Improved Rates and Coding for the MIMO Two-Way Relay Channel

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***Abstract*—The Gaussian multiple-input multiple-output two-way relay channel is considered. By applying linear pre- and post-processing, the channel matrices are transformed into triangular form having equal diagonals. Over the obtained triangular channels, dirty-paper coding is applied, yielding parallel symmetric scalar two-way relay channels; thus, reducing the coding task to that of coding over the scalar symmetric two-way relay channel. Any existing coding technique can then be readily applied over these resulting channels. This technique allows to obtain new achievable rates in the symmetric case.**

I. Introduction

The two-way relay channel (TWRC) [1] is composed of two terminals exchanging information only via a third-party relay. This simple scenario, which is composed of a multiple-access (MAC) section from the two terminals to the relay, and a broadcast (BC) section, from the relay to the two terminals, manifests many basic principles encountered in more complex networks, and therefore gained much attention lately.

A traditional communication approach over the TWRC is decode-and-forward (DF). Here, the network is treated at two different levels: a physical-layer local code translates the wireless channels into "bit-pipes", over which network coding is applied. Namely, a MAC code is used over the first section, allowing the relay to decode the messages of both terminals. Then, the relay uses a BC code to convey to the terminals a common message; assuming that this was decoded correctly, each user obtains the desired message from the common message and from their own message, as in network coding. DF has the advantage that the noise of the MAC stage is "cleaned" by the relay rather than being accumulated. Even though DF proves optimal in the limit of low signal-to-noise ratio (SNR), turning the MAC channel into bit-pipes incurs a loss, which increases as the SNRs grow.

Other approaches do not turn all of the links to bit-pipes, but rather leverage the physical properties of at least some of the links. In compress-and-forward (CF), the relay merely compresses its received signal. However, since each of the terminals knows its transmitted signal, CF can utilize "remote Wyner–Ziv coding" [2] for both messages simultaneously, i.e., each terminal tries to recover the message signal sent by the other user with its own message signal serving as decoder side-information [3]. CF, thus, defers decoding from the relay to the terminals, that may have better conditions; this comes at the price of noise accumulation. It turns out that CF outperforms DF for a sufficiently high BC capacity.[1] Performance can be further improved by incorporating layers of CF and DF; see [3].

Structured physical-layer network coding (sPNC) approaches [5]–[7] aim to avoid noise accumulation, without turning the MAC channel into bit-pipes. This is accomplished by using structured codes, where the sum of codewords is also a codeword. Thus, the relay can decode a "sum-message" and forward it. Though outperformed by DF and CF at low SNRs, sPNC becomes optimal in the limit of high SNR.

We note that the CF approach over the TWRC has been somewhat overshadowed by sPNC and DF. Nevertheless, as stated above, it offers better performance for certain SNR values. In Section II we revisit these known techniques for the Gaussian single-input single-output (SISO) TWRC, providing a detailed comparison. The symmetric setting, in which the channel quality of the users is equal and the desired rates are equal as well, is of special interest. In this setting, the best known achievable rate region is given by time-sharing between DF, CF and sPNC.[2] We find that, in the symmetric setting, this time-sharing strategy has a gap from the cut-set upper bound on the capacity of at most 0.2625 bits (per complex stream). In this work we consider the extension of these techniques to the multiple-input-multiple-output (MIMO) case. We note that the difficulty of the task greatly varies between the different strategies.

DF generalizes to the MIMO case in a straightforward manner, using any scheme for the MIMO MAC and (common-message) BC channels. As for CF, even though an information-theoretic expression for the achievable rates can be formulated, its explicit evaluation in the MIMO case is hard in general, let alone code construction. Thus, suboptimal scalar approaches have been suggested [8], [9].

The generalization of sPNC to the MIMO case is a very different issue, as this approach is specifically tailored to scalar additive channels. Thus, some form of decomposition of the channel into parallel subchannels is required. Two techniques have been proposed. The first technique, by Yang et

[1] The CF approach always outperforms amplify-and-forward (AF) over the TWRC, in contrast to other network topologies.

