Sutivong and Sigurjónsson [10]), to be equal to the interference-free MAC channel, by applying dirty paper coding by both users.

Philosof, Zamir and Erez [11] considered a binary modulo-additive version of this channel ("binary DMAC"), depicted also in Figure 1:

$$Y = X_1 \oplus X_2 \oplus S \oplus Z\,, \tag{7}$$

where $X_1, X_2, S, Z \in \mathbb{Z}_2$. The input ("power") constraints are

$$\frac{1}{n} w_H(\mathbf{x}_i) \le q_i\,, \qquad i = 1, 2\,, \tag{8}$$

where $0 \le q_1, q_2 \le 1/2$. The noise $Z$ has $\mathrm{Bernoulli}(\varepsilon)$ distribution and is independent of $S, X_1, X_2$; the state information $S \sim \mathrm{Bernoulli}\,(1/2)$ is referred to as "interference". The rates, $R_1$ and $R_2$, of the two users are given by $R_i = \frac{1}{n} \log |\mathcal{W}_i|$, where $\mathcal{W}_i$ is the set of messages of user $i$.

In [11], the capacity for two different scenarios is derived:

- *The binary doubly-dirty MAC*: in this scenario $S = S_1 \oplus S_2$, where $S_1, S_2 \sim \mathrm{Bernoulli}(1/2)$ are independent and known non-causally to encoders 1 and 2, respectively. The capacity region of this channel is given by the set of all rate pairs $(R_1, R_2)$ satisfying:

$$\mathcal{C}(q_1, q_2) \triangleq \Big\{ (R_1, R_2) : \\ R_1 + R_2 \le \mathrm{uce}\big\{ \max\left[ H_b(q_{\min}) - H_b(\varepsilon) \right] \big\} \Big\}\,,$$

where $q_{\min} \triangleq \min(q_1, q_2)$ and the upper convex envelope operation is w.r.t. $q_1$ and $q_2$.

- *The binary MAC with a single informed user*: in this scenario $S$ is known only to user 1. The capacity region of this channel is given by the set of all rate pairs $(R_1, R_2)$ satisfying:

$$\mathcal{C}(q_1, q_2) \triangleq \mathrm{cl\ conv} \Big\{ (R_1, R_2) : \\ R_2 \le H_b(q_2 \circledast \varepsilon) - H_b(\varepsilon) \\ R_1 + R_2 \le H_b(q_1) - H_b(\varepsilon) \Big\}\,. \tag{9}$$

Unlike in the Gaussian case, in which the common interference capacity region is the same as the interference-free region, and is achieved using *stationary* inputs, in the *binary* DMAC, we shall see that there is a loss.

In this work we consider the binary MAC with *common* interference, depicted in Figure 1, where the interference $S$ is known non-causally at both encoders. We assume that the interference is "strong" ($S \sim \mathrm{Bernoulli}\,(1/2)$).

This is the worst-case interference, as any other distribution of the interference can be transformed into a uniform one by incorporating dithering at the receiver's end.

We show that using the dirty-paper strategies that achieve (5) in (6), along with successive decoding of the messages ("onion peeling") allows to achieve a rate region of the binary DMAC (7), that is equal to the binary clean MAC capacity region up to a loss which stems from the loss seen in the point-to-point case (4),(5). Moreover, we show that these strategies achieve the sum-capacities of the binary clean MAC and dirty MAC with common interference, which are equal to the sum-capacities of these channels when full cooperation between the encoders is allowed, unlike in the Gaussian MAC with common interference.

To simplify the treatment we concentrate on the noiseless case, i.e., $Z = 0$.

The paper is organized as follows: We first consider the binary clean MAC in Section II and then turn to treating the binary MAC with common interference in Section III.

## II. Clean MAC

In this section we consider the "clean" binary modulo-additive channel:

$$Y = X_1 \oplus X_2 \oplus Z \tag{10}$$

with input constraints $q_1, q_2$.

The capacity region of this channel contains the capacity region of the binary DMAC (7), and therefore serves as an outer bound. Furthermore, the capacity achieving strategies are useful also for the DMAC case, as is discussed in Section III. As mentioned earlier, we concentrate on the "noiseless case" ($Z = 0$).

The binary additive model (10) is a special case of the general ("clean") MAC channel, the capacity region of which is known to be [12], [13]:

$$\mathcal{C} \triangleq \mathrm{cl\ conv} \Big\{ (R_1, R_2) : \; R_1 \le I(X_1; Y | X_2) \\ R_2 \le I(X_2; Y | X_1) \\ R_1 + R_2 \le I(X_1, X_2; Y) \Big\}\,, \tag{11}$$

where the closure and convex hull operations are taken over all product distributions $p_1(x_1) p_2(x_2)$ on $\mathcal{X}_1 \times \mathcal{X}_2$.

In the Gaussian additive MAC, any point within its capacity region can be achieved using *Gaussian stationary* inputs. Hence the convex hull operation is superfluous (see, e.g., [3]). In the binary case, however, the use of stationary inputs is not optimal and convex hull is necessary to achieve the capacity region envelope. To see this, we rewrite (11) explicitly for the binary case:

$$\mathcal{C} \triangleq \text{cl conv} \Big\{(R_1, R_2) : R_i \leq H_b(X_i)\,, \quad i = 1, 2 \\ R_1 + R_2 \leq H_b(X_1 \oplus X_2)\Big\}\,, \tag{12}$$

where the closure and the convex hull are taken over all admissible distributions of the form $p_1(x_1)p_2(x_2)$ on $\{0,1\}\times\{0,1\}$, such that the input constraints (8) are satisfied.

One easily verifies that, by allowing only stationary inputs in (12), .i.e., relinquishing the convex hull, the sum-rate $R_1 + R_2$ cannot exceed

$$R_1 + R_2 \leq H_b(q_1 \circledast q_2)\,, \tag{13}$$

which is suboptimal, as indicated by the following theorem.

*Theorem 1 (Sum-Rate Capacity of the Binary Clean MAC):* The sum-capacity of the binary noiseless modulo-additive MAC (10) with input constraints $\frac{1}{n} w_H(\mathbf{x_i}) \leq q_i$, $i = 1, 2$, is:

$$C_{\text{clean}}^{\text{sum}} = H_b^+(q_1 + q_2)\,, \tag{14}$$

where $H_b^+(q) \triangleq H_b\left(\min\{q, 1/2\}\right)$.

*Proof: Direct:* Using time-sharing one can divide each block into two parts: in the first $\alpha n$ block samples user 1 spends all of its "power" to convey his private message, while user 2 is silent (transmits zeros), whereas in the remaining $(1-\alpha)n$ block samples user 2 spends all of its transmission "power" to convey his message, while user 1 is silent. This leads to the sum-rate

$$R_1 + R_2 = \alpha H_b^+\left(\frac{q_1}{\alpha}\right) + (1-\alpha) H_b^+\left(\frac{q_2}{1-\alpha}\right),$$

which is equal to $H_b^+(q_1 + q_2)$ for $\alpha = \frac{q_1}{q_1+q_2}$.
*Converse: Full cooperation* between the transmitters can only increase the sum-capacity. Full cooperation transforms the problem into a point-to-point problem of transmitting over a binary clean channel with power constraint $\frac{1}{n} w_H(\mathbf{x}) \leq q_1 + q_2$, the capacity of which is $H_b^+(q_1 + q_2)$. ∎

Thus, the sum-capacity of the binary (clean) MAC (14) is strictly greater than the best achievable rate using only stationary inputs (13).

*Remark 1:*

- The sum-capacity of (10) can be shown, using the same methods, to be:
$$C_{\text{clean}}^{\text{sum}} = H_b^+\Big((q_1 + q_2) \circledast \varepsilon\Big) - H_b(\varepsilon)\,.$$
- If we allow full cooperation between the transmitters, the capacity of the channel does not outperform (14), as pointed out in the converse part of the proof. In the Gaussian case, on the other hand, the sum-capacity of the MAC channel is equal to $\frac{1}{2}\log(1 + \text{SNR}_1 + \text{SNR}_2)$, which is strictly smaller than the full-cooperation capacity, $\frac{1}{2}\log(1 + \text{SNR}_1 + \text{SNR}_2 + 2\sqrt{\text{SNR}_1\text{SNR}_2})$. This dissimilarity stems from the difference of the alphabets that we work with in both problems and the nature of the addition. In the binary case, no "coherence" can be attained by transmitting the same message, and additional power can only assist in exploiting more time slots within a block. In the Gaussian case, on the other hand, cooperation allows additional coherence gain, which cannot be achieved in the binary case.

