Binary Dirty MAC with Common Interference

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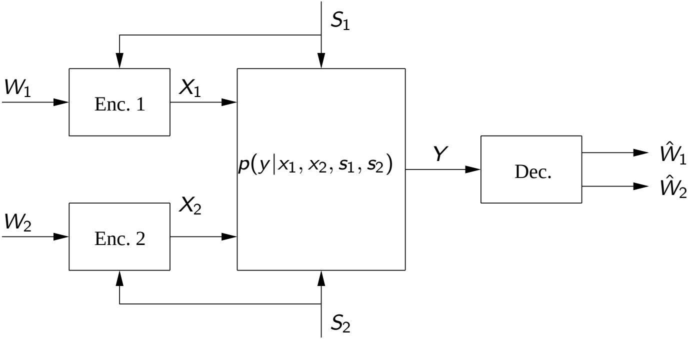
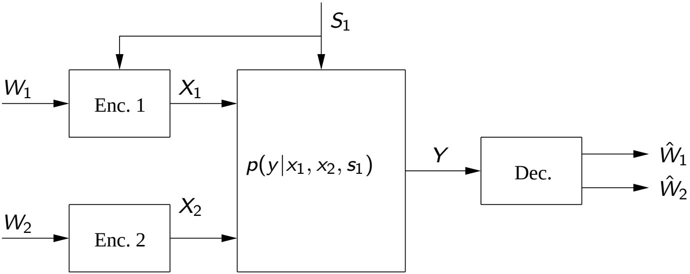
MAC with Two Correlated States



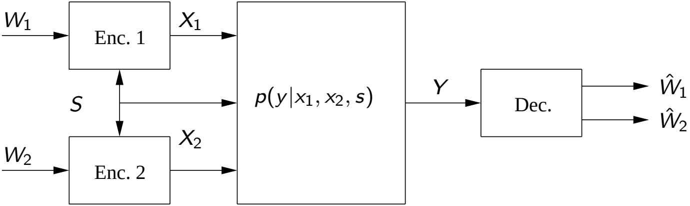
- X_1, X_2 - Channel inputs.
- S_1, S_2 - Channel states known (non-causally) to encoders $1, 2$, respectively.
- S_1, S_2 might be dependent.
- Y - Channel Output.

MAC with a Single-Informed User



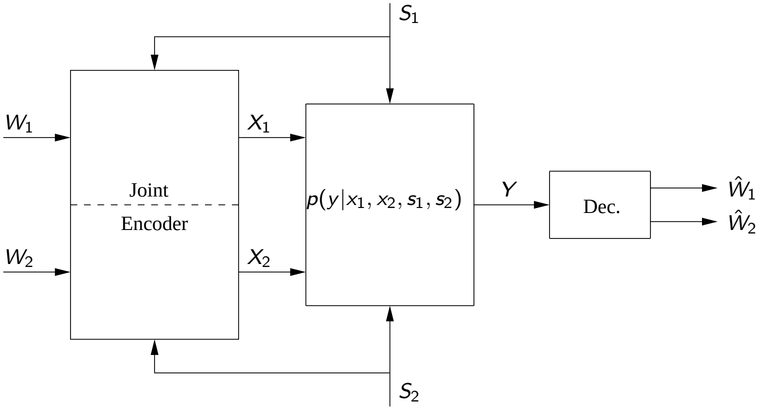
- X_1, X_2 - Channel inputs.
- S_1 - Channel state known (non-causally) to encoders 1 only.
- Y - Channel Output.

MAC with Common State



- X_1, X_2 - Channel inputs.
- S - Channel state known (non-causally) to both encoders.
- Y - Channel Output.

Outer-Bound: Cooperating Encoders



Remark

Not tight in general!

