Control over Noisy Communication Media

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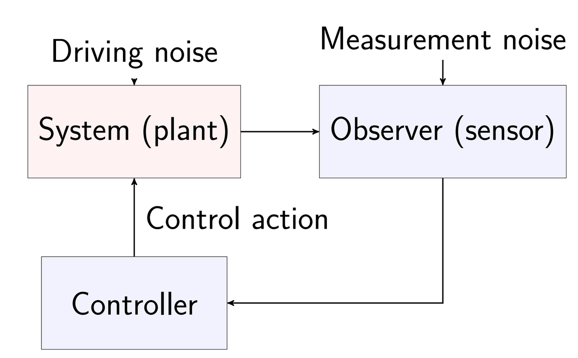
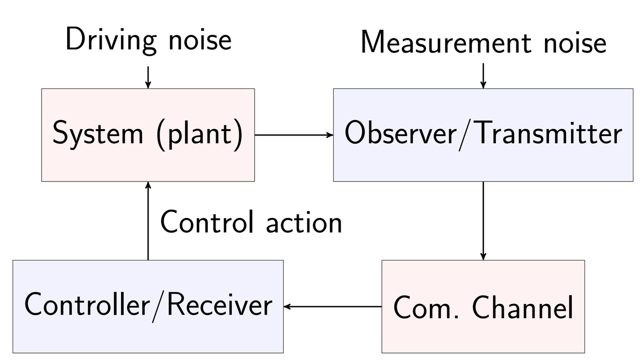
Networked Control Setting and Approaches

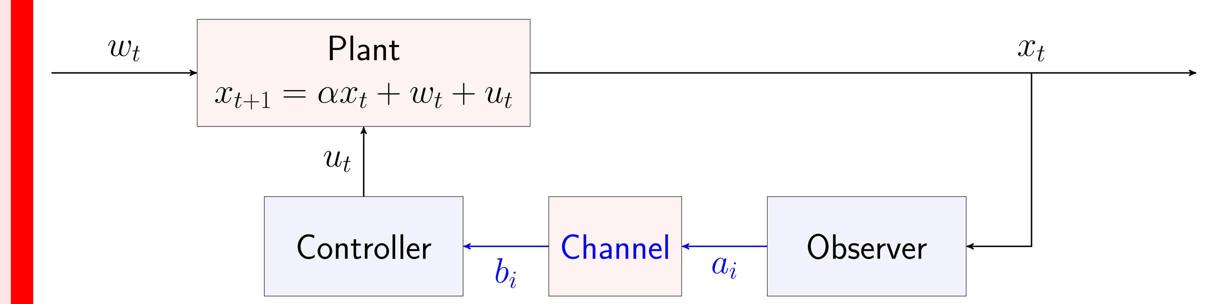
Traditional versus Networked Control





Linear Quadratic Gaussian Control over Noisy Channels

$x_{t+1} = \alpha x_t + u_t + w_t, \quad w_t \sim$ i.i.d. $\mathcal{N}(0, W), \quad |\alpha| > 1$



LQG cost: $\bar{J}_T = \mathbb{E}\left[\sum_{t=1}^{T-1}\left(\mathsf{Q}_t x_t^2 + \mathsf{R}_t u_t^2\right) + \mathsf{Q}_T x_T^2\right]$

LQG Control over Gaussian Channels

Control rate ≠ Communication rate!

- How fast the plant dynamic is ⇒ Control sampling rate
- Bandwidth available ⇒ Communication signaling rate
- Communication rate can be much higher in practice
- Assume N channel uses per one control sample

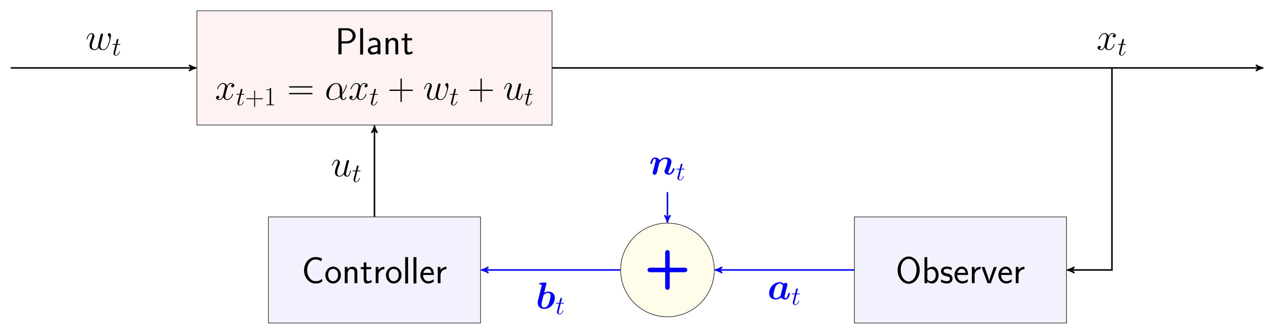
Scalar LQG system

$x_{t+1} = \alpha x_t + u_t + w_t$

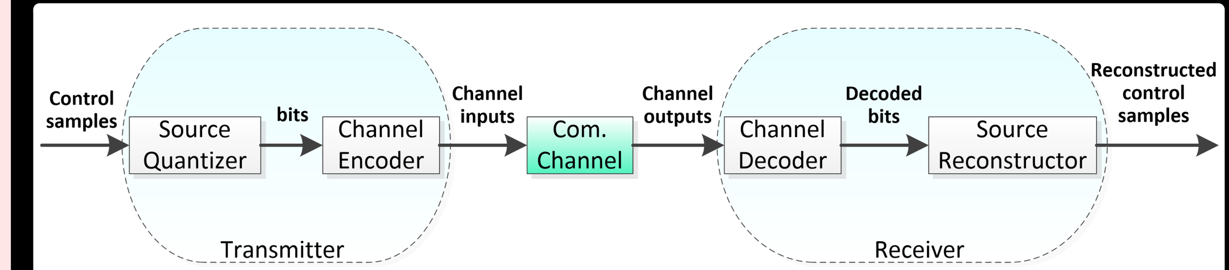
Scalar Gaussian channel

$\boldsymbol{b}_t = \boldsymbol{a}_t\left(x^t, u^{t-1}\right) + \boldsymbol{n}_t \quad \updownarrow N$

Power constraint: $\mathbb{E}\left[\boldsymbol{a}_t^2\right] \le NP$



Source–Channel Separation and Analog Codes



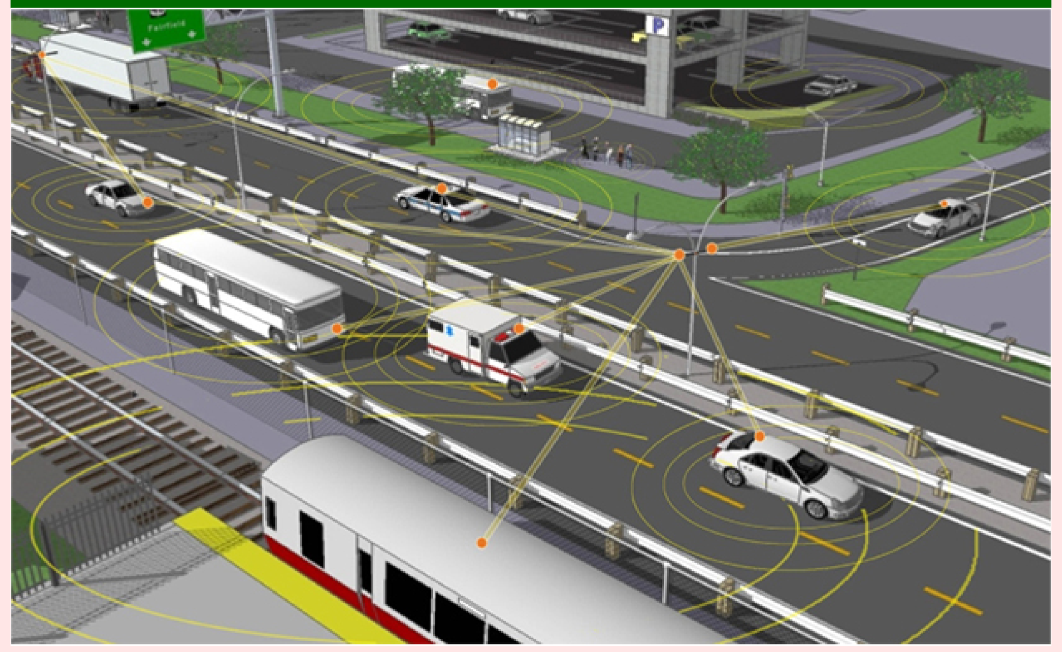
Com. is broken into two tasks → Bits serve as an interface:

1) Quantization: Batch of control samples → Block of bits
2) Channel coding: Block of bits → Batch of channel inputs

- Breaks down design & analysis tasks into two simpler tasks
- Optimal when block lengths **(=delay!)** go to infinity
- **Suboptimal for control!**
- **Better alternative**: Analog maps (*no going through bits*)

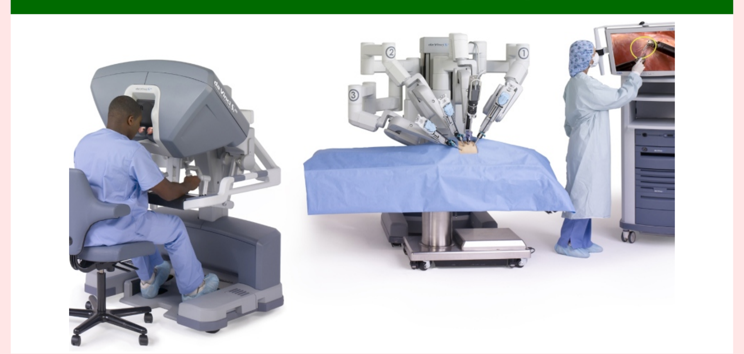
Applications: Cyber-Physical Systems and the Internet of Things

Self-Driving Cars

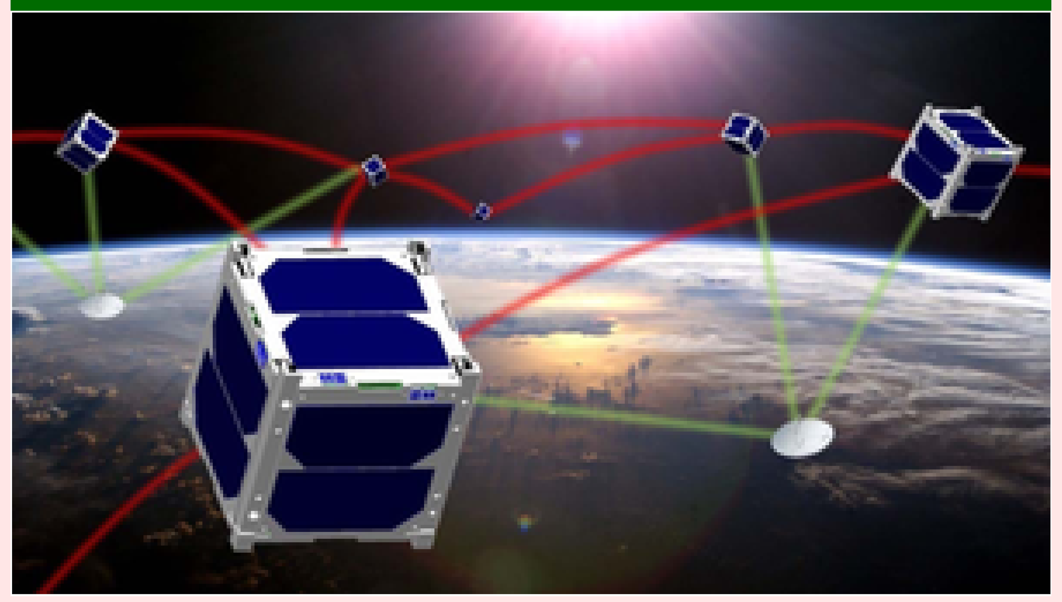


Internet of Things

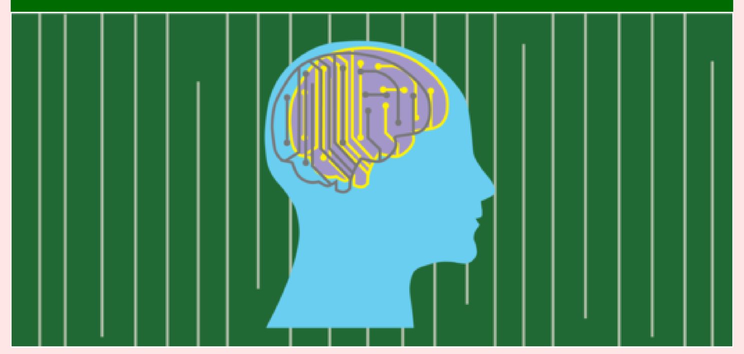
Remote Surgery



Small Satellites



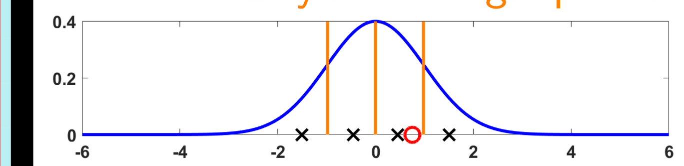
Neuroscience



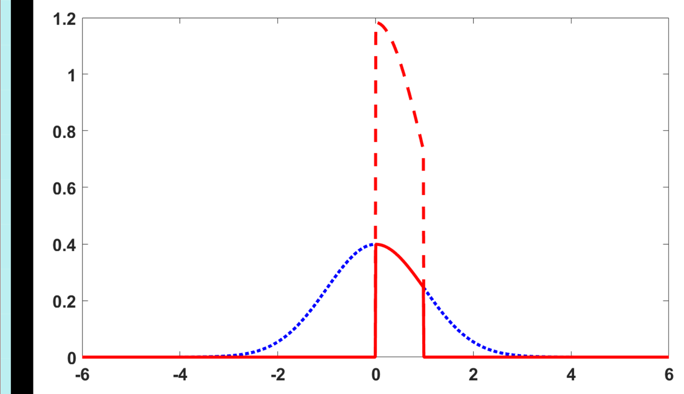
Control over a Bit-Pipe

Channel = Finite-rate noiseless

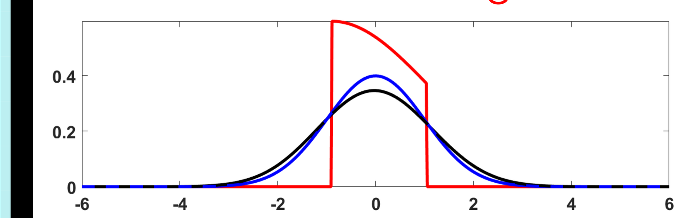
- Log-concave PDF↘ Lloyd-Max Alg. optimal!



