Network Modulation:

Transmission Technique for MIMO Networks

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Joint work with:
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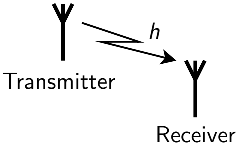
Talk Outline

- Novel MIMO multicast scheme
  - Two-user: via new joint decomposition of two matrices
  - Multi-user: via algebraic space–time coding structure
- Various applications
- New information-theoretic results

MIMO Multicast (Closed Loop): State of the Art

	Unicast	Multicast
Theory		
SISO		
MIMO		

Single-Input Single-Output (SISO) Unicast



$$y = hx + z$$

- x – Input of power 1
- y_i – Output
- h – Channel gain
- z – White Gaussian noise $\sim \mathcal{CN}(0,1)$
- Optimal communication rate (capacity): $C = \log(1 + |h|^2)$
- **Good practical codes that approach capacity are known!**

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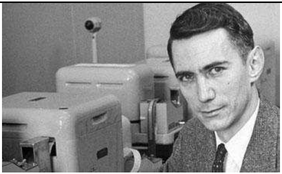
Theory	 ✓	
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MIMO		

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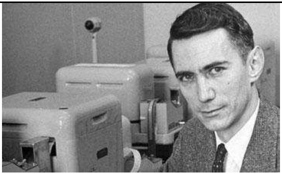
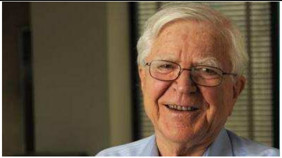
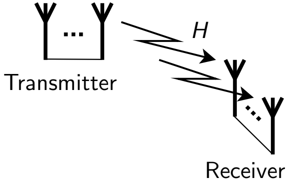
Theory	 ✓	
SISO	 ✓	
MIMO		

Multiple-Input Multiple-Output (MIMO) Unicast



$$\mathbf{y} = H\mathbf{x} + \mathbf{z}$$

- $\mathbf{x}$ – Input vector of power $1 \cdot N_t$
- $\mathbf{y}$ – Output vector
- H – Channel matrix
- $H_{k\ell}$ – Gain from transmit-antenna ℓ to receive-antenna k.
- z – White Gaussian noise $\sim \mathcal{CN}(\mathbf{0}, I)$
- Optimal rate (capacity): $C = \max_{C_\mathbf{x}} \log(1 + HC_\mathbf{x}H^\dagger) \approx \log(1 + HH^\dagger)$

MIMO Multicast (Closed Loop): State of the Art

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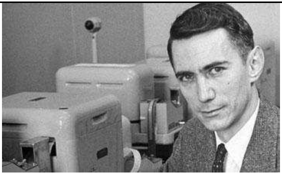
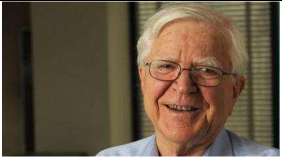
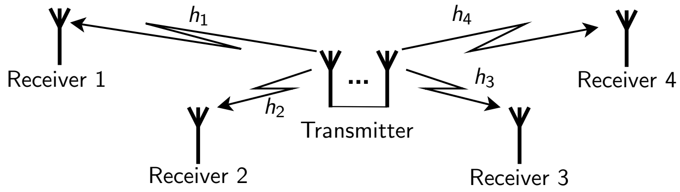
Theory	 ✓	
SISO	 ✓	
MIMO	 ✓	

SISO Multicast



$$y_i = h_i x + z_i \quad i = 1, \ldots, K$$

- x – Input of power 1
- y_i – Output of user i
- h_i – Channel gain to user i
- z_i – White Gaussian noise $\sim \mathcal{CN}(0,1)$
- Optimal rate (capacity): $C = \min_i \log(1 + |h_i|^2)$

MIMO Multicast (Closed Loop): State of the Art

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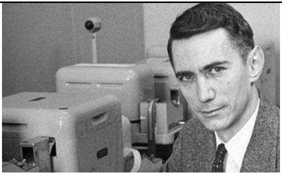
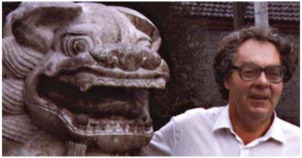
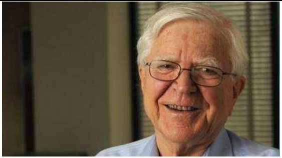
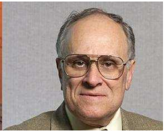
Theory	 ✓	 ✓
SISO	 ✓	
MIMO	  ✓	

