Network Modulation: Transmission Technique for MIMO Networks

Anatoly Khina

Joint work with:
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Yuval Kochman – HUJI
Gregory W. Wornell – MIT

ACC Workshop, Feder Family Award Ceremony
February 27th, 2012
Talk Outline

- Novel MIMO multicast scheme
  - Two-user: via new joint decomposition of two matrices
  - Multi-user: via algebraic space–time coding structure
- Various applications
- New information-theoretic results
## MIMO Multicast (Closed Loop): State of the Art

<table>
<thead>
<tr>
<th>Theory</th>
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Single-Input Single-Output (SISO) Unicast

\[ y = hx + z \]

- \( x \) – Input of power 1
- \( y_i \) – Output
- \( h \) – Channel gain
- \( z \) – White Gaussian noise \( \sim \mathcal{CN}(0, 1) \)

Optimal communication rate (capacity):
\[ C = \log(1 + |h|^2) \]

Good practical codes that approach capacity are known!
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Multiple-Input Multiple-Output (MIMO) Unicast

\[ y = Hx + z \]

- \( x \) – Input vector of power \( 1 \cdot N_t \)
- \( y \) – Output vector
- \( H \) – Channel matrix
- \( H_{k\ell} \) – Gain from transmit-antenna \( \ell \) to receive-antenna \( k \).
- \( z \) – White Gaussian noise \( \sim \mathcal{CN}(0, I) \)
- Optimal rate (capacity): \( C = \max_{\mathbf{C}_x} \log(1 + H\mathbf{C}_x H^\dagger) \approx \log(1 + HH^\dagger) \)
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![Image](anatoly_khina.png)

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SISO Multicast

\[
y_i = h_i x + z_i \quad i = 1, \ldots, K
\]

- \(x\) – Input of power 1
- \(y_i\) – Output of user \(i\)
- \(h_i\) – Channel gain to user \(i\)
- \(z_i\) – White Gaussian noise \(\sim \mathcal{CN}(0, 1)\)
- Optimal rate (capacity): \(C = \min_i \log(1 + |h_i|^2)\)
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Gaussian MIMO Multicast

\[
y_i = H_i x + z_i \quad i = 1, \ldots, K
\]

- \(x\) – \(N_t \times 1\) input vector of power \(N_t \cdot 1\)
- \(y_i\) – Output vector of user \(i\)
- \(H_i\) – Channel matrix to user \(i\)
- \(z_i\) – White Gaussian noise vector \(\sim \mathcal{CN}(0, I)\)
- “Closed loop” (Full channel knowledge everywhere)
Optimal Achievable Rate (Capacity)

Multicasting capacity

\[ C = \max_{C_X} \min_{i=1,\ldots,K} \log \left\{ \det \left( I + H_i C_X H_i^\dagger \right) \right\} \]

- Optimization over covariance matrices \( C_X \) satisfying the power constraint

White Input / High SNR

\[ C_{WI} \approx \min_{i=1,\ldots,K} \log \left\{ \det \left( I + H_i H_i^\dagger \right) \right\} \]
### MIMO Multicast (Closed Loop): State of the Art

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Summary: Multicast is (Almost) Everywhere...

Network Modulation

- Multicast
  - Rateless Codes
  - Permutated Channels
- Full-Duplex Relay
- Parallel Relay
- Half-Duplex Relay
- Two-Way Relay
- Gaussian Source Multicast (JSCC)
Unicast

\[ y = Hx + z \]

White-input capacity

\[ C_{WI} = \log \left\{ \det \left( I + HH^\dagger \right) \right\} \]

• But how is this rate achieved?
Practical (Capacity-Achieving) Unicast Approaches

(Info. Theory) White-input capacity

\[ C_{WI} = \log \{ \det \left( I + HH^\dagger \right) \} \]

(Comm.) Achieving this rate with a practical scheme

- Singular value decomposition (SVD)
- QR decomposition (GDFE / V-BLAST)
- Geometric mean decomposition (GMD)
- Dirty-paper coding (DPC)
Practical (Capacity-Achieving) Unicast Approaches