- $f(x_1 - \hat{x}_1 | x_1 \in$ cell $i)$



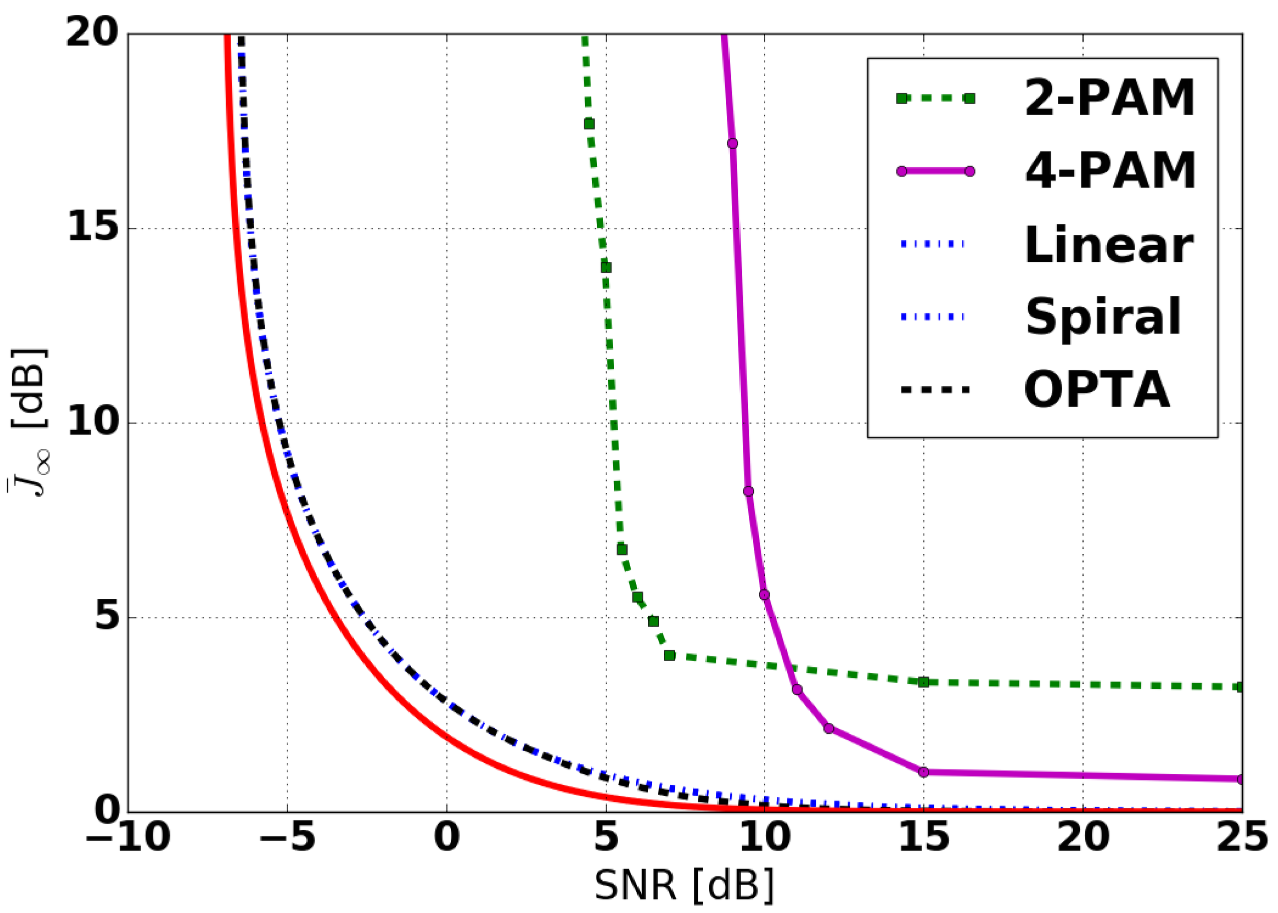
- $\alpha(x_1 - \hat{x}_1) + w_1$
Convolution of log-concave PDFs = log-concave!



