Decode-and-Forward for the Gaussian Relay Channel via Standard AWGN Coding and Decoding

Anatoly Khina — Tel Aviv University
Joint work with:
Or Ordentlich, Uri Erez — Tel Aviv University
Yuval Kochman — Hebrew University
Gregory W. Wornell — MIT

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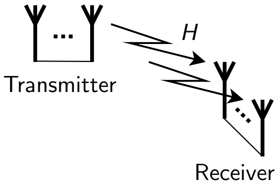
MIMO Unicast (Point-to-Point)



$$\mathbf{y} = H\mathbf{x} + \mathbf{z}$$

$$\mathbf{z} \sim \mathcal{CN}(0, I)$$

White-input capacity

$$\mathcal{C}_{\mathrm{WI}} = \log\left\{\det\left(I + HH^{\dagger}\right)\right\}$$

But how can this rate be achieved in practice?

Black Box Approach

SVD: $H = QDV^\dagger$

- Q and V — unitary
- Apply V at Tx and Q at Rx

$$D = \begin{pmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & d_{n-1} & 0 \\ 0 & \cdots & 0 & 0 & d_n \end{pmatrix}$$

- Results in scalar sub-channels (each with a different SNR)

QR: $H = QR$

- Q — unitary
- Apply Q at Rx (no SP at Tx)

$$R = \begin{pmatrix} d_1 & * & * & \cdots & * \\ 0 & d_2 & * & \cdots & * \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{n-1} & * \\ 0 & 0 & \cdots & 0 & d_n \end{pmatrix}$$

- Off-diagonal elements canceled via successive decoding

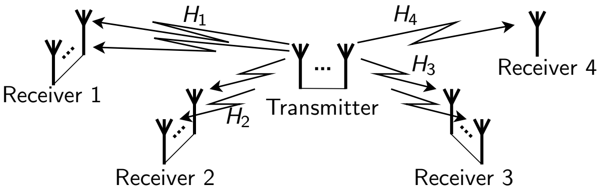
Effective Sub-Channels

$$y_1 = d_1 x_1 + z_1$$
$$y_2 = d_2 x_2 + z_2$$
$$\vdots$$
$$y_n = d_n x_n + z_n$$

Both approaches reduce to the **"black box"** task:

- **Coding over P2P AWGN scalar channels**
- **Any "off-the-shelf" (fixed SNR) P2P codes!**

Gaussian MIMO Multicast



$$\mathbf{y}_i = H_i \mathbf{x} + \mathbf{z}_i \,, \quad i = 1, \ldots, K$$

Multicast capacity

$$\mathcal{C} = \max_{C_X} \min_{i=1,\ldots,K} \log \left\{ \det \left(I + H_i C_X H_i^\dagger \right) \right\}$$

- **White-input capacity:**

$$\mathcal{C}_{\mathrm{WI}} = \min_{i=1,\ldots,K} \log \left\{ \det \left(I + H_i H_i^\dagger \right) \right\}$$

How to achieve this using a black box approach?

Joint Equi-Diagonal Triangularization (JET)

Black Box for Multicast (Multiple Users)?

- **SVD fails!** — joint diagonalization is **not possible**
- **QR fails!** — $\text{diag}(R_1) \neq \text{diag}(R_2) \rightarrow$ **loses in rate!**

JET (New Joint Unitary Decomposition)

H_1 and H_2 can be jointly decomposed as:

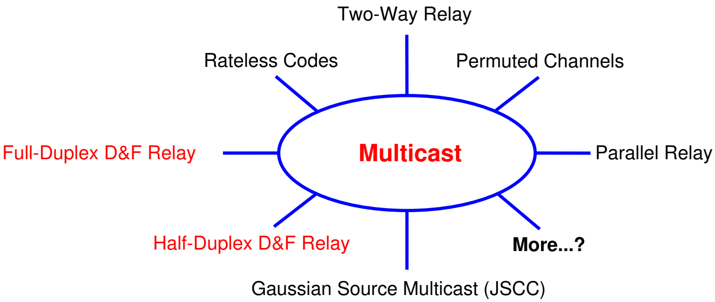
$$H_i = Q_i \underbrace{\begin{pmatrix} d_1 & * & * