

Decomposing the MIMO Broadcast Channel

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Abstract—The problem of transmitting a common message over a multiple-input multiple-output (MIMO) Gaussian broadcast channel with multiple receivers is well understood in terms of capacity. Nevertheless, existing optimal (capacity-achieving) schemes for this scenario require joint decoding of the multiple streams transmitted, entailing high computational complexity. In this paper, a low-complexity scheme requiring only single stream decoding is proposed. The scheme uses a matrix decomposition, which allows, by linear pre- and post-processing, to simultaneously transform both channel matrices to triangular forms, where the diagonal entries of both channels are equal. In conjunction with successive interference cancellation at each receiver, parallel channels are created, over each of which scalar coding and decoding may be used. We prove that this channel transformation conserves mutual information, and hence any sub-optimality of the proposed scheme is governed solely by the gap-to-capacity of the scalar coding scheme over the parallel channels.

Index Terms—Broadcast channel, MIMO channel, geometric mean decomposition, successive decoding, MMSE, zero-forcing, GDFE, successive interference cancellation.

I. INTRODUCTION

The multiple-input multiple-output (MIMO) Gaussian broadcast (BC) channel has gained much attention over the past decade. Unlike the single-input single-output (SISO) case, the Gaussian MIMO BC channel is not degraded. Nevertheless, capacity regions were established for some scenarios, such as private-messages only, and for a common message with a single private message, and bounds were derived for others, see [1]–[5] and references therein.

In this work we concentrate on the common-message problem, the capacity which is long known to equal the (worst-case) capacity of the compound channel [6], with the compound parameter being the channel matrix index.

Direct capacity-achieving implementation for MIMO channels involves joint decoding of all antenna signals. In point-to-point (single-user) communication, such high-complexity schemes, e.g., using bit-interleaved coded modulation (BICM) in conjunction with sphere detection, essentially require the same resources as if working in an “open loop” mode. Thus, the complexity involved is similar to that required for approaching the isotropic mutual information of the channel, when only the receiver knows the channel. Nonetheless, it is well known that for the MIMO point-to-point channel, when the channel is known at both ends, capacity may be approached

with greatly reduced complexity by applying “scalar coding” - the combination of scalar additive white Gaussian noise (AWGN) codebooks, linear processing, and possibly also successive interference cancellation (SIC). Such solutions include using the singular-value decomposition (SVD) to establish parallel virtual AWGN channels, see [7]. Alternatively, SIC-based schemes such as generalized decision feedback equalization (GDFE) and Vertical Bell-Laboratories Space-Time coding (V-BLAST) may be used, see [8], [9]. Similarly, scalar-coding (in this case dirty-paper coding) techniques also achieve the private-message MIMO BC capacity, see, e.g., [10].

In the presence of a common message, however, to our knowledge, no scalar capacity-approaching coding solutions are known. QR-based schemes fail, since requiring the individual streams to be simultaneously decodable at all the receivers implies that the rate *per stream* is governed by the smallest of the corresponding diagonal elements, (potentially) inflicting an unbounded rate penalty. Adapting SVD to this scenario is problematic since the decomposition requires multiplying by a channel-dependent (unitary) matrix at the encoder, which prevents from using this decomposition for more than one channel simultaneously.¹ As a result of these difficulties, other techniques were proposed, which are suboptimal in general, see, e.g., [12], [13].

In this work, we present an optimal successive-decoding (low-complexity) scheme for a two-user common-message Gaussian MIMO BC channel. The result is derived by introducing a decomposition, novel to the best of our knowledge, which allows to simultaneously decompose any two non-singular square matrices with equal determinants into triangular forms with *equal diagonals*, by multiplying by the *same unitary* matrix on the right and different *unitary* matrices on the left.² The proposed scheme is based upon using this decomposition, in conjunction with SIC and good *scalar* AWGN codes.³

By applying the above decomposition directly to the channel

¹Indeed, the generalized singular-value decomposition (GSVD) [11] allows to use a single transformation for two different channels at one of the ends, but for each virtual parallel channel it yields a different gain for each user, thus not solving the inefficiency mentioned above. In fact, using GSVD may result in *worse* performance than using a QR-based decoder without any transformation at the encoder.