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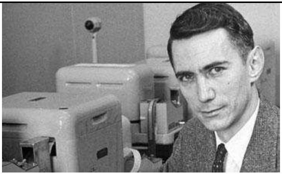
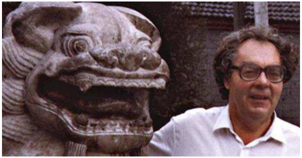
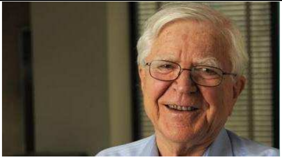
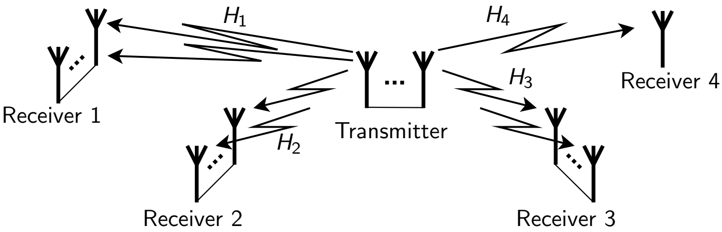
Theory	 ✓	 ✓
SISO	 ✓	 ✓
MIMO	 ✓	

Gaussian MIMO Multicast



$$\mathbf{y}_i = H_i \mathbf{x} + \mathbf{z}_i \quad i = 1, \ldots, K$$

- $\mathbf{x}$ – $N_t \times 1$ input vector of power $N_t \cdot 1$
- $\mathbf{y}_i$ – Output vector of user i
- H_i – Channel matrix to user i
- $\mathbf{z}_i$ – White Gaussian noise vector $\sim \mathcal{CN}(\mathbf{0}, I)$
- "Closed loop" (Full channel knowledge everywhere)

Optimal Achievable Rate (Capacity)

Multicasting capacity

$$\mathcal{C} = \max_{C_X} \min_{i=1,\ldots,K} \log\left\{\det\left(I + H_i C_X H_i^\dagger\right)\right\}$$

- Optimization over covariance matrices C_X satisfying the power constraint

White Input / High SNR

$$\mathcal{C}_{\mathrm{WI}} \approx \min_{i=1,\ldots,K} \log\left\{\det\left(I + H_i H_i^\dagger\right)\right\}$$

MIMO Multicast (Closed Loop): State of the Art

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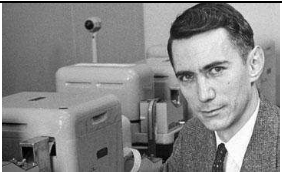
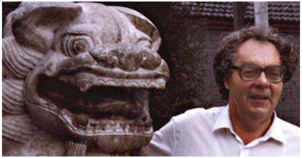
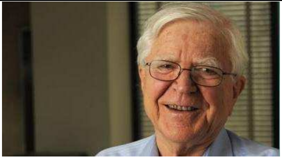
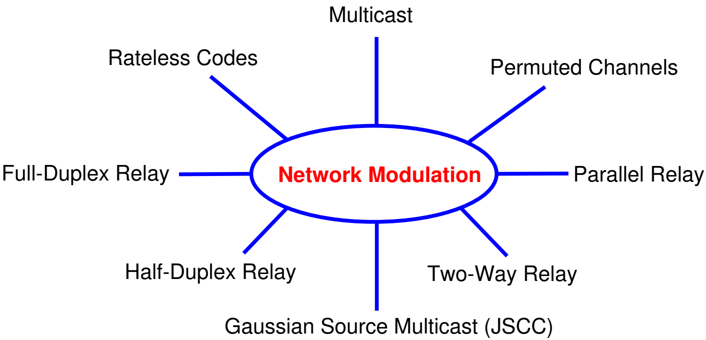
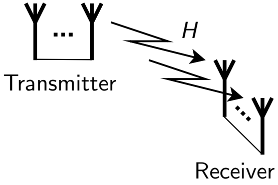
Theory	 ✓	 ✓
SISO	 ✓	 ✓
MIMO	 ✓	 ?

Summary: Multicast is (Almost) Everywhere...



Unicast



$$\mathbf{y} = H\mathbf{x} + \mathbf{z}$$

White-input capacity

$$\mathcal{C}_{\mathrm{WI}} = \log\left\{\det\left(I + HH^{\dagger}\right)\right\}$$

- But how is this rate achieved?