[2] Although in principle pDF could improve performance, evaluating the expressions shows that in the symmetric setting it does not.

al. [10], relies on the generalized singular value decomposition (GSVD) [11]. The GSVD results in triangular matrices with proportional rows (though with different diagonal values); the column proportionality allows to recover linear combinations of the messages (similar to the non-symmetric sPNC technique of [6]) using successive interference cancellation (SIC). The second technique, proposed in [12], allows to triangularize both channel matrices, such that the resulting diagonals are equal, using the joint equi-diagonal triangularization (JET) [13], and together with dirty-paper coding (DPC) employs the symmetric scalar sPNC of [5] over the resulting parallel sub-channels. When the target rates of the two terminals are close ("symmetric case"), the JET-based scheme achieves better performance, whereas when the two rates differ substantially, the performance of the GSVD-based scheme is superior.

In this work, we concentrate on the symmetric case, where the channels of the users have the same quality (yet, for MIMO, the matrices can be very different from each other) and the desired rates are equal. In Section III-B, we find that having equal diagonals is advantageous not only for sPNC, but also for CF. We further propose an improvement of the JET-based scheme of [12], by allowing DF, CF or sPNC (or optimal time-sharing between them) over each subchannel, according to its parameters. Finally, we demonstrate the performance of the proposed technique for a parallel channels example.

II. Background: Communication Strategies for the SISO Two-Way Relay Channel

In this section we consider the Gaussian SISO TWRC and describe the communication approaches mentioned in Section I along with the cut-set outer bound, both for the symmetric and the non-symmetric cases.

A. Channel Model

The TWRC consists of two terminals and a relay. We define the channel model as follows. Transmission takes place in two phases, each one, w.l.o.g., consisting of N channel uses. At each time instance n in the first phase, terminal i ($i = 1, 2$) transmits a signal $x_{i,n}$ and the relay receives y_n according to some memoryless MAC channel $W_{\text{MAC}}(y|x_1, x_2)$. At each time instance n in the second phase, the relay transmits a signal x_n and terminal i ($i = 1, 2$) receives $y_{i,n}$ according to some memoryless BC channel $W_{\text{BC}}(y_1, y_2|x)$. Before transmission begins, terminal i possesses an independent message of rate R_i, unknown to the other nodes; at the end of the two transmission phases, each terminal should be able to decode, with arbitrarily low error probability, the message of the other terminal. The closure of all achievable pairs (R_1, R_2) is the capacity region of the network.

In the Gaussian SISO setting, the MAC phase of this channel is given by

$$y = h_1 x_1 + h_2 x_2 + z\,,$$

where, w.l.o.g., x_1 and x_2 are subject to the same power constraint P, and z is additive white Gaussian noise (AWGN) of power 1.

The exact nature of the BC channel is not material in the context of this work. We characterize it by its "side-information rate region" $\mathcal{C}_{\text{BC}}$ [1], which corresponds to the private-message capacity rate-region over the BC channel where each decoder knows the message intended for the other decoder. This rate region is equal to the closure of the convex hull of all rate-pairs (R_1, R_2) satisfying:

$$\begin{aligned} R_1 &\le I(X; Y_2|X_2)\,, \\ R_2 &\le I(X; Y_1|X_1)\,, \end{aligned}$$

for some product distributions $p(x)p(y_1|x)p(y_2|x)$.

Note that in the symmetric setting, $R \triangleq R_1 = R_2$, the optimum achievable rate is equal to the common-message capacity C_{common} of the BC channel *with no side-information* at the decoders.

B. Communication Schemes in the Symmetric-Rate Setting

Here we specialize to the symmetric case:

$$\begin{aligned} R &\triangleq R_1 = R_2 \\ h &\triangleq h_1 = h_2\,, \end{aligned} \tag{2a}$$

and, without loss of generality, take $h = 1$.

By the min-cut max-flow theorem, one cannot achieve a rate greater than the point-to-point capacities of the MAC links or the common-message capacity of the BC channel [1]:

$$R_{\text{CS}} = \min \left\{ \log (1 + P)\,, C_{\text{common}} \right\}\,. \tag{3}$$

In the DF approach, the relay decodes both messages with sum-rate $2R$. Instead of forwarding both messages, it can use a network-coding approach and XOR them. Then, each terminal can XOR out its own message to obtain the desired one. The resulting rate is given by:

$$R_{\text{DF}} = \min \left\{ \frac{1}{2} \log (1 + 2P)\,, C_{\text{common}} \right\}\,. \tag{4}$$