²This decomposition applies as well to the more general (non-square) case, as discussed in the sequel.

³For an example of the use of successive decoding of virtual channels in point-to-point MIMO schemes, see [14].

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matrices, the proposed scheme can approach the common-message capacity in the high signal-to-noise ratio (SNR) limit. For general SNR, applying the same decomposition to modified matrices, amounts to optimal linear pre- and post-processing, corresponding to beamforming and minimum mean-squared error (MMSE) estimation, respectively. This in turn allows us to establish a capacity-achieving scheme for any two (not necessarily same-rank or square) channel matrices.

Beyond the pure common-message scenario, for the two-user case, the proposed approach can also be used for transmission of a combination of private and common messages; in which case the best known achievable rate region may be achieved.

The rest of the paper is organized as follows. In Section II we present the aforementioned decomposition. In Section III we formally define the communication problem, and in Section IV we apply the decomposition to derive a “zero-forcing” (ZF) scheme that is optimal at high SNR. In Section V we prove that a variant of the scheme is optimal at any SNR. Finally, we discuss extensions in Section VI.

II. JOINT EQUI-DIAGONAL TRIAGONALIZATION

To construct the joint equi-diagonal triangularization we use the geometric mean decomposition (GMD) [15], according to which any matrix A of dimensions $m \times n$ and rank k can be decomposed as

$$A = U_1 D U_2^\dagger, \quad (1)$$

where U_1 and U_2 are $m \times k$ and $n \times k$ (respectively) matrices with orthonormal columns and D is a $k \times k$ upper-triangular matrix with a *constant* diagonal equal to the geometric mean of the non-zero singular values of A . We shall also use the QR and RQ decompositions (see, e.g., [16]) according to which A can be factorized as

$$A = QR, \quad (2)$$

or alternately as

$$A = \tilde{R} V^T, \quad (3)$$

where Q and V are $m \times m$ and $n \times n$ unitary matrices, respectively, and R and \tilde{R} are $m \times n$ “generalized upper triangular” matrices (matrices with zero entries beneath the main diagonal, i.e., $[R]_{ij} = 0$ for $i > j$) with non-negative diagonals.

These decompositions allow to prove the following theorem.

Theorem 1: Let A_1 and A_2 be complex-valued matrices, of dimensions $m_1 \times n$ and $m_2 \times n$, respectively. Assume that $m_1, m_2 \geq n$ and that the matrices have full rank (rank n). Then A_1 and A_2 can be jointly decomposed as

$$\begin{aligned} A_1 &= a_1 U_1 D_1 V^\dagger \\ A_2 &= a_2 U_2 D_2 V^\dagger, \end{aligned} \quad (4)$$

and

$$a_i \triangleq \sqrt[n]{\prod_{j=1}^n \sigma_{i,j}},$$

where U_1 , U_2 and V are unitary of dimensions $m_1 \times m_1$, $m_2 \times m_2$ and $n \times n$, respectively; D_1 and D_2 are generalized upper-triangular matrices with non-negative *equal* diagonal elements, and where $\{\sigma_{i,j}\}_{j=1}^n$ are the singular values of A_i .

Remark 1: The condition on the dimension of the matrices is not necessary, neither do they need to be full rank. It suffices that both have the same rank k , regardless of the dimensions m_1 , m_2 and n (though in that case, the coefficients a_1 and a_2 may vary). However, these conditions greatly simplify the proof, and are sufficient for proving our main result in Section V.

Remark 2: The decomposition in Theorem 1 is not unique in general. This stems from, inter alia, the fact that GMD, which is used in the construction of the decomposition of Theorem 1, is not unique in general; see [15].

Remark 3: In the real case, viz. when A_1 , A_2 are real-valued matrices, the joint triangularization of Theorem 1 is achieved with *orthogonal* (real) matrices U_1, U_2, V and *real-valued* matrices D_1, D_2 .