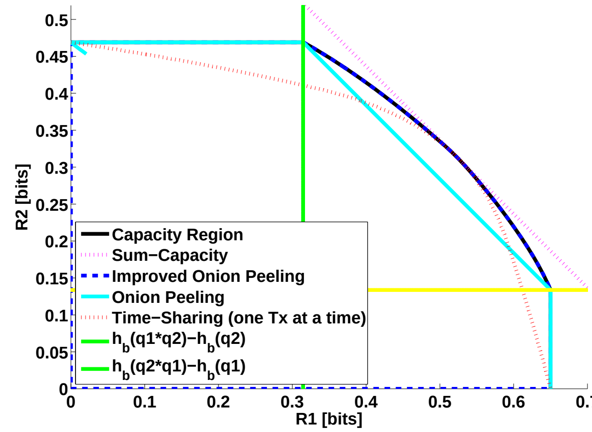

Fig. 2. Rate Regions for the binary clean MAC and input constraints: $q_1 = 1/6,\ q_2 = 1/10$.

To find the capacity region of (12) explicitly, we replace the convex hull with a time-sharing variable $Q$, with alphabet of size $|\mathcal{Q}| = 2$ (see, e.g., [14]).

$$\mathcal{C} \triangleq \bigcup \Big\{(R_1, R_2) : R_1 \leq H_b(X_1|Q) \\ R_2 \leq H_b(X_2|Q) \\ R_1 + R_2 \leq H_b(X_1 \oplus X_2|Q)\Big\}\,, \tag{15}$$

where the union is over all admissible Markov chains $X_1 \leftrightarrow Q \leftrightarrow X_2$, satisfying the input constraints $EX_i \leq q_i$, $i = 1, 2$.

*Remark 2:* Note that $X_1$ and $X_2$ are not independent in (15), but rather independent *given* the time-sharing parameter $Q$.

Deriving an explicit analytical expression for (15) is hard, and numerical solutions are needed instead.

Using the scheme proposed in the proof of Theorem 1, in which only one user transmits at each time instance, whereas the other user remains silent, is suboptimal in general, as is illustrated in Figure 2. Exploring the capacity region in (15), we note that the corner points of the pentagons, which constitute the capacity region, i.e., points that satisfy one of the first two inequalities and the third one with equality in (15), can be achieved by incorporating the "successive cancellation" (or "onion peeling") method, in which the decoder treats the message of one of the users as noise, recovers the message of the other user, and subtracts it to recover the remaining message.

Due to the time-sharing variable $Q$ of cardinality 2, two such strategies need to be considered, to achieve a general point in the capacity region (15), such that the power constraints are satisfied on the average. Nevertheless, we examine these rates for stationary points (viz. $P(Q=0)=1$) , to obtain better understanding. Thus, both users transmit simultaneously at all times, such that user 1 uses all of its available power $EX_1 = q_1$, whereas user 2 uses only some portion of its power $EX_2 = q_2'$ $(0 \le q_2' \le q_2)$. User 1 treats $q_2'$ as noise and can achieve a rate of $R_1 = H_b(q_1 \circledast q_2') - H_b(q_2')$. After recovering the message of user 1, it can be subtracted, such that user 2 sees a clean point-to-point channel and hence can achieve a rate of $R_2 = H_b(q_2')$. Note that even though using this strategy alone the capacity region cannot be achieved, it does achieve certain rate pairs which cannot be achieved by simple time-sharing, like the one used in Lemma 1 and depicted in Figure 2.