In the CF approach, the noisy sum of the messages, transmitted by the sources, is quantized at the relay, using remote Wyner–Ziv coding [2], with each terminal using its transmitted message as decoder side-information. The achievable rate using this scheme is [3]

$$R_{\text{CF}} = \log \left(1 + P \upharpoonleft\!\upharpoonright P_{\text{common}}\right)\,, \tag{5}$$

where P_{common} is the effective SNR of the BC phase that satisfies $C_{\text{common}} = \log(1 + P_{\text{common}})$, and

$$A \upharpoonleft\!\upharpoonright B \triangleq \frac{AB}{1 + A + B}\,.$$

In the sPNC approach [5], both terminals transmit codewords generated from the same lattice code. Due to the linearity property of the lattice code, the sum of the two codewords is a valid lattice codeword. This sum is decoded at the relay and sent to the terminals. Each terminal, then, recovers the sum codeword and subtracts from it its own lattice

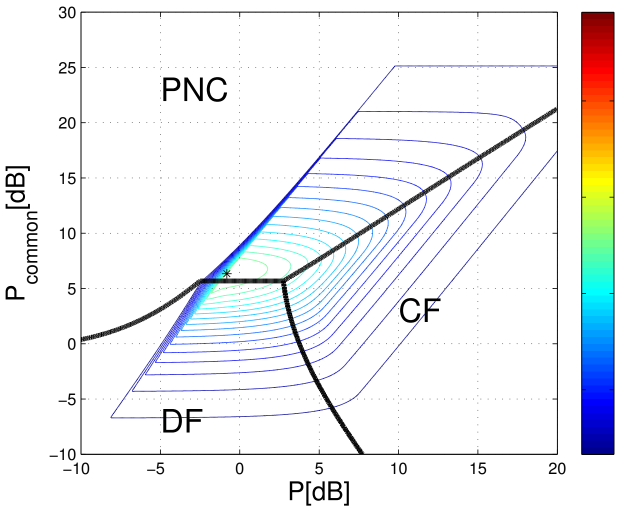

Fig. 1: Performance of different schemes in the symmetric SISO setting. The P vs. P_{common} plain is partitioned into areas where the individual schemes (DF, CF and sPNC) are superior. Then, the performance of optimal time-sharing is compared with the cut-set upper bound. The rounded contour lines show the difference, increasing inwardly until the point marked by an asterisk, where the maximal difference of 0.2625 bits is reached.

codeword, to obtain the codeword transmitted by the other terminal. The rate achievable using this scheme is given by:

$$R_{\text{PNC}} = \min\left\{ \left[\log\left(\frac{1}{2} + P\right)\right]^{+}, C_{\text{common}} \right\}, \qquad (6)$$

where $[x]^{+} \triangleq \max\{0, x\}$.

Both the CF rate (5) and the sPNC rate (6) are within one bit from the cut-set bound (3), as shown in [3] and [6], respectively. Fortunately, the worst-case parameters for the different schemes are different, and time-sharing can further improve performance; we find numerically, that time-sharing between the rates (4), (5) and (6) is within 0.2625 bits from optimality (and the guaranteed fraction of capacity is at least 78.77%), see Figure 1 (and [14, Figure 2b]).

Remark 1: We note that sPNC is very sensitive to synchronization, which may be an obstacle in practice: as sPNC relies on the linearity of the codebook by decoding the sum codewords transmitted by the two terminals, perfect symbol-synchronization is required. Using a cyclic codebook allows to support small synchronization skews, at the price of a loss in performance which grows with the skew size. The performance of the DF and CF strategies, on the other hand, is invariant to synchronization and hence may be a better candidate in certain real-world communication scenarios.

C. *Communication Schemes in the Asymmetric Setting*

We briefly recall generalizations beyond the symmetric setting. The cut-set outer-region is given by all rate-pairs $(R_1, R_2) \in \mathcal{C}_{\text{BC}}$, satisfying:

$$R_i \leq \log\left(1 + |h_i|^2 P\right), \quad i = 1, 2.$$

The extension of DF is straightforward. As for CF, there may be an advantage in using another layer, which is to be decoded by only one of the users. Such a layer can be combined with DF and/or a CF layer intended for both users, in which case it will compress a refinement of the signal received at the relay. See [3], [14] for details and for rate expressions.