Proof: Before considering the general case, we start by proving the theorem for the case when the matrices are square.

square case ($m_1 = m_2 = n$):

In this case A_1 and A_2 are invertible. Denote $a_i \triangleq \sqrt[n]{|\det(A_i)|}$. Thus, one may apply the GMD to the matrix B , defined below:

$$B \triangleq \frac{A_1}{a_1} \begin{pmatrix} A_2 \\ a_2 \end{pmatrix}^{-1} = U_1 R U_2^\dagger, \quad (5)$$

where the matrices U_1, U_2, R are as in (1).

Now, apply RQ decompositions to $U_i^\dagger A_i / a_i$ ($i = 1, 2$) to achieve

$$U_i^\dagger \frac{A_i}{a_i} = D_i V_i^\dagger, \quad (6)$$

where D_i and V_i are as in (3). By substituting (6) into (5) we have

$$U_1 D_1 V_1^\dagger V_2 D_2^{-1} U_2^\dagger = U_2 R U_2^\dagger,$$

which is equivalent to

$$V_1^\dagger V_2 = D_1^{-1} R D_2. \quad (7)$$

We note that the l.h.s. of (7) is unitary, whereas its r.h.s. is an upper-triangular matrix with a non-negative diagonal. An equality between such matrices can hold only if both matrices are equal to the identity matrix of the appropriate dimensions ($n \times n$). Thus, we have

$$V \triangleq V_1 = V_2.$$

$$[D_1]_{ii} = [R]_{ii} [D_2]_{ii}, \quad i = 1, \dots, n,$$

where $[X]_{ij}$ denotes the (i, j) entry of the matrix X .

Since the diagonal of R is constant and is equal to the geometric mean of the singular values of B , which, in turn, is equal to $[R]_{ii} = 1$ (for all i), we have

$$[D_1]_{ii} = [D_2]_{ii},$$

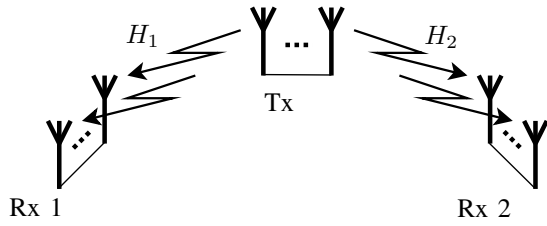


Fig. 1. Two-user MIMO BC channel. Even though both users may have similar channel quality, the actual channel matrices H_1 , H_2 differ.

Thus, we established the desired decomposition (4).

Larger row dimension case ($m_1, m_2 \geq n$):

We start by decomposing A_i using QR decompositions:

$$A_i = Q_i R_i, \quad i = 1, 2,$$

where Q_i and R_i are as in (2). Since A_i is full-rank and $m_i \geq n$, the diagonal elements of R_i are all (strictly) positive and the entries on its lower $(m_i - n)$ rows are all zeros. Denote the $n \times n$ upper sub-matrices of R_i by \tilde{R}_i . Since \tilde{R}_i are non-singular square matrices, invoking the proof for the square case (above), we may decompose them as

$$\begin{aligned} \tilde{R}_1 &= a_1 \tilde{U}_1 \tilde{D}_1 V^\dagger \\ \tilde{R}_2 &= a_2 \tilde{U}_2 \tilde{D}_2 V^\dagger. \end{aligned}$$

Now, construct the augmented unitary matrices W_i :

$$W_i \triangleq \begin{pmatrix} \tilde{U}_i & 0 \\ 0 & I_{m_i - n} \end{pmatrix},$$

and the generalized triangular matrices D_i of dimensions $m_i \times n$:

$$D_i \triangleq \begin{pmatrix} \tilde{D}_i \\ 0 \end{pmatrix}.$$

Thus, we arrive at the desired decomposition of A_1 and A_2 (4), with $U_i \triangleq Q_i W_i$. ■

III. PROBLEM DEFINITION: COMMON-MESSAGE MIMO BC

The two-user Gaussian MIMO BC channel, depicted in Figure 1, is given by

$$\mathbf{y}_i = H_i \mathbf{x} + \mathbf{n}_i, \quad i = 1, 2, \quad (8)$$

where \mathbf{x} is the complex-valued channel input vector of size $N_t \times 1$, \mathbf{y}_i ($i = 1, 2$) are the received vectors of sizes $N_r^i \times 1$, H_i are the $N_r^i \times N_t$ complex channel matrices and \mathbf{n}_i are mutually-independent identically-distributed circularly-symmetric complex Gaussian random vectors of sizes N_r^i , i.e., $\mathbf{n}_i \sim \mathcal{N}(0, I_{N_r^i})$. The transmit signal \mathbf{x} is subject to an average total power constraint $E[\mathbf{x}^\dagger \mathbf{x}] \leq P$.⁴