Practical (Capacity-Achieving) Unicast Approaches

(Info. Theory) White-input capacity

$$\mathcal{C}_{\mathrm{WI}} = \log\left\{\det\left(I + HH^{\dagger}\right)\right\}$$

(Comm.) Achieving this rate with a practical scheme

- Singular value decomposition (SVD)
- QR decomposition (GDFE / V-BLAST)
- Geometric mean decomposition (GMD)
- Dirty-paper coding (DPC)

Practical (Capacity-Achieving) Unicast Approaches

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- **Singular value decomposition (SVD)**
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Singular Value Decomposition (SVD)

$$H = Q \Lambda V^\dagger$$

- Q, V – Unitary
- Λ – Diagonal
- Parallel AWGN SISO sub-channels $\rightarrow$ "off-the-shelf" codes
- Diagonal of Λ = SISO channel gains $\Rightarrow R_i = \log\left(1 + \lambda_i^2\right)$

Generalization to Multicast?

- Precoding matrix V depends on channel matrix H
- Which V to take??
- **Bottleneck problem** $(\Lambda_1 \neq \Lambda_2)$

$\Downarrow$

~~SVD~~

QR Decomposition (GDFE/V-BLAST)

$$H = QT$$

- Q – Unitary
- T – Upper-triangular matrix
- Successive interference cancellation
- Parallel AWGN SISO channels
- Diagonal of T – SISO channel gains

Remark

Both zero-forcing and MMSE (capacity-achieving) solutions exist.

QR Based Scheme

Scheme

- **Channel:** $\mathbf{y} = H\mathbf{x} + \mathbf{z} = QT\mathbf{x} + \mathbf{z}$
- **Transmitter:** $\mathbf{x}$ – SISO codebooks
- **Receiver:** $\tilde{\mathbf{y}} = Q^\dagger \mathbf{y} = T\mathbf{x} + Q^\dagger \mathbf{z}$
- $\tilde{\mathbf{z}} = Q^\dagger \mathbf{z} \sim \mathcal{CN}(0, I_N)$

Example for 2×2

$$\begin{aligned}
\tilde{y}_1 &= [T]_{11} x_1 + \overbrace{[T]_{12} x_2}^{\text{Interference}} + \tilde{z}_1 \\
\tilde{y}_2 &= 0\ x_1 + [T]_{22} x_2 + \tilde{z}_2
\end{aligned}$$

QR Based Scheme

Generalization of QR based solution to Multicast?

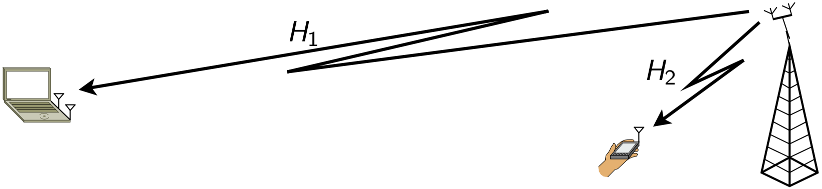
- T depends on H.
- For two channel matrices H_1 and H_2:
  $\text{diag}(T_1) \neq \text{diag}(T_2) \Rightarrow$ **different sub-channel gains!**
- **Bottleneck problem:**
  - **Info. Theory:** $\sum_{j=1}^{N_t} \log\left(|[T_1]_{jj}|^2\right) = \sum_{j=1}^{N_t} \log\left(|[T_2]_{jj}|^2\right)$ ✓
  - **Comm.:** $R_j = \log\left(\left|\min\left\{[T_1]_{jj}, [T_2]_{jj}\right\}\right|^2\right)$ X
- Can we have equal diagonals?

QR Based Scheme

Idea

- SVD uses both Q and V
- QR uses only Q
- Can V help in QR case to achieve equal diagonals?
- **YES!**

Illustrative Example



$$H_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad H_2 = \begin{bmatrix} \sqrt{99} & 0 \end{bmatrix}$$

- $C_1^{\mathrm{WI}} = 2\log\left(1+3^2\right) = \log\left(1+\left(\sqrt{99}\right)^2\right) = C_2^{\mathrm{WI}}$
- Send same signal over both antennas

  $\Downarrow$

  Losses half of the rate at High SNR!
- What if we add a precoding matrix V? How to choose V?

