Binary Dirty MAC with Common Interference

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MAC with Two Correlated States

- \( X_1, X_2 \) - Channel inputs.
- \( S_1, S_2 \) - Channel states known (non-causally) to encoders 1, 2, respectively.
- \( S_1, S_2 \) might be dependent.
- \( Y \) - Channel Output.
MAC with a Single-Informed User

- $X_1, X_2$ - Channel inputs.
- $S_1$ - Channel state known (non-causally) to encoders 1 only.
- $Y$ - Channel Output.
MAC with Common State

- $X_1, X_2$ - Channel inputs.
- $S$ - Channel state known (non-causally) to both encoders.
- $Y$ - Channel Output.
Outer-Bound: Cooperating Encoders

\[ p(y|x_1, x_2, s_1, s_2) \]

Remark

Not tight in general!
Achievable Region for the MAC with Common State

\[ \mathcal{R} \triangleq \text{cl conv} \left\{ (R_1, R_2) : R_1 \leq I(U; Y|V) - I(U; S|V) \right. \]
\[ \left. \quad R_2 \leq I(V; Y|U) - I(V; S|U) \right. \]
\[ \left. \quad R_1 + R_2 \leq I(U, V; Y) - I(U, V; S) \right. \]
\[ \text{For } (U, V) \text{ satisfying: } (U, X_1) \leftrightarrow S \leftrightarrow (V, X_2) \]
\[ (U, V) \leftrightarrow (X_1, X_2, S) \leftrightarrow Y \}, \]

where \( \text{cl} \) and \( \text{conv} \) denote the close and convex-hull operations.
Gaussian Dirty MAC

**Point-to-point case (dirty paper channel, Costa ’83)**

\[ Y = X + S + Z \]

- Interference \( S \) induces no loss in rate.
- Costa’s auxiliary \( U = \alpha S \), where \( \alpha = \frac{\text{SNR}}{1 + \text{SNR}} \) achieves capacity.

**Dirty MAC with Common Interference (Gel’fand and Pinsker ’84)**

- Choosing Costa-like auxiliaries achieves same capacity region as that of the “clean” MAC.
- The convex-hull is superfluous \( \iff \) Time-sharing is not needed!
- Sum-capacity is strictly smaller than the rate achievable when encoders can cooperate, e.g., in the equal SNR case:
  - Sum-capacity: \( \frac{1}{2} \log (1 + 2\text{SNR}) \).
  - Cooperating encoders rate: \( \frac{1}{2} \log (1 + 4\text{SNR}) \).
Binary Dirty MAC with Common Interference

\[ Y = X_1 \oplus X_2 \oplus S \oplus Z \]

- \( Z \sim \text{Bernoulli}(\varepsilon) \) - Noise.
- \( S \sim \text{Bernoulli}(1/2) \) - Known to both encoders (non-causally).
- Input ("power") constraints: \( \frac{1}{n} w_H(x_i) \leq q_i \).
Clean (interference-free) capacity:

\[ C_{\text{clean}} = H_b(q \otimes \epsilon) - H_b(\epsilon), \]

where \( q_1 \otimes q_2 \triangleq (1 - q_1)q_2 + q_1(1 - q_2) \).

Dirty capacity

(Barron, Chen, Wornell; Zamir, Shamai, Erez):

\[ C_{\text{noncausal dirty}} = \max \{H_b(q) - H_b(\epsilon), 0\}. \]

Loss due to interference even in the point-to-point setting!

Noiseless case

In the noiseless case \((\epsilon = 0 \iff Z \equiv 0)\):

\[ C_{\text{clean}} = C_{\text{noncausal dirty}} = H_b(\min\{q, 1/2\}) \triangleq H^+_b(q) \]
For simplicity, we shall concentrate on the noiseless case:

\( \varepsilon = 0 \iff Z = 0 \)

**Scheme (for both “clean” and “dirty” binary MACs)**

Divide each block of size \( n \) into two sub-blocks:

- During the first \( \alpha n \) time-slots user 1 sends its message using all of its power and user 2 is silent:
  \[
  R_1 = \alpha H_b^+ \left( \frac{q_1}{\alpha} \right).
  \]

- In the remaining \( (1 - \alpha)n \) time-slots user 2 sends its message using all of its power and user 1 is silent:
  \[
  R_2 = (1 - \alpha) H_b^+ \left( \frac{q_2}{1 - \alpha} \right).
  \]
Sum-Capacity

Sum-rate of proposed scheme

- \( R_1 + R_2 = \alpha H_b^+ \left( \frac{q_1}{\alpha} \right) + (1 - \alpha) H_b^+ \left( \frac{q_2}{1-\alpha} \right) \).

- Choose \( \alpha = \frac{q_1}{q_1+q_2} \): \( R_1 + R_2 = H_b^+ (q_1 + q_2) \).

Cooperating encoders (Upper-Bound)

- Cooperation between encoders \( \Downarrow \)

- P2P problem with input ("power") constraint \( q_1 + q_2 \).

- Capacity of the P2P scheme:
  \[ R_{UB} = H_b^+ (q_1 + q_2) \]
Sum-Capacity

**Sum-capacity (noiseless)**

\[ C_{\text{clean}} = C_{\text{dirty}}^{\text{noncausal}} = H^+_b (q_1 + q_2) \]

**Sum-Capacity (noisy)**

- **Clean MAC sum-capacity:**

\[ C_{\text{sum}}^{\text{clean}} = H^+_b \left( (q_1 + q_2) \otimes \varepsilon \right) - H_b(\varepsilon). \]

- **Dirty MAC sum-capacity:**

\[ C_{\text{sum}}^{\text{dirty}} = \max \left\{ H^+_b (q_1 + q_2) - H_b(\varepsilon), 0 \right\}. \]
Differences from Gaussian Case

- Cooperation does not increase (sum) capacity. (Holds for both clean and dirty)
- Time-sharing is essential to achieve capacity. (Holds for both clean and dirty)
- In the noisy case, presence of interference reduces capacity.
Clean MAC capacity-achieving strategies

- Time-sharing between “onion-peeling” strategies.
- “Onion-peeling” strategy:
  1. first user treats message of other user as noise and decodes its message.
  2. The message of first user is peeled from the output $y$.
  3. The message of the other user is decoded.

Dirty MAC strategies

Using the same techniques for the dirty case

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Loss due to interference

(Even in the noiseless case (but not in sum-rate)!)
Dirty MAC Rate Region for $q_1 = 1/6$, $q_2 = 1/10$
For a general “clean” two-user MAC, time-sharing between 2 onion-peeling strategies is needed.

**Binary two-user MAC:** Time-sharing between:
- One onion-peeling strategy: User that is peeled first transmits with all of its power.
- Other user transmits with all of its remaining power.

A similar time-sharing for the binary dirty MAC suffices as well.