Decode-and-Forward for the Gaussian Relay Channel via Standard AWGN Coding and Decoding

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Black Box Approach to Point-to-Point MIMO Channels: $y = Hx + z, \quad z \sim \mathcal{CN}(0, I)$

SVD

- $H = QDV^\dagger$
- Q and V — unitary
- Apply V at Tx and Q at Rx
- $D = \begin{pmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & d_{n-1} & 0 \\ 0 & \cdots & 0 & 0 & d_n \end{pmatrix} \Rightarrow \begin{matrix} y_1 = d_1 x_1 + z_1 \\ y_2 = d_2 x_2 + z_2 \\ \vdots \\ y_n = d_n x_n + z_n \end{matrix}$
- Results in parallel scalar sub-channels (each sub-channel has a different SNR)

QR

- $H = QT$
- Q — unitary
- Apply Q at Rx (no SP is required at Tx)
- $T = \begin{pmatrix} t_1 & * & * & \cdots & * \\ 0 & t_2 & * & \cdots & * \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & t_{n-1} & * \\ 0 & 0 & \cdots & 0 & t_n \end{pmatrix} \Rightarrow \begin{matrix} y_1^{\text{eff}} = t_1 x_1 + z_1 \\ y_2^{\text{eff}} = t_2 x_2 + z_2 \\ \vdots \\ y_n^{\text{eff}} = t_n x_n + z_n \end{matrix}$
- Off-diagonal elements are canceled via successive interference cancellation (SIC)

GMD

- $H = QTV^\dagger$
- Q and V — unitary
- Apply V at Tx and Q at Rx
- $T = \begin{pmatrix} t & * & * & \cdots & * \\ 0 & t & * & \cdots & * \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & t & * \\ 0 & 0 & \cdots & 0 & t \end{pmatrix} \Rightarrow \begin{matrix} y_1^{\text{eff}} = t x_1 + z_1 \\ y_2^{\text{eff}} = t x_2 + z_2 \\ \vdots \\ y_n^{\text{eff}} = t x_n + z_n \end{matrix}$
- Off-diagonal elements are canceled via SIC
- **Constant diag.** $\Rightarrow$ same code over all sub-channels

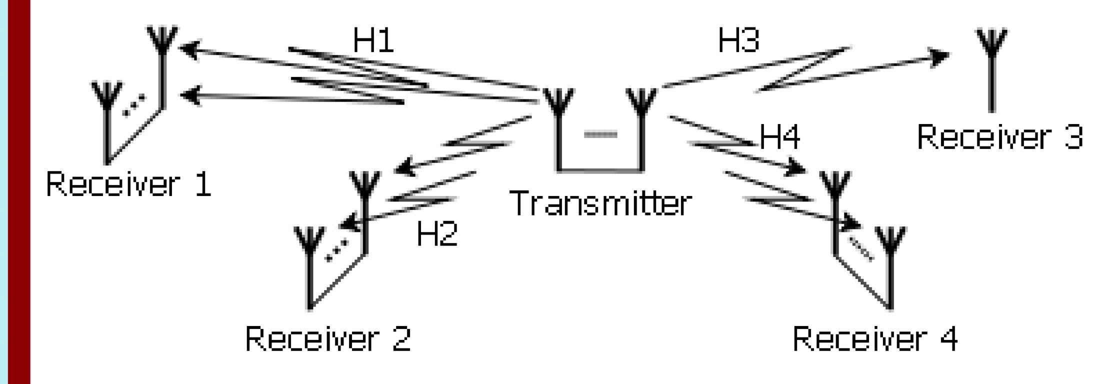
All three approaches reduce to the **"black box"** task:

- **Coding over single-user AWGN scalar channels**
- **Any "off-the-shelf" (fixed SNR) single-user codes apply**

How to implement MIMO multicast via a black box approach?

Multicast

- Same information is transmitted to all users



Joint Equi-Diagonal Triangularization (JET)

- H_1 and H_2 – $N \times N$ non-singular matrices
- $\det(H_1) = \det(H_2)$
- H_1 and H_2 can be jointly decomposed as:

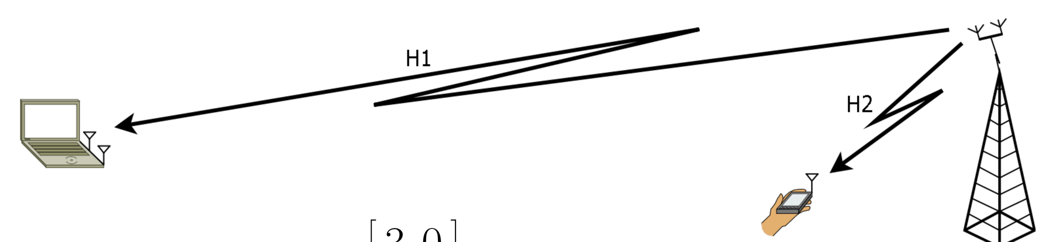
$$H_1 = Q_1 T_1 V^\dagger$$
$$H_2 = Q_2 T_2 V^\dagger$$

- Q_1, Q_2, V – unitary
- T_1 and T_2 are upper-triangular with **equal** diagonals

Remarks

- Can be extended to non-square matrices
- Works for general SNRs by decomposing $\begin{pmatrix} H_i C_x^{1/2} \\ I \end{pmatrix}$
- JET is **possible for more users** via time-extensions...

Illustrative Example



$$H_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad H_2 = \begin{bmatrix} \sqrt{99} & 0 \end{bmatrix}$$

- $C_1^{\text{WI}} = 2\log(1+3^2) = \log\left(1 + \left(\sqrt{99}\right)^2\right) = C_2^{\text{WI}}$
- Send same signal over both antennas

$\Downarrow$

Losses half of the rate at High SNR!

Using JET

Matrix V is applied to $\begin{bmatrix} H_i \\ I_{N_t} \end{bmatrix}$ (MMSE variant for general SNR):

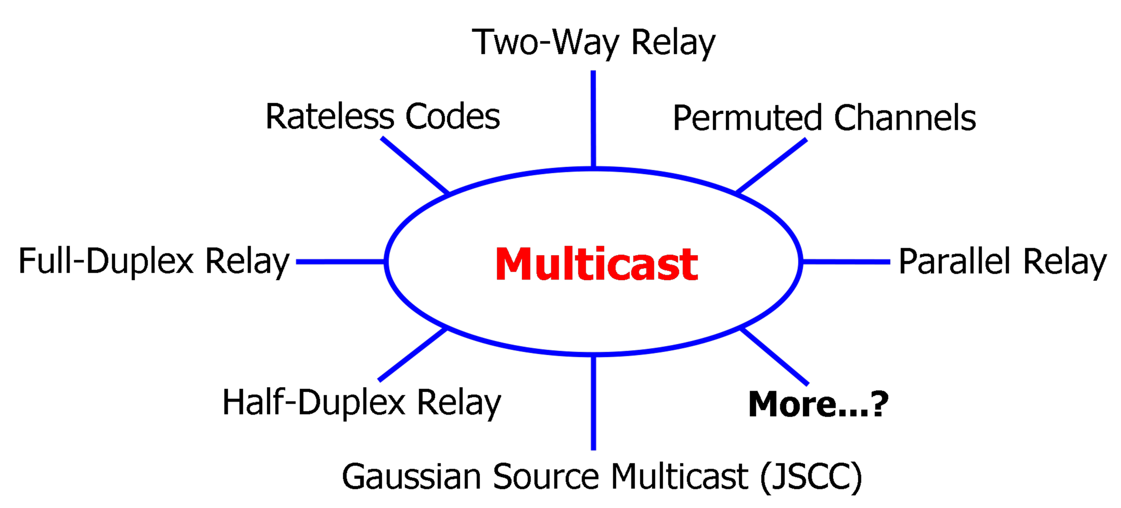
$$\begin{bmatrix} H_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 0.286 & -0.905 \\ 0.905 & 0.286 \\ 0.095 & -0.301 \\ 0.301 & 0.095 \end{bmatrix}}^{Q_1} \overbrace{\begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{10} \end{bmatrix}}^{T_1} \overbrace{\begin{bmatrix} 0.302 & 0.954 \\ -0.954 & 0.302 \end{bmatrix}}^{V^\dagger}$$