⁴Alternatively, one can consider an input covariance constraint $C_{\mathbf{x}} \triangleq E[\mathbf{x} \mathbf{x}^\dagger] \preceq C$, where by $C_1 \preceq C_2$ we mean that the matrix $(C_2 - C_1)$ is positive semi-definite and C^\dagger denotes the Hermitian transpose of C .

The common-message capacity is the maximum achievable rate of a codebook that can be decoded with vanishing error probability by both users. It is given by the compound-channel (worst-case) capacity expression:

$$C = \max_{C_{\mathbf{x}}} \min_{i=1,2} \log \left\{ \det \left(I + H_i C_{\mathbf{x}} H_i^\dagger \right) \right\}, \quad (9)$$

where the maximization is over all covariance matrices $C_{\mathbf{x}}$ subject to the power constraint.

IV. ZERO-FORCING SCHEME

In the common-message problem above, assume that the number of receive antennas at each user is at least as large as the number of transmit antennas, and that both channel matrices are full rank, i.e. have rank N_t .⁵ Assume that both matrices lead to the same capacity in the high-SNR limit and w.l.o.g. use the normalization

$$\det \left(H_i H_i^\dagger \right) = 1, \quad i = 1, 2.$$

It is easy to verify that in the high-SNR limit, the capacity $C(P)$ under a power constraint P satisfies

$$\lim_{P \rightarrow \infty} \left[C(P) - N_t \log \frac{P}{N_t} \right] = 0. \quad (10)$$

Thus no unequal power-allocation (beamforming or “water-filling”) and no MMSE estimation are needed, in this limit.

By applying the joint triangularization of Theorem 1 to both channel matrices H_i , normalized by the transmit power, we have

$$\sqrt{P} H_i = U_i D_i V^\dagger, \quad i = 1, 2, \quad (11)$$

where U_1, U_2 and V are unitary, and D_1 and D_2 are $N_r^1 \times N_t$ and $N_r^2 \times N_t$ (respectively) generalized upper-triangular with the same real positive diagonal \mathbf{d} satisfying

$$\prod_{j=1}^{N_t} \frac{d_j}{\sqrt{P}} = 1. \quad (12)$$

We shall use this decomposition to transform the MIMO BC channel into k parallel virtual SISO BC channels with respective gains d_j , $j = 1, \dots, N_t$, and input power $1/N_t$ over each, allowing transmission rates greater than

$$R_j = \log \frac{d_j^2}{N_t}. \quad (13)$$

The resulting total rate,

$$R \triangleq \sum_{j=1}^{N_t} R_j = N_t \log \frac{P}{N_t} \quad (14)$$

is indeed optimal in the high SNR limit, c.f. (10). We now describe in detail a transmission scheme which allows to achieve this rate.

⁵These assumptions will be dropped when we present the optimal MMSE scheme in the next section. Even for the zero-forcing scheme the only necessary assumption is that the matrices have the same rank $k \leq \min(N_t, N_r^1, N_r^2)$. See Remark 1 after Theorem 1.

Split the total rate into sub-messages of power $1/N_t$ and rates R_j as specified in (13), and use k independent codebooks of equal power $1/N_t$, each of which is capacity-achieving for an AWGN channel of appropriate rate. Denote the vector formed by taking the n -th element of each codebook by $\tilde{\mathbf{x}}$ (we suppress here and onward the time index n and reserve the vector notation for the spatial dimension). Using Theorem 1 form (11), the corresponding transmit vector is given by:

$$\mathbf{x} = \sqrt{P}V\tilde{\mathbf{x}}. \quad (15)$$

Since V is unitary, the power constraint P is satisfied. At receiver i we may obtain