*Remark 3:*

- When using this onion peeling strategy, user 2 does not exploit all of its power, but only a portion $0 \le q_2' \le q_2$. Hence a "residual" power of $q_2 - q_2'$ is left unexploited. This implies that this strategy is not optimal (except when $q_2' = q_2$) as is, and a way to exploit this residual power needs to be constructed.
- As mentioned earlier, time-sharing between such onion peeling strategies allows to achieve capacity. However, numerical evidence suggest that a simpler scheme suffices to achieve the capacity region, as is depicted in Figure 2. In this scheme we divide the transmission block into two parts, where in the first we use onion peeling, such that the user being "pilled" first uses all of its power, whereas the other user uses a portion of its power in the first sub-block, and transmits with its remaining power in the second-block (whereas the other user is silent). If we denote by $\alpha$ the block portion allotted to onion peeling and by $q_2'$ $(0 \le q_2' \le q_2)$ the power of user 2, used during this period, this scheme supplies us with the following achievable rates:

$$\begin{aligned} R_1 &= \alpha H_b^+\left(\frac{q_1}{\alpha} \circledast \frac{q_2'}{\alpha}\right) - \alpha H_b^+\left(\frac{q_2'}{\alpha}\right) , \\ R_2 &= \alpha H_b^+\left(\frac{q_2'}{\alpha}\right) + (1-\alpha) H_b^+\left(\frac{q_2 - q_2'}{1-\alpha}\right) , \end{aligned} \tag{16}$$

  Or in the noise case:

$$\begin{aligned} R_1 &= \alpha H_b^+\left(\frac{q_1}{\alpha} \circledast \frac{q_2'}{\alpha} \circledast \varepsilon\right) - \alpha H_b^+\left(\frac{q_2'}{\alpha} \circledast \varepsilon\right) , \\ R_2 &= \alpha H_b^+\left(\frac{q_2'}{\alpha} \circledast \varepsilon\right) + (1-\alpha) H_b^+\left(\frac{q_2 - q_2'}{1-\alpha} \circledast \varepsilon\right) \\ &\quad - H_b(\varepsilon) , \end{aligned}$$

  in a similar manner.
- The roles of user 1 and user 2 are not symmetric: the achievable rate pairs, using onion peeling, when user 2 is peeled, differ from the rate pairs that are achieved when user 1 is peeled. Hence, by switching roles between the two users, one may achieve additional rate points.

## III. Dirty MAC with Common Interference

We adopt the strategies introduced in Section II to the dirty case (7) (depicted also in Figure 1), and derive an achievable rate region.

Similarly to the clean MAC case, the sum-capacity of the binary DMAC with common interference is equal to the capacity of this channel when both encoders can fully cooperate, as indicated by the following theorem.

*Theorem 2 (Sum-Rate Capacity of DMAC with Common SI):* The sum-capacity of the binary noiseless modulo-additive MAC with common interference (7) and input constraints $\frac{1}{n} w_H(\mathbf{x_i}) \le q_i$, $i = 1, 2$, is:

$$C_{\text{dirty}}^{\text{sum}} = H_b^+(q_1 + q_2) . \tag{17}$$

*Proof: Direct:* We repeat the proof of Lemma 1, only now the point-to-point BSC capacity (4) should be replaced by the binary dirty paper channel capacity (5). Nevertheless, in the noiseless case ($Z = 0$), there is no difference between the two expressions, and thus

$$R_1 + R_2 = H_b^+(q_1 + q_2) .$$

*Converse:* Again, like in the proof of Lemma 1, we allow *full cooperation* between the transmitters, which in turn transforms the problem into a point-to-point channel, the capacity of which is $H_b^+(q_1 + q_2)$. ∎

*Remark 4:*

- In the presence of noise, the sum-capacity of this channel is

$$C_{\text{Dirty}}^{\text{sum}} = \text{uch} \max\left\{ H_b^+(q_1+q_2) - H_b(\varepsilon), 0 \right\} .$$

- In the noiseless case, the sum-capacities of the binary clean and dirty MACs are equal. However, in the presence of noise $Z$, the sum-capacity of the dirty MAC channel is strictly smaller than that of the clean MAC channel (for $q_1 + q_2 < \frac{1}{2}$). This difference stems from the capacity loss, due to the presence of interference, of the point-to-point setting (4),(5)
- As was mentioned in Remark 1, if we allow full cooperation between the transmitters, the capacity of the channel cannot exceed (17), in contrast to the Gaussian case, in which additional "coherence gain" can be achieved.