$$\begin{bmatrix} H_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \sqrt{99} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 0.949 & -0.300 \\ 0.905 & -0.030 \\ 0.302 & 0.954 \end{bmatrix}}^{Q_2} \overbrace{\begin{bmatrix} \sqrt{10} & -9 \\ 0 & \sqrt{10} \end{bmatrix}}^{T_2} \overbrace{\begin{bmatrix} 0.302 & 0.954 \\ -0.954 & 0.302 \end{bmatrix}}^{V^\dagger}$$

- $Q_1^\dagger Q_1 = Q_2^\dagger Q_2 = V^\dagger V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\text{diag}(T_1) = \text{diag}(T_2) = \left[\sqrt{10} \ \sqrt{10}\right]^T$

Parallel SISO channels with equal gains for both users!

Multicast is (Almost) Everywhere...



Gaussian Rateless Coding

$$y = \alpha x + z,$$

- α **is unknown at Tx** but **is known at Rx**
- Rx sends NACKS until it is able to recover the message
- α can take only a finite number of values: $\alpha_1, \alpha_2, \ldots$
- Can be represented as a MIMO multicast problem

Example $\alpha \in \{\alpha_1, \alpha_2\}$, $\alpha_1 > \alpha_2$

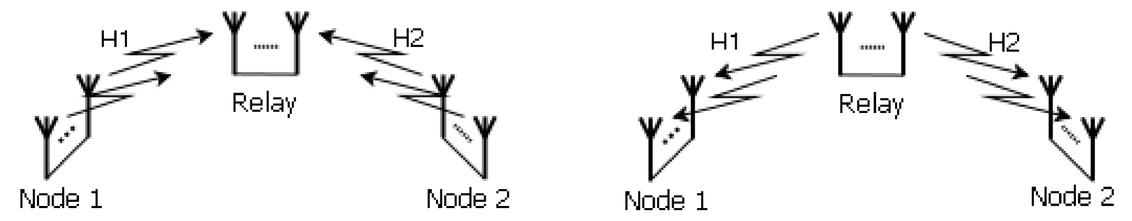
- $C_1 = 2C_2$
- **Effective Matrices in MIMO Multicast Scheme:**

$$H_1^{\text{eff}} = \begin{pmatrix} \alpha_1 & 0 \end{pmatrix}$$
$$H_2^{\text{eff}} = \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{pmatrix}$$

- Coincides with the solution of [Erez, Trott, Wornell]
- Works for MIMO channels H_1, H_2 (replacing α_1, α_2)

MIMO Two-Way Relay (New Achievable)

- Two nodes want to exchange messages via a relay



(a) MAC Phase (b) Broadcast Phase

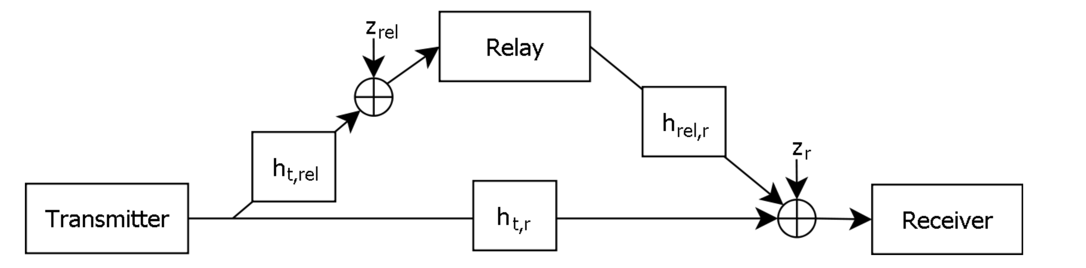
MAC Phase

- Apply JET to H_1 and H_2 (roles of V and Q switched)
- Use dirty-paper coding to pre-cancel off-diagonal elements (Replaces successive interference cancellation of multicast)

Broadcast (Multicast!) Phase

- Use MIMO multicast scheme

Relay Channel



Half-Duplex Relay

- Relay can receive or transmit but **not both**
- Decode-and-forward implementation: "**rateless relay**"

Effective Matrices in MIMO Multicast Scheme:

$$H_1^{\text{eff}} = \left(\sqrt{P_1} h_{t,\text{rel}} \ 0 \cdots 0\right)$$

$$H_2^{\text{eff}} = \begin{pmatrix} \sqrt{P_1} h_{t,r} & 0 & \cdots & 0 \\ 0 & \begin{matrix}\sqrt{P_2} h_{t,r} \\ + \sqrt{P_{\text{rel}}} h_{\text{rel},r}\end{matrix} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \begin{matrix}\sqrt{P_2} h_{t,r} \\ + \sqrt{P_{\text{rel}}} h_{\text{rel},r}\end{matrix} \end{pmatrix}$$

Full-Duplex Relay

- Relay can receive and transmit **simultaneously**
- Decode-and-forward implementation (*previous works*): *Special* code constructions
- But... **"Off-the-shelf" codes** suffice!

Effective Matrices in MIMO Multicast Scheme:

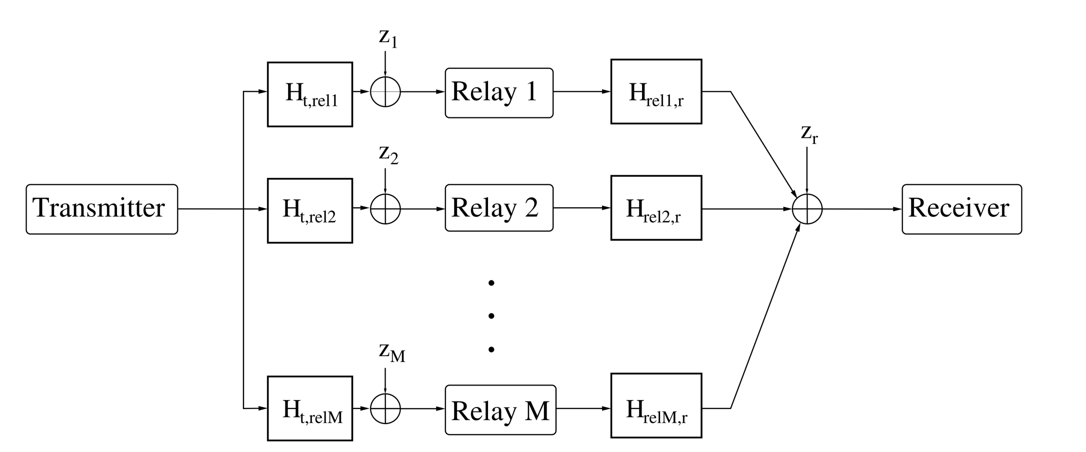
$$H_{\text{rel}}^{\text{eff}} = \sqrt{2}\left(\sqrt{1-\rho^2} h_{t,\text{rel}} \ 0\right)$$

$$H_r^{\text{eff}} = \sqrt{2}\begin{pmatrix} \sqrt{1-\rho^2} h_{t,r} & 0 \\ 0 & \frac{\rho h_{t,r} + h_{\text{rel},r}}{\sqrt{((1-\rho^2) h_{t,r}^2 P + 1}} \end{pmatrix}$$

- $0 \le \rho^2, 1-\rho^2 \le 1$— power portions allocated to consecutive sub-messages in transmitted signal by Tx

More Relays with/without Line-of-Sight

- Assume more relays with no line-of-sight (LoF)



Decode-and-Forward

- Similar to two-way relay scheme (MAC $\longleftrightarrow$ BC)
- **MAC phase:** Use MIMO multicast scheme
- **Broadcast phase:** Apply JET and use DPC
- **More relays + LoF:** Combine with previous scheme