(Info. Theory) White-input capacity

\[ C_{WI} = \log \left\{ \det \left( I + HH^\dagger \right) \right\} \]

(Comm.) Achieving this rate with a practical scheme

- **Singular value decomposition (SVD)**
- **QR decomposition** (GDFE / V-BLAST)
- **Geometric mean decomposition** (GMD)
- **Dirty-paper coding** (DPC)
Singular Value Decomposition (SVD)

\[ H = Q \Lambda V^\dagger \]

- \( Q, V \) – Unitary
- \( \Lambda \) – Diagonal
- Parallel AWGN SISO sub-channels \( \rightarrow \) “off-the-shelf” codes
- Diagonal of \( \Lambda = \) SISO channel gains \( \Rightarrow R_i = \log (1 + \lambda_i^2) \)

Generalization to Multicast?

- Precoding matrix \( V \) depends on channel matrix \( H \)
- Which \( V \) to take??
- Bottleneck problem \( (\Lambda_1 \neq \Lambda_2) \)

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QR Decomposition (GDFE/V-BLAST)

\[ H = QT \]

- \( Q \) – Unitary
- \( T \) – Upper-triangular matrix
- Successive interference cancellation
- Parallel AWGN SISO channels
- Diagonal of \( T \) – SISO channel gains

Remark

Both zero-forcing and MMSE (capacity-achieving) solutions exist.
QR Based Scheme

**Scheme**

- **Channel:** \( y = Hx + z = QTx + z \)
- **Transmitter:** \( x \) – SISO codebooks
- **Receiver:** \( \tilde{y} = Q^\dagger y = Tx + Q^\dagger z \)
- \( \tilde{z} = Q^\dagger z \sim \mathcal{CN}(0, I_N) \)

**Example for 2 \times 2**

\[
\begin{align*}
\tilde{y}_1 &= [T]_{11}x_1 + [T]_{12}x_2 + \tilde{z}_1 \\
\tilde{y}_2 &= 0x_1 + [T]_{22}x_2 + \tilde{z}_2
\end{align*}
\]
Generalization of QR based solution to Multicast?

- $T$ depends on $H$.

- For two channel matrices $H_1$ and $H_2$:
  \[ \text{diag}(T_1) \neq \text{diag}(T_2) \Rightarrow \text{different sub-channel gains!} \]

- Bottleneck problem:
  - Info. Theory: \[ \sum_{j=1}^{N_t} \log \left( |[T_1]_{jj}|^2 \right) = \sum_{j=1}^{N_t} \log \left( |[T_2]_{jj}|^2 \right) \checkmark \]
  - Comm.: \[ R_j = \log \left( \min \{|[T_1]_{jj}, [T_2]_{jj}|^2 \} \right) \ X \]

- Can we have equal diagonals?

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QR Based Scheme

Idea

- SVD uses both $Q$ and $V$
- QR uses only $Q$
- Can $V$ help in QR case to achieve equal diagonals?
- YES!
Illustrative Example

\[ H_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad H_2 = \begin{bmatrix} \sqrt{99} & 0 \end{bmatrix} \]

- \( C_{WI}^1 = 2 \log (1 + 3^2) = \log \left( 1 + (\sqrt{99})^2 \right) = C_{WI}^2 \)
- Send same signal over both antennas

\[ \downarrow \]

**Losses half of the rate at High SNR!**

- What if we add a precoding matrix \( V \)? How to choose \( V \)?

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Joint Equi-Diagonal Triangularization (JET)

Theorem [Kh., Kochman, Erez; Allerton2010, SP2012]

- $H_1$ and $H_2$ – $N \times N$ non-singular matrices
- $\det(H_1) = \det(H_2)$
- $H_1$ and $H_2$ can be jointly decomposed as:

\[
H_1 = Q_1 T_1 V^\dagger
\]
\[
H_2 = Q_2 T_2 V^\dagger
\]

- $Q_1, Q_2, V$ – unitary
- $T_1$ and $T_2$ are upper-triangular with equal diagonals