Achievable Region for the MAC with Common State

$$\mathcal{R} \triangleq \mathrm{cl}\,\mathrm{conv}\Big\{(\mathrm{R}_1,\mathrm{R}_2) : \begin{aligned} R_1 &\le I(U;Y|V) - I(U;S|V) \\ R_2 &\le I(V;Y|U) - I(V;S|U) \\ R_1+R_2 &\le I(U,V;Y) - I(U,V;S) \end{aligned}$$

$$\text{For } (U,V) \text{ satisfying: } (U,X_1) \leftrightarrow S \leftrightarrow (V,X_2)$$

$$(U,V) \leftrightarrow (X_1,X_2,S) \leftrightarrow Y\Big\},$$

where cl and conv denote the close and convex-hull operations.

Gaussian Dirty MAC

Point-to-point case (dirty paper channel, Costa '83)

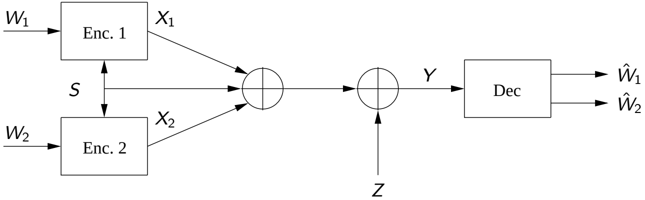
$$Y = X + S + Z$$

- Interference S induces no loss in rate.
- Costa's auxiliary $U = \alpha S$, where $\alpha = \frac{\mathsf{SNR}}{1+\mathsf{SNR}}$ achieves capacity.

Dirty MAC with Common Interference (Gel'fand and Pinsker '84)

- Choosing Costa-like auxiliaries achieves same capacity region as that of the "clean" MAC.
- The convex-hull is superfluous $\Leftrightarrow$ Time-sharing is not needed!
- Sum-capacity is strictly smaller than the rate achievable when encoders can cooperate, e.g., in the equal SNR case:
  - Sum-capacity: $\frac{1}{2}\log\left(1+2\mathsf{SNR}\right)$.
  - Cooperating encoders rate: $\frac{1}{2}\log\left(1+4\mathsf{SNR}\right)$.

Binary Dirty MAC with Common Interference



$$Y = X_1 \oplus X_2 \oplus S \oplus Z$$

- $Z \sim \mathrm{Bernoulli}(\varepsilon)$ - Noise.
- $S \sim \mathrm{Bernoulli}(1/2)$ - Known to both encoders (non-causally).
- Input ("power") constraints: $\frac{1}{n} w_H(\mathbf{x}_i) \leq q_i$.

Point-to-Point Capacities

- **Clean (interference-free) capacity:**

  $$C_{\text{clean}} = H_b(q \circledast \varepsilon) - H_b(\varepsilon),$$

  where $q_1 \circledast q_2 \triangleq (1 - q_1)q_2 + q_1(1 - q_2)$.

- **Dirty capacity (Barron, Chen, Wornell; Zamir, Shamai, Erez):**

  $$C_{\text{dirty}}^{\text{noncausal}} = \mathrm{uch}\max\{\mathrm{H_b}(\mathrm{q}) - \mathrm{H_b}(\varepsilon), 0\}.$$

Loss due to interference even in the point-to-point setting!

Noiseless case

In the noiseless case ($\varepsilon = 0 \Leftrightarrow Z \equiv 0$):

$$C_{\text{clean}} = C_{\text{dirty}}^{\text{noncausal}} = H_b(\min\{q, 1/2\}) \triangleq H_b^+(q)$$

Sum-Capacity

Remark

For simplicity, We shall concentrate on the noiselss case:

$$\varepsilon = 0 \Leftrightarrow Z \equiv 0$$

Scheme (for both "clean" and "dirty" binary MACs)

Divide each block of size n into two sub-blocks:

- During the first αn time-slots user 1 sends its message using all of its power and user 2 is silent:

$$R_1 = \alpha H_b^+\left(\frac{q_1}{\alpha}\right).$$

- In the remaining $(1-\alpha)n$ time-slots user 2 sends its message using all of its power and user 1 is silent:

$$R_2 = (1-\alpha)H_b^+\left(\frac{q_2}{1-\alpha}\right).$$

Sum-Capacity

Sum-rate of proposed scheme

- $R_1 + R_2 = \alpha H_b^+\left(\frac{q_1}{\alpha}\right) + (1-\alpha) H_b^+\left(\frac{q_2}{1-\alpha}\right)$.
- Choose $\alpha = \frac{q_1}{q_1+q_2}$: $R_1 + R_2 = H_b^+(q_1 + q_2)$.