The sPNC approach in the non-symmetric case achieves rate-pairs $(R_1, R_2) \in \mathcal{C}_{\text{BC}}$, satisfying [6]

$$R_i \leq \left[\log\left(\frac{|h_i|^2}{|h_1|^2 + |h_2|^2} + |h_i|^2 P\right)\right]^{+}. \qquad (7)$$

III. Communication Strategies for the MIMO Two-Way Relay Channel

In this section we consider the symmetric-rate setting (2a), and extend the cut-set outer bound and the different schemes of Section II to the MIMO case.

A. *Channel Model*

We consider a Gaussian MIMO setting, where terminal i ($i = 1, 2$) has $N_{t;i}$ transmit antennas and the relay has M_r receive antennas, during the MAC phase. Denoting vectors by boldface, the MAC channel is given by:

$$\boldsymbol{y} = H_1\mathbf{x}_1 + H_2\mathbf{x}_2 + \mathbf{z},$$

where H_i are $M_r \times N_{t;i}$ matrices, $\mathbf{z}$ is circularly-symmetric white Gaussian noise with unit variance, and the inputs are subject to some input covariance matrix constraints, the most common being an individual-power constraint (constraint on the diagonal elements of the covariance matrices) and total power constraint (constraint on the trace of the covariance matrices). We denote by K_i ($i = 1, 2$) the input covariance matrix used by terminal i during transmission.

We assume that the number of transmit antennas at each node $N_{t;i}$ is at least as large as the number of receive antennas M_r, and that the matrices H_1 and H_2 are full-rank, i.e., have rank M_r.[3] We further assume, w.l.o.g., that the products of the singular values, of each of the channel matrices, are equal to 1, or equivalently that:

$$\left|H_i H_i^{\dagger}\right| = 1, \quad i = 1, 2,$$

where $|\cdot|$ denotes the determinant.

As in the SISO case, the exact nature of the BC channel is not material and we characterize it using its common-message capacity C_{common}.

B. *Communication Schemes in the Symmetric-Rate Setting*

The cut-set bound, in this case, is given by

$$R_{\text{CS}} = \min\left\{C_1, C_2, C_{\text{common}}\right\},$$

where

$$C_i \triangleq \max_{K_i} \log\left|I + H_i K_i H_i^{\dagger}\right|, \quad i = 1, 2,$$

[3]For the cases in which the matrices are not full-rank or have more receive antennas, see the treatment in [15].

are the individual capacities of the MIMO links, and the maximization is carried over all K_i subject to the covariance-matrix constraints.

The achievable rate using the DF approach is equal to

$$C_{\text{DF}} = \min\left\{C_{\text{MAC}}, C_{\text{common}}\right\} ,$$

where

$$\begin{aligned} C_{\text{MAC}} = \max_{K_1,K_2} \min\Big\{ &\log\left|I + H_1K_1H_1^\dagger\right|, \\ &\log\left|I + H_2K_2H_2^\dagger\right| \;, \\ &\frac{1}{2}\log\left|I + H_1K_1H_1^\dagger + H_2K_2H_2^\dagger\right|\Big\} , \end{aligned}$$

and the maximization is carried over all admissible input covariance matrices K_1 and K_2 satisfying the power constraints.

Two independent works extended sPNC to the MIMO case, relying on two different joint unitary matrix triangularizations. The first, proposed by Yang et al. [10], relies on the GSVD [11]. Applying this decomposition to the effective channel matrices $H_iK_i^{1/2}$, we have:[4]

$$\begin{aligned} H_1K_1^{1/2} &= ULD_1V_1^\dagger \\ H_2K_2^{1/2} &= ULD_2V_2^\dagger , \end{aligned}$$

where U, V_1 and V_2 are unitary matrices, L is a lower-triangular matrix, and D_1 and D_2 are diagonal matrices with positive values satisfying $D_1^2+D_2^2 = I$. Define $L_1 \triangleq LD_1$ and $L_2 \triangleq LD_2$ and denote their diagonals by $\mathbf{d}_1^{\text{GSVD}} \triangleq \operatorname{diag}\{L_1\}$ and $\mathbf{d}_2^{\text{GSVD}} \triangleq \operatorname{diag}\{L_1\}$. In terms of these values, we have the following achievable rates, which are an improved variant of Theorem 1 in [10].