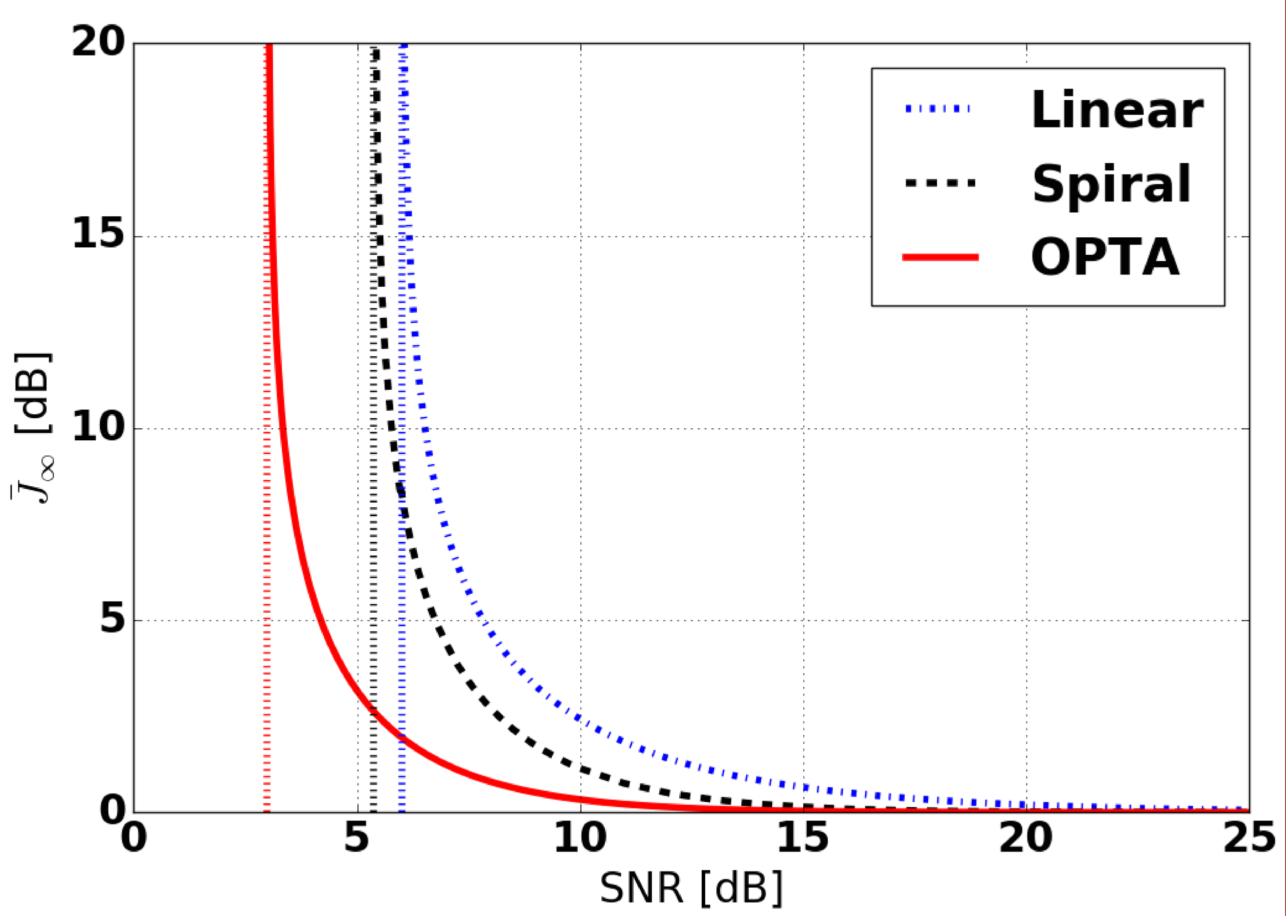
- Log-concave PDF↘ Lloyd-Max Alg. optimal!

Performance Comparison

- $\alpha = 1.2, W_1 = 1, \mathsf{Q}_t \equiv 1, \mathsf{R}_t = 0$



- $\alpha = 3, W_1 = 1, \mathsf{Q}_t \equiv 1, \mathsf{R}_t = 0$



- **Higher-order moments:** Separation-based schemes fail!
- Analog maps can stabilize **all moments** and guarantee **a.s. stability**!
- Better curves can be devised

Joint Source–Channel Analog Code Design

- N Gaussian channel uses per one control sample

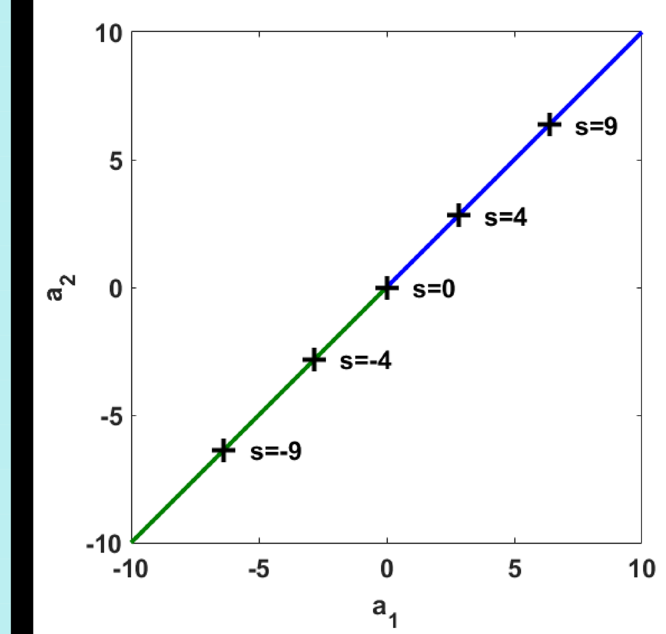
Rate match ($N = 1$): One channel use per one control sample