Cooperating encoders (Upper-Bound)

Cooperation between encoders

$$\Downarrow$$

P2P problem with input ("power") constraint $q_1 + q_2$.

- Capacity of the P2P scheme:

$$R_{\mathrm{UB}} = H_b^+(q_1 + q_2)$$

Sum-Capacity

Sum-capacity (noiseless)

$$C_{\text{clean}} = C_{\text{dirty}}^{\text{noncausal}} = H_b^+ \left(q_1 + q_2\right)$$

Sum-Capacity (noisy)

- **Clean MAC sum-capacity:**

$$C_{\text{clean}}^{\text{sum}} = H_b^+ \Big((q_1 + q_2) \circledast \varepsilon \Big) - H_b(\varepsilon) \,.$$

- **Dirty MAC sum-capacity:**

$$C_{\text{dirty}}^{\text{sum}} = \text{uch}\max \left\{ H_b^+(q_1 + q_2) - H_b(\varepsilon), 0 \right\} \,.$$

Differences from Gaussian Case

- Cooperation does not increase (sum) capacity.
  (Holds for both clean and dirty)
- Time-sharing is essential to achieve capacity.
  (Holds for both clean and dirty)
- In the noisy case, presence of interference reduces capacity.

Achievable Region

Clean MAC capacity-achieving strategies

- Time-sharing between "onion-peeling" strategies.
- "Onion-peeling" strategy:
  1. first user treats message of other user as noise and decodes its message.
  2. The message of first user is peeled from the output y.
  3. The message of the other user is decoded.

Dirty MAC strategies

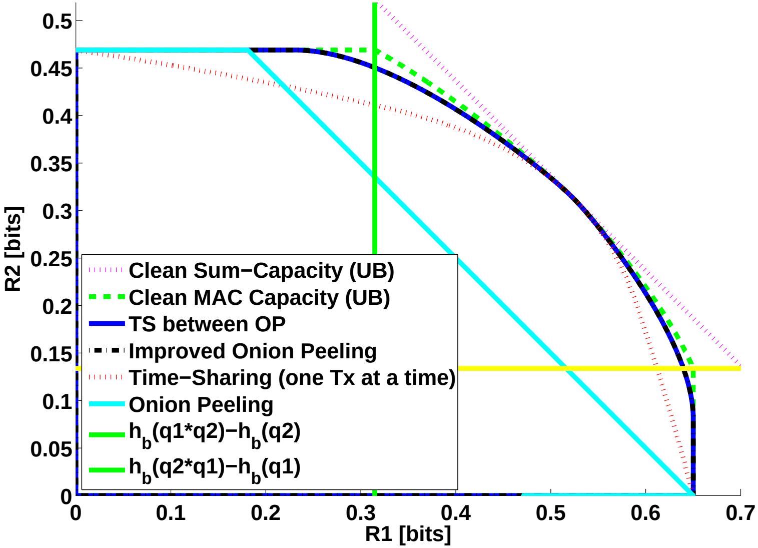
Using the same techniques for the dirty case

$\Downarrow$

Loss due to interference

(Even in the noiseless case (*but not in sum-rate!*)!)

Dirty MAC Rate Region for $q_1 = 1/6, \ q_2 = 1/10$



One Onion Peeling Strategy Suffices

- For a general "clean" two-user MAC, time-sharing between 2 onion-peeling strategies is needed.
- Binary two-user MAC: Time-sharing between:
  - One onion-peeling strategy: User that is peeled first transmits with all of its power.
  - Other user transmits with all of its remaining power.
- A similar time-sharing for the binary dirty MAC suffices as well.