Joint Equi-Diagonal Triangularization (JET)

Theorem [Kh., Kochman, Erez; Allerton2010, SP2012]

- H_1 and H_2 – $N \times N$ non-singular matrices
- $det(H_1) = det(H_2)$
- H_1 and H_2 can be jointly decomposed as:

$$H_1 = Q_1 T_1 V^\dagger$$
$$H_2 = Q_2 T_2 V^\dagger$$

- Q_1, Q_2, V – unitary
- T_1 and T_2 are upper-triangular with **equal** diagonals

For $det(H_1) > det(H_2)$:

$$H_1 = \sqrt[N]{\det(H_1)} Q_1 T_1 V^\dagger$$
$$H_2 = \sqrt[N]{\det(H_2)} Q_2 T_2 V^\dagger$$

Illustrative Example

Matrix V is applied to $\begin{bmatrix} H_i \\ I_{N_t} \end{bmatrix}$ (MMSE variant):

$$\begin{bmatrix} H_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 0.286 & -0.905 \\ 0.905 & 0.286 \\ 0.095 & -0.301 \\ 0.301 & 0.095 \end{bmatrix}}^{Q_1} \overbrace{\begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{10} \end{bmatrix}}^{T_1} \overbrace{\begin{bmatrix} 0.302 & 0.954 \\ -0.954 & 0.302 \end{bmatrix}}^{V^\dagger}$$

$$\begin{bmatrix} H_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \sqrt{99} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 0.949 & -0.300 \\ 0.905 & -0.030 \\ 0.302 & 0.954 \end{bmatrix}}^{Q_2} \overbrace{\begin{bmatrix} \sqrt{10} & -9 \\ 0 & \sqrt{10} \end{bmatrix}}^{T_2} \overbrace{\begin{bmatrix} 0.302 & 0.954 \\ -0.954 & 0.302 \end{bmatrix}}^{V^\dagger}$$

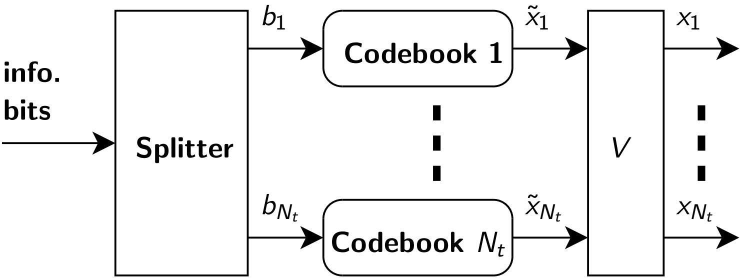
- $Q_1^\dagger Q_1 = Q_2^\dagger Q_2 = V^\dagger V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\mathrm{diag}(T_1) = \mathrm{diag}(T_2) = \begin{bmatrix} \sqrt{10} & \sqrt{10} \end{bmatrix}^T$

$$\Downarrow$$

Parallel SISO channels with equal gains for both users!

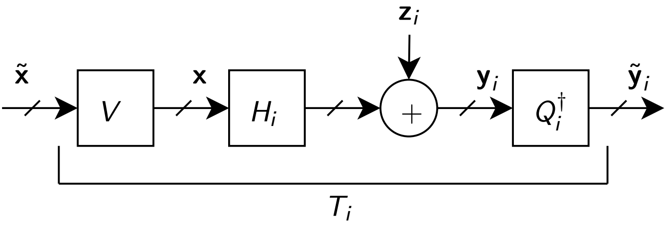
Network Modulation: MIMO Multicast Scheme

Transmitter:



Network Modulation: MIMO Multicast Scheme

Effective Channel:

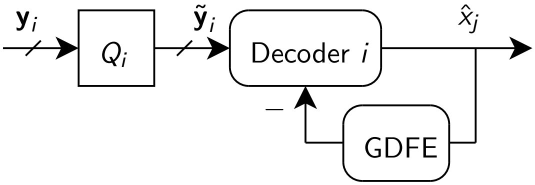


$$\tilde{\mathbf{y}}_i = T_i \tilde{\mathbf{x}}_i + \tilde{\mathbf{z}}_i$$

$$\tilde{\mathbf{z}}_i = Q^\dagger \mathbf{z}_i \sim \mathcal{CN}(0, I)$$

Network Modulation: MIMO Multicast Scheme

Receiver:



Extensions

Extension: Optimal MMSE (Capacity-Achieving) Scheme

- Holds for channel matrices of any dimension and rank (not equal between H_1 and H_2)
- Based upon an extension of the decomposition to non-square matrices
- Similar to the extension of V-BLAST from zero-forcing to MMSE
- For non-white input covariance matrix $C_{\mathbf{x}}$, decompose:

$$\begin{bmatrix} H_i C_{\mathbf{x}}^{1/2} \\ I_{N_t} \end{bmatrix}$$

- Any number of codebooks $\geq$ number of Tx antennas

Multiple Users

Problem

- We have used V to triangularize two matrices.
- **What to do for more??**

Is 2 just a bit more than 1?
Or... Is 2 a simplified ∞?