Theorem 1: For any admissible input covariance matrices, the following symmetric rate is achievable:

$$\begin{aligned} R_{\text{PNC}}^{\text{GSVD}} &= \min\left\{R_{\text{PNC},1}^{\text{GSVD}} ,\, R_{\text{PNC},2}^{\text{GSVD}} ,\, C_{\text{common}}\right\} \\ R_{\text{PNC},i}^{\text{GSVD}} &= \sum_{j=1}^{M_r}\left[\log\left(\frac{\left|d_{i,j}^{\text{GSVD}}\right|^2}{\left|d_{1,j}^{\text{GSVD}}\right|^2 + \left|d_{2,j}^{\text{GSVD}}\right|^2} + \left|d_{i,j}^{\text{GSVD}}\right|^2\right)\right]^+ . \end{aligned}$$

where $\{d_{i,j}^{\text{GSVD}}\}$ are given by the GSVD defined above.

Proof sketch: By applying V_1 and V_2 at encoders 1 and 2 (in addition to $K_1^{1/2}$ and $K_2^{1/2}$), respectively, and $U^\dagger$ at the decoder, we attain the effective channel matrices L_1 and L_2. L_1 and L_2 are equal to products of the same lower-triangular matrix and different diagonal matrices, and thus are lower-triangular with proportional rows. This proportionality, in turn, allows to utilize SIC. Using asymmetric sPNC (7) over the resulting channels with gains $\left\{d_{1,j}^{\text{GSVD}}, d_{2,j}^{\text{GSVD}}\right\}$, achieves (9). After the decoding of each scalar stream, the coarse (shaping) lattice is decoded over the reals, to facilitate SIC from not-yet decoded subchannels. This is possible with arbitrarily small error [16], allowing to recover the sum over the reals of the two lattice codewords. ■

[4] $K^{1/2}$ is any matrix B satisfying $BB^\dagger = K_i$, and can be found, e.g., via the Cholesky decomposition.

The second extension of sPNC, proposed in [12], relies on applying the JET [13] to the effective channel matrices. The JET of the effective channel matrices $H_iK_i^{1/2}$ is given by:

$$\begin{aligned} H_1K_1^{1/2} &= UL_1V_1^\dagger \\ H_2K_1^{1/2} &= UL_2V_2^\dagger , \end{aligned}$$

where U, V_1 and V_2 are unitary matrices, and L_1 and L_2 are lower-triangular with equal diagonals $\mathbf{d}^{\text{JET}}$:

$$\mathbf{d}^{\text{JET}} \triangleq \operatorname{diag}(L_1) = \operatorname{diag}(L_2) . \tag{11}$$

While [12] applies symmetric sPNC (6) to the resulting scalar channels, in this work we generalize the result to any symmetric scalar strategy, as follows.

Theorem 2: Let $R(d, C)$ be an achievable symmetric rate for the SISO TWRC (2), with MAC gains $h_1 = h_2 = d$ and common-message BC capacity C. Then, the following symmetric rate is achievable:

$$R^{\text{JET}} = \sum_{j=1}^{M_r} R(d_j^{\text{JET}}, R_j)$$

for any non-negative rates R_j satisfying $\sum_{j=1}^{M_r} R_j \leq C_{\text{common}}$.

Proof sketch: By applying V_1 and V_2 at encoders 1 and 2 (in addition to $K_1^{1/2}$ and $K_2^{1/2}$), respectively, and $U^\dagger$ at the decoder, we attain the effective channel matrices L_1 and L_2. The equal diagonals of L_1 and L_2 allow to cancel out their off-diagonal elements via DPC, resulting in symmetric scalar subchannels with gains (11). Over the resulting symmetric scalar subchannels, a SISO TWRC strategy is used. ■

Substituting the symmetric scalar sPNC rates (6) we get [12, Thm. 1] as a special case. Alternatively, DF, CF or any time-sharing of schemes can be used.

Remark 2: While the transformation to scalar channels is necessary for sPNC, DF and CF can also work over a vector channel. Although the analysis and implementation may be very complicated, it may yield some performance improvement. To that end, one may apply sPNC to some of the subchannels, and another scheme jointly over the others. In that case, balancing diagonal values in the non-sPNC block is not imperative; see [13] for a block version of the JET.