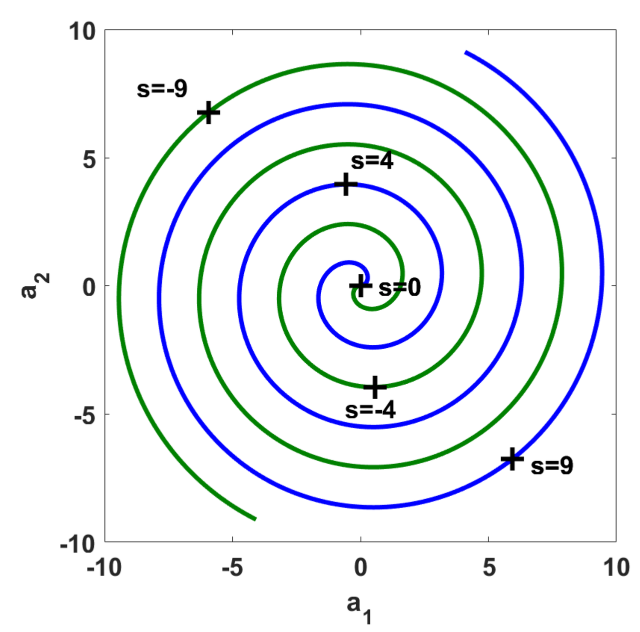
- **Linear analog transmission is optimal:** $a_t = x_t$
  - ▷ Equality up to power adjustment (=known constant)
- **Conclusion:** No coding is needed!

Rate mismatch: $N = 2$ channel uses per one control sample

- Linear scheme gains nothing over $N = 1$ case: $\mathrm{SNR}^{\mathrm{eff}} = 2\mathrm{SNR}$ ☹
- Info.-theoretic asymptopia (delay → ∞): $1 + \mathrm{SNR}^{\mathrm{eff}} = (1 + \mathrm{SNR})^2$

$$\begin{cases} a_1(s) = \frac{1}{\sqrt{2}}s \\ a_2(s) = \frac{1}{\sqrt{2}}s \end{cases} \qquad \begin{cases} a_1(s) = s\cos(2s) \\ a_2(s) = s\sin(2s)\,\mathrm{sign}(s) \end{cases}$$





- Different parametrization (pre-map) further helps: $s \to |s|^\beta$
- **Achieves $\mathrm{SNR}^{\mathrm{eff}} \propto \mathrm{SNR}^2$ ⇒ Better stabilizability and cost**

Channel Anytime-Reliable Tree Codes

How to generate a good code?

- Tree codes proved to exist [Schulman IT'96]
  - ▷ Non-constructive proof
- $P_e(t, d)$ - First-error prob. d steps back, at time t
- Has to drop faster than a^{2d} [Sahai-Mitter IT'06]: $P_e(t, d) < A2^{-(2\log\alpha+\epsilon)d}, \quad \forall t, \forall d$
- Important step towards practicality ⇒ *exist w.h.p.* [Sukhavasi-Hassibi AC'16]

How to decode?

- Optimal decoding is difficult $\sim O(2^N)$
- Sequential decoding [Wozencraft '57]
- Guarantees [Jelinek's Book '68]
- *Guarantees w.h.p.* [Kh.-Halbawi-Hassibi ISIT'16]

Universality [Kh.-Halbawi-Hassibi, submitted IT'17]

- Guarantees w.h.p. ⇒ Universality w.r.t. channel