- How one buys more **degrees of freedom**?
- And and what price?

Multiple Users

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Space–Time Coding to the Rescue!

Space–Time Coding Structure

$$H_i = Q_i T_i V^\dagger \qquad \textcolor{red}{\mathbf{X}}$$

- **Bunch two channel uses together:**

$$\overbrace{\begin{pmatrix} H_i & \mathbf{0} \\ \mathbf{0} & H_i \end{pmatrix}}^{\mathcal{H}_i} = \overbrace{\begin{pmatrix} Q_i & \mathbf{0} \\ \mathbf{0} & Q_i \end{pmatrix}}^{\mathcal{Q}_i} \overbrace{\begin{pmatrix} T_i & \mathbf{0} \\ \mathbf{0} & T_i \end{pmatrix}}^{\mathcal{T}_i} \overbrace{\begin{pmatrix} V^\dagger & \mathbf{0} \\ \mathbf{0} & V^\dagger \end{pmatrix}}^{\mathcal{V}} \qquad \textcolor{red}{\mathbf{X}}$$

- $\mathcal{H}_i$ have a block-diagonal structure.
- Use general $\mathcal{Q}_i$, $\mathcal{V}$ (*not* block-diagonal):

$$\overbrace{\begin{pmatrix} H_i & \mathbf{0} \\ \mathbf{0} & H_i \end{pmatrix}}^{\mathcal{H}_i} = \left(\mathcal{Q}_i\right) \left(\mathcal{T}_i\right) \left(\mathcal{V}\right)^\dagger \qquad \textcolor{blue}{\checkmark}$$

- Exploiting off-diagonal **0**s enables JET of **more users!**

Space–Time Coding Structure

Space–Time Coding Structure

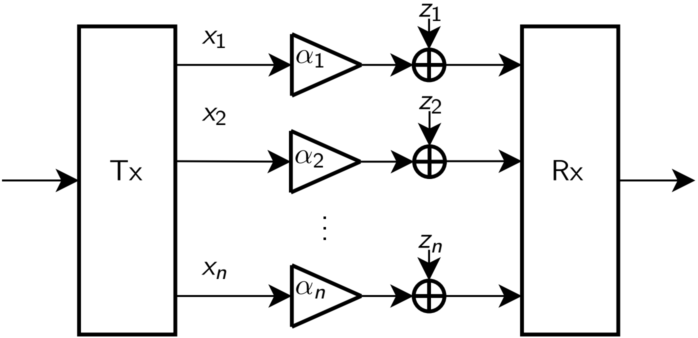
[Kh., Hitron, Erez ISIT2011][Livni, Kh., Hitron, Erez ISIT2012]

- Any number of users K
- Any number of antennas at each node
- Joint **constant-diagonal** triangularization of K matrices
- Process jointly N_t^{K-1} symbols
- Prefix–suffix loss of N_t^{K-1} symbols total
- **Numerical evidence**: Can be improved!

Applications

Gaussian Permuted Parallel Channels

- General channels: [Willems, Gorokhov][Hof, Sason, Shamai]



- Gains $\{\alpha_i\}$ are known
- **Order** of gains is **not known at Tx**, but **known at Rx**

Equivalent Problem

Be optimal for all permutation-orders simultaneously.

Gaussian Permuted Parallel Channels

Special case of MIMO multicasting problem!

n! effective channel matrices:

$$H_i \triangleq \begin{pmatrix} \alpha_{\pi_i(1)} & 0 & \cdots & 0 \\ 0 & \alpha_{\pi_i(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{\pi_i(n)} \end{pmatrix}, \qquad \pi_i \in S_n, \quad i = 1, ..., n!$$

Optimal precoding matrices [Hitron, Kh., Erez ISIT2012]

- **2 gains:** Hadamard/DFT; *Single channel use*
- **3 gains:** DFT; *Single channel use*
- **4-6 gains:** Quaternion-based matrices; *Two channel uses*
- **• • •**