For $\det(H_1) > \det(H_2)$:

\[
H_1 = \sqrt[N]{\det(H_1)} Q_1 T_1 V^\dagger
\]
\[
H_2 = \sqrt[N]{\det(H_2)} Q_2 T_2 V^\dagger
\]
Illustrative Example

Matrix $V$ is applied to $\begin{bmatrix} H_i \\ I_{N_t} \end{bmatrix}$ (MMSE variant):

$\begin{bmatrix} H_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} Q_1 \\ T_1 \\ V^\dagger \end{bmatrix}$

$\begin{bmatrix} H_2 \\ l_2 \end{bmatrix} = \begin{bmatrix} \sqrt{99} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} Q_2 \\ T_2 \\ V^\dagger \end{bmatrix}$

- $Q_1^\dagger Q_1 = Q_2^\dagger Q_2 = V^\dagger V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\text{diag}(T_1) = \text{diag}(T_2) = \begin{bmatrix} \sqrt{10} & \sqrt{10} \end{bmatrix}^T$

$\downarrow$

Parallel SISO channels with equal gains for both users!
Network Modulation: MIMO Multicast Scheme

Transmitter:

- info. bits
- Splitter
- Codebook 1
- $b_1$ to $\tilde{x}_1$
- $b_{N_t}$ to $\tilde{x}_{N_t}$
- $x_1$ to $x_{N_t}$
Network Modulation: MIMO Multicast Scheme

Effective Channel:

\[ \tilde{y}_i = T_i \tilde{x}_i + \tilde{z}_i \]

\[ \tilde{z}_i = Q_i^\dagger z_i \sim \mathcal{CN}(0, I) \]
Network Modulation: MIMO Multicast Scheme

Receiver:

\[ y_i \xrightarrow{Q_i} \tilde{y}_i \xrightarrow{\text{Decoder } i} \hat{x}_j \]

\[ \hat{y}_i \xrightarrow{\text{GDFE}} \]

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Extensions
Extension: Optimal MMSE (Capacity-Achieving) Scheme

- Holds for channel matrices of any dimension and rank (not equal between $H_1$ and $H_2$)

- Based upon an extension of the decomposition to non-square matrices

- Similar to the extension of V-BLAST from zero-forcing to MMSE

- For non-white input covariance matrix $C_x$, decompose:

$$
\begin{bmatrix}
H_i C_x^{1/2} \\
I_{N_t}
\end{bmatrix}
$$

- Any number of codebooks $\geq$ number of Tx antennas
Problem

- We have used $V$ to triangularize two matrices.
- **What to do for more??**

  **Is 2 just a bit more than 1?**
  **Or... Is 2 a simplified $\infty$?**

- How one buys more **degrees of freedom**?
- And and what price?
Problem

- We have used $V$ to triangularize two matrices.
- **What to do for more??**
  
  **Is 2 just a bit more than 1?**
  **Or... Is 2 a simplified $\infty$?**

- How one buys more **degrees of freedom**?
- And and what price?

**Space–Time Coding to the Rescue!**
Space–Time Coding Structure

\[ H_i = Q_i T_i V^\dagger \]

- Bunch two channel uses together:

\[ \begin{pmatrix} H_i & 0 \\ 0 & H_i \end{pmatrix} = \begin{pmatrix} Q_i & 0 \\ 0 & Q_i \end{pmatrix} \begin{pmatrix} T_i & 0 \\ 0 & T_i \end{pmatrix} \begin{pmatrix} V^\dagger & 0 \\ 0 & V^\dagger \end{pmatrix} \]

- \( H_i \) have a block-diagonal structure.
- Use general \( Q_i, V \) (not block-diagonal):

\[ \begin{pmatrix} H_i \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} Q_i \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} T_i \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} V^\dagger \\ 0 \\ 0 \end{pmatrix} \]

- Exploiting off-diagonal 0s enables JET of more users!
Space–Time Coding Structure

[Kh., Hitron, Erez ISIT2011][Livni, Kh., Hitron, Erez ISIT2012]

- Any number of users $K$
- Any number of antennas at each node
- Joint constant-diagonal triangularization of $K$ matrices
- Process jointly $N_t^{K-1}$ symbols
- Prefix–suffix loss of $N_t^{K-1}$ symbols total