C. Comparison of Decompositions

The GSVD- and JET-based approaches both translate the MIMO problem into parallel SISO ones, and both become optimal in the limit of high SNR (assuming full-rank channel matrices). The GSVD-based scheme also carries over to the asymmetric case. However, we note that the JET-based scheme has the following advantages:

1) **Use of any strategy.** Since the JET approach uses DPC, any strategy can be used over the subchannels; the decoder for each subchannel will receive an input signal as if this were the only channel. In contrast, the GSVD approach uses SIC, where the task of canceling inter-channel interference is left to the relay. In order to cancel out interference, the relay thus needs to decode.

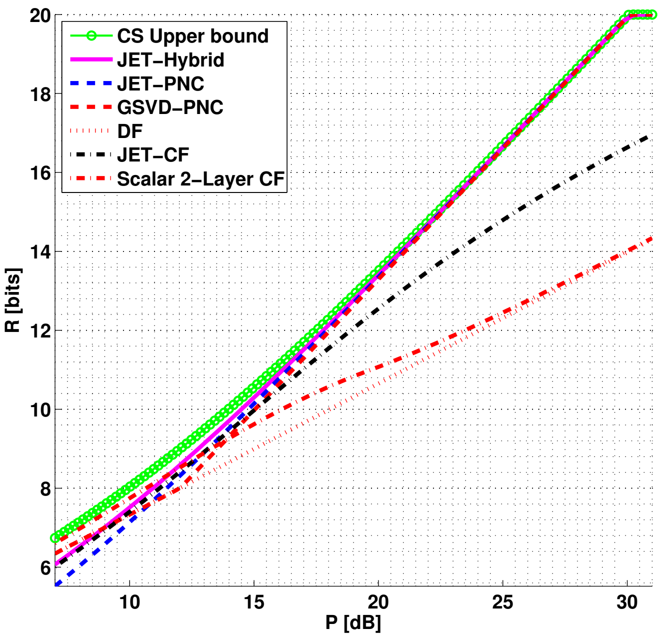

Fig. 2: Performance of the proposed strategies for $C_{\text{common}} = 20$ bits.

This is the case for DF and, as Theorem 1 shows, for sPNC as well; however, CF cannot be used.

2) **Symmetric subchannels.** The JET approach gives rise to *symmetric* scalar channels, while GSVD gives asymmetric ones.[5] Symmetry allows for simpler schemes. Furthermore, asymmetric techniques seem to have inherent losses, since they require a trade-off between "strong" and "weak" signals. While we do not prove that balanced channels are always better, this seems to be the case, as illustrated in the example below.

The different techniques are illustrated in the following simple example.

Example 1: Consider a Gaussian MIMO TWRC with a MAC phase comprising two parallel asymmetric channels

$$H_1 = \begin{pmatrix} 1/4 & 0 \\ 0 & 4 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 4 & 0 \\ 0 & 1/4 \end{pmatrix},$$

and a common-message BC capacity of $C_{\text{common}} = 20$ bits, where the terminals are subject to a per-antenna individual power constraint P.

Figure 2 depicts the different achievable rates of Section III as a function of P.

In contrast to the case of general channel matrices, in the case of parallel channels (corresponding to diagonal channel matrices), all the scalar asymmetric techniques of Section II-C can be used. Nonetheless, one observes that these techniques are inferior to their symmetric counterparts (resulting after applying the JET). This gap is especially pronounced, if we compare the optimum asymmetric strategy with the optimal JET-based hybrid strategy.

[5]In fact, the GSVD provides the "most spread diagonal ratios" out of all possible joint unitary matrix triangularizations of given two matrices; see [13].

IV. Discussion

The decomposition used in Theorem 2 is of a zero-forcing flavor. Namely, we do not allow residual interference below the diagonal. Nevertheless, it is well known that zero-forcing techniques suffer from noise enhancement and can therefore be improved by balancing between residual interference and the physical (Gaussian) noise. Indeed, at low SNR the MMSE variant of DF, achieved using MMSE V-BLAST outperforms the JET-based scheme of Theorem 2. Constructing an MMSE variant for the proposed JET-based scheme is more challenging and is left as future research.

In the case where there are additive interferences known at the terminals (but not to the relay), the result of Theorem 2 still holds, as it combines naturally with dirty-paper coding. The performance of the GSVD-based scheme of Theorem 1, on the other hand, deteriorates (the rate goes to zero in the extreme case of very strong interferences) as it is based on successive cancellation of the decoded messages.

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