$$\tilde{\mathbf{y}}_i \triangleq U_i^\dagger \mathbf{y}_i = D_i \tilde{\mathbf{x}} + \tilde{\mathbf{n}}_i, \quad i = 1, 2, \quad (16)$$

where $\tilde{\mathbf{n}}_i \triangleq U_i^\dagger \mathbf{n}_i$ are circularly-symmetric complex Gaussian with covariance matrices I_{N_t} , since U_i are unitary. Hence, we transformed the channel into an effective triangular channel. Note that for the last sub-channel, N_t , we have a virtual AWGN channel of capacity R_{N_t} . It can thus be decoded with arbitrarily low probability of error, and subtracted out for the sake of decoding the subchannel $N_t - 1$. In general, we can use SIC:

$$[\mathbf{y}'_i]_j = [\tilde{\mathbf{y}}_i]_j - \sum_{l=j+1}^{N_t} [D_i]_{j,l} [\hat{\mathbf{x}}_i]_l, \quad (17)$$

where $[\hat{\mathbf{x}}_i]_l$ is the decoded l -th codebook at receiver i . Assuming correct decoding, we obtain an effective AWGN channel of equal rate R_j to each user,

$$[\mathbf{y}'_i]_j = d_j [\tilde{\mathbf{x}}]_j + [\tilde{\mathbf{n}}_i]_j, \quad i = 1, 2.$$

We conclude that the total rate (14) is indeed achievable by a scheme which only uses scalar coding, linear operations and successive interference cancellation. Of course, if one of the matrices yields larger capacity, the performance corresponding with the smaller is achievable; the proposed decomposition of Theorem 1 will yield a fixed sub-channel gain a_2/a_1 which will not be used in a common-message setting (but can be utilized when adding a private message to the ‘‘strong’’ user via dirty-paper coding, see Section VI).

Remark 4: In comparison, consider applying the QR decomposition to the channel matrices at the decoder, without using a suitable matrix V at the encoder. In this case, the triangular matrices D_1 and D_2 will have arbitrary diagonals \mathbf{d}_1 and \mathbf{d}_2 , each satisfying (12). Using these matrices along with SIC and scalar coding over the parallel channels, will achieve a rate according to the minimum of $[\mathbf{d}_1]_j$ and $[\mathbf{d}_2]_j$ for the j -th channel. Thus, we obtain an achievable rate of

$$N_t \log \frac{P}{N_t} + \sum_{j=1}^{N_t} \log \frac{\min \{ [\mathbf{d}_1]_j^2, [\mathbf{d}_2]_j^2 \}}{P}.$$

Comparing to (10), this approach will result in a loss in the high SNR limit, unless the diagonals are equal.

V. OPTIMAL MMSE SCHEME

The scheme described in Section IV is of a zero-forcing nature, in the sense that the inter-channel interference is completely cancelled. Although this approach is optimal in the high-SNR limit, an MMSE receiver which strikes a balance between residual interference and noise can improve performance. We now present a scheme which achieves the optimum rate (9). The derivation follows the general lines of the extension from the geometric mean decomposition (GMD) to the universal channel decomposition (UCD) as developed in [14], which in turn builds upon the MMSE version of V-BLAST [17].

For a given channel input covariance matrix $C_{\mathbf{x}}$ (of trace at most P), let $F_i = H_i \sqrt{C_{\mathbf{x}}}$ for $i = 1, 2$. We define augmented matrices and apply to them the decomposition of Theorem 1:

$$\begin{bmatrix} F_i \\ I \end{bmatrix} \triangleq G_i = \alpha_i U_i D_i V^\dagger \quad i = 1, 2, \quad (18)$$

where the identity matrix I has dimensions $N_t \times N_t$. Without loss of generality, assume that $|\alpha_1^2| \geq |\alpha_2^2| = 1$. The transmission is given by

$$\mathbf{x} = \sqrt{C_{\mathbf{x}}}V\tilde{\mathbf{x}}, \quad (19)$$

where $\tilde{\mathbf{x}}$ is the codeword vector as in Section IV, for codebooks of power $1/N_t$ and rates R_j (13). At each receiver we compute

$$\tilde{\mathbf{y}}_i = \tilde{U}_i \mathbf{y}_i, \quad (20)$$

where \tilde{U}_i consists of the first N_t rows of U_i . Finally, successive decoding is performed as in (17). Define the signal-to-interference and noise ratio (SINR) for the decoding of the j -th codebook in decoder i in this process to be:

$$S_{i,j} = \text{Var} \left([\tilde{\mathbf{x}}]_j \middle| \tilde{\mathbf{y}}_i, [\tilde{\mathbf{x}}]_{j+1}^{N_t} \right). \quad (21)$$

The following shows optimality of the scheme.