The capacity region of the "single informed user" (9) serves as an inner bound for the capacity region of the common interference dirty MAC. To improve the achievable region of our channel of interest, we allow time-sharing between "single informed user" strategies, where the informed user is alternately user 1 or user 2. Note that by this only the user that is pilled first needs to know the interference. This is also true for the sum-capacity achieving strategy presented in the proof of Theorem 2.

*Remark 5:*

- The strategies used in [11], to achieve the capacity region of the single informed user (9), can be viewed as onion peeling, where user 2 assumes a point-to-point dirty paper channel and input constraint $0 \leq q_2' \leq q_2$; and user 1 treats the signal of user 2, $X_2$, as noise, and uses dirty paper coding of the form (5). The achievable rates, using this strategy, are of the form:

$$R_1 = H_b(q_1) - H_b(q_2')\,,$$
$$R_2 = H_b(q_2')\,,$$

where since $q_2'$ can take any value in the interval $[0, q_2]$, the single informed user capacity (9) is achieved by time-sharing between such strategies (where user 1 is always pilled first, since user 2 is ignorant of the interference sequence).

- Using such "stationary" strategies alone (with no time-sharing), one cannot hope to achieve the sum-capacity of Lemma 2 (or the whole capacity region of the single-informed user problem (9)), since there is an average residual power of $q_2 - q_2'$, for each sample, left un-exploited. As in the clean MAC case Section II, we conjecture that rather than using time-sharing between two "onion peeling" strategies, a simplified scheme that divides the transmission blocks into two parts, where in the first sub-block "onion peeling" is performed, where the user that is pilled first exploits all of "power", and in the second sub-block the other user transmits with all of its remaining power. This allows to achieve rate pairs of the form:

$$R_1 = \alpha H_b^+\left(\frac{q_1}{\alpha}\right) - \alpha H_b^+\left(\frac{q_2'}{\alpha}\right)\,,$$
$$R_2 = \alpha H_b^+\left(\frac{q_2'}{\alpha}\right) + (1-\alpha) H_b^+\left(\frac{q_2 - q_2'}{1-\alpha}\right)\,, \quad (18)$$

where $q_2' \in [0, q_2]$. See Figure 3. In the noisy case, this scheme achieves the following rates:

$$R_1 = \alpha H_b^+\left(\frac{q_1}{\alpha}\right) - \alpha H_b^+\left(\frac{q_2'}{\alpha} \circledast \varepsilon\right)\,,$$
$$R_2 = \alpha H_b^+\left(\frac{q_2'}{\alpha} \circledast \varepsilon\right) + (1-\alpha) H_b^+\left(\frac{q_2 - q_2'}{1-\alpha} \circledast \varepsilon\right) - H_b(\varepsilon)\,.$$

- Even in the noiseless case ($Z = 0$) the achievable rate region of the dirty channel (18) is properly contained in its corresponding clean counterpart (16) (as depicted in Figure 3), in contrast to the point-to-point setting, in which the capacities are equal in the absence of noise (4), (5). The gap between the two regions stems from the fact that in the first sub-block, the user being peeled first, treats the signal of the other user as *noise*, in the presence of *interference*. Hence the achievable rate during this stage is strictly smaller, due to the point-to-point loss of binary dirty paper coding (4), (5).
- This strategy is asymmetric in user 1 and user 2, as was explained in Remark 3.

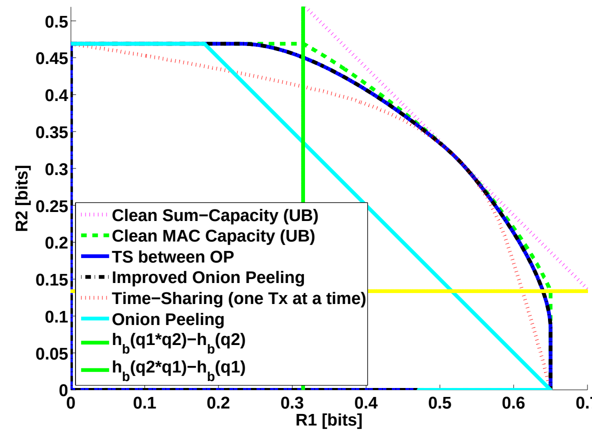


Fig. 3. Rate regions for the binary DMAC with common interference and input constraints $q_1 = 1/6,\ q_2 = 1/10$.

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