Gaussian Rateless (Incremental Redundancy) Coding

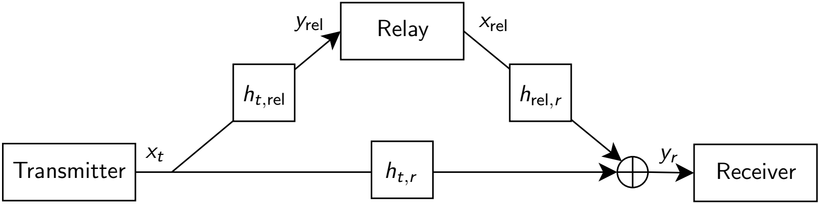
$$y = \alpha x + z \,,$$

- α is **unknown at Tx** but is **known at Rx**
- Rx sends NACKS/ACKS until it is able to recover the message
- Assume α can take only a finite number of values: $\alpha_1, \alpha_2, \ldots$
- Can be represented as a MIMO multicasting problem [Kh., Kochman, Erez, Wornell ITW2011]

Example $\alpha \in \{\alpha_1, \alpha_2\}$, $\alpha_1 > \alpha_2$

- $C_1 = 2C_2$
- Effective matrices: $H_1 = \begin{pmatrix} \alpha_1 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{pmatrix}$
- Coincides with the solution of [Erez, Trott, Wornell]
- Works for MIMO channels H_1, H_2 (replacing α_1, α_2)

Half-Duplex Relay



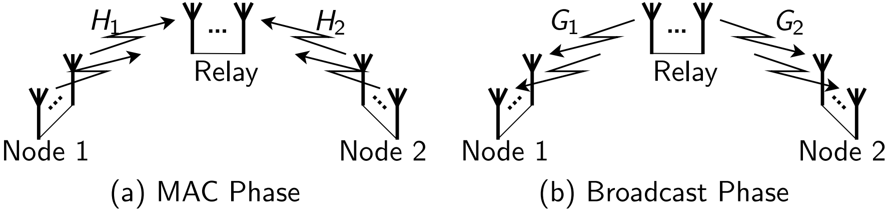
- **Half-duplex:** Relay can receive or transmit but **not both**
- Decode-and-forward implementation: "**rateless relay**"

Effective Matrices: [Kh., Kochman, Erez, Wornell ITW2011]

$$\mathcal{H}_1 = \begin{bmatrix} \sqrt{P_1} h_{t,rel} & 0 & \cdots & 0 \end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix} \sqrt{P_1} h_{t,r} & 0 & \cdots & 0 \\ 0 & \begin{matrix} \sqrt{P_2} h_{t,r} \\ + \sqrt{P_{rel}} h_{rel,r} \end{matrix} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \begin{matrix} \sqrt{P_2} h_{t,r} \\ + \sqrt{P_{rel}} h_{rel,r} \end{matrix} \end{bmatrix}$$

MIMO Two-Way Relay (New Achievable) [Kh., Kochman, Erez ISIT2011]

- Two nodes want to exchange messages via a relay



MAC Phase

- Apply JET to H_1 and H_2 (roles of V and Q switched)
- Use dirty-paper coding to pre-cancel off-diagonal elements (Replaces successive interference cancellation of broadcast)

Broadcast (Multicast!) Phase

- Use previously discussed multicasting scheme

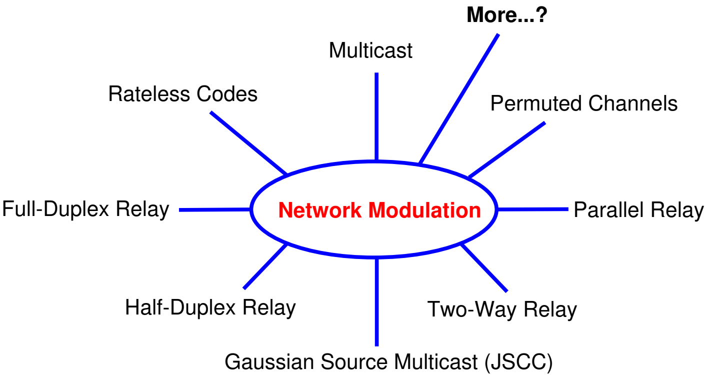
MIMO Multicasting of a Gaussian Source

[Kochman, Kh., Erez ICASSP2011][Kh., Kochman, Erez SP2012]

- Separation does not hold!
- Different triangularization is needed
- $(N_t - 1)$ sub-channels with **equal** diagonal values (last gain may differ)
- Combine with hybrid digital–analog scheme

- Decomposition possible under a “generalized Weyl condition”
- When decomposition is possible: **New achievable distortion!**
- For 2 transmit-antennas: **Optimum performance!**

Summary: Multicast is (Almost) Everywhere...



Even now, me talking to you...