**Numerical evidence:** Can be improved!
Applications
**Gaussian Permutated Parallel Channels**

- **General channels:** [Willems, Gorokhov][Hof, Sason, Shamai]

\[
\begin{align*}
X_1 \rightarrow \alpha_1 & \rightarrow \mathcal{C}_1 \\
X_2 \rightarrow \alpha_2 & \rightarrow \mathcal{C}_2 \\
\vdots \\
X_n \rightarrow \alpha_n & \rightarrow \mathcal{C}_n \\
\end{align*}
\]

- **Gains** \(\{\alpha_i\}\) are known
- **Order** of gains is **not known at Tx**, but **known at Rx**

**Equivalent Problem**

Be optimal for all permutation-orders simultaneously.

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Gaussian Permutated Parallel Channels

Special case of MIMO multicasting problem!

**n! effective channel matrices:**

\[
H_i \triangleq \begin{pmatrix}
\alpha_{\pi_i(1)} & 0 & \cdots & 0 \\
0 & \alpha_{\pi_i(2)} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \alpha_{\pi_i(n)}
\end{pmatrix}, \quad \pi_i \in S_n, \quad i = 1, \ldots, n!
\]

Optimal precoding matrices [Hitron, Kh., Erez ISIT2012]

- **2 gains:** Hadamard/DFT; *Single channel use*
- **3 gains:** DFT; *Single channel use*
- **4-6 gains:** Quaternion-based matrices; *Two channel uses*

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Gaussian Rateless (Incremental Redundancy) Coding

\[ y = \alpha x + z , \]

- \( \alpha \) is **unknown at Tx** but is **known at Rx**
- Rx sends NACKS/ACKS until it is able to recover the message
- Assume \( \alpha \) can take only a finite number of values: \( \alpha_1, \alpha_2, ... \)
- Can be represented as a MIMO multicasting problem
  [Kh., Kochman, Erez, Wornell ITW2011]

**Example** \( \alpha \in \{\alpha_1, \alpha_2\}, \alpha_1 > \alpha_2 \)

- \( C_1 = 2C_2 \)
- Effective matrices: \( H_1 = \begin{pmatrix} \alpha_1 & 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{pmatrix} \)
- Coincides with the solution of [Erez, Trott, Wornell]
- Works for MIMO channels \( H_1, H_2 \) (replacing \( \alpha_1, \alpha_2 \))

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**Half-Duplex Relay**

- **Half-duplex**: Relay can receive or transmit but **not both**
- Decode-and-forward implementation: “**rateless relay**”

**Effective Matrices**: [Kh., Kochman, Erez, Wornell ITW2011]

\[
H_1 = \begin{bmatrix}
\sqrt{P_1} h_{t,rel} & 0 & \ldots & 0
\end{bmatrix},
H_2 = \begin{bmatrix}
\sqrt{P_1} h_{t,r} & 0 & \ldots & 0 \\
0 & \sqrt{P_2} h_{t,r} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \sqrt{P_2} h_{t,r} + \sqrt{P_{rel}} h_{rel,r}
\end{bmatrix}
\]
MIMO Two-Way Relay (New Achievable) [Kh., Kochman, Erez ISIT2011]

- Two nodes want to exchange messages via a relay

**MAC Phase**
- Apply JET to $H_1$ and $H_2$ (roles of $V$ and $Q$ switched)
- Use dirty-paper coding to pre-cancel off-diagonal elements
  (Replaces successive interference cancellation of broadcast)

**Broadcast (Multicast!) Phase**
- Use previously discussed multicasting scheme
MIMO Multicasting of a Gaussian Source

Separation does not hold!

Different triangularization is needed

\((N_t - 1)\) sub-channels with equal diagonal values
(last gain may differ)

Combine with hybrid digital–analog scheme

Decomposition possible under a “generalized Weyl condition”

When decomposition is possible: New achievable distortion!

For 2 transmit-antennas: Optimum performance!

[Kochman, Kh., Erez ICASSP2011][Kh., Kochman, Erez SP2012]
Summary: Multicast is (Almost) Everywhere...

Even now, me talking to you...