Theorem 2: Let V , U_1 and U_2 be given by (18). Let \mathbf{d} be the diagonal of the corresponding D_1 or D_2 , and let \tilde{U}_i be the first N_t rows of U_i . The rates (13) and SINRs (21) are related by

$$R_j = \log(1 + S_{1,j}) = \log(1 + S_{2,j}) \quad \forall j = 1, \dots, N_t \quad (22)$$

and furthermore

$$\sum_{j=1}^{N_t} R_j = \log \left\{ \det \left(I + H_i C_{\mathbf{x}} H_i^\dagger \right) \right\}. \quad (23)$$

Proof: The received signal is

$$\mathbf{y}_i = F_i V \tilde{\mathbf{x}} + \mathbf{z} \triangleq \tilde{F}_i + \mathbf{z}.$$

Recalling (18), we can define an augmented matrix for the \tilde{F}_i channels:

$$\begin{aligned} \tilde{G}_i &\triangleq \begin{bmatrix} \tilde{F}_i \\ I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & V^\dagger \end{bmatrix} G_i V = \begin{bmatrix} I & 0 \\ 0 & V^\dagger \end{bmatrix} U_i D_i \\ &\triangleq W_i D_i. \end{aligned}$$

We now note that W_i is unitary, and thus we have arrived at a QR decomposition of \tilde{G}_i . Also note that the first N_t rows of W_i equal \tilde{U}_i . Following the exposition in [14, Section III-D] (see also e.g. [17, Section II]), each of the decoders is exactly the MMSE-VBLAST decoder for \tilde{F}_i . Now (22) follows (for each decoder) by [14, Lemma III.3]. Furthermore,

$$\sum_{j=1}^{N_t} R_j = \log \det \left(I + \tilde{F}_i \tilde{F}_i^\dagger \right) = \log \det \left(I + F_i F_i^\dagger \right),$$

where the first equality follows by [14, Corollary III.4], and the second holds since V is unitary. Since this is equivalent to (23), the proof is completed. ■

By (22), the successive decoding procedure will succeed with arbitrarily low probability of error for rates approaching R_j . By (23), the sum of the codebook rates equals the mutual information over the channels. Taking $C_{\mathbf{x}}$ be the covariance matrix maximizing (9), capacity is achieved.

VI. DISCUSSION AND GENERALIZATIONS

In this work, the problem of broadcasting over a MIMO channel was considered. In the special case where the channel matrices are both diagonal, this is equivalent to broadcasting over SISO channels of different colors (i.e., different frequency responses), where the channel spectra are piecewise constant (but since the dimension is not restricted, any “well-behaved” spectrum can be approached). Thus, we have also presented a practical optimal scheme for colored broadcasting (of course, it is straightforward to do that in the fortunate special case where the channel is degraded). Interestingly, the scheme gives up the orthogonality between frequency bins and creates dependence which can be successively cancelled; this can be viewed as “frequency-domain decision feedback equalization”.

As mentioned in the introduction, the proposed scheme can be extended to the case where both common and private messages are present. For one private message, the capacity is given by superposition, and this approach also yields the best known achievable rate for two messages, see [5]. Since the joint triangularization is information-lossless, we can always use it, and then add on top of it a private-message layer using dirty-paper coding. Interestingly, in that case we would have interference cancellation both at the encoder and the decoders.

Another natural extension would be to the case of more than two receivers; this is currently under research.

The MIMO BC problem is just one example of a network setting where jointly decomposing two channel matrices yields simple transmission schemes. Another example is the rateless problem, addressed in [18]. Moreover, in some scenarios this approach can help in deriving new achievability results; see [19] for such work regarding the Gaussian two-way MIMO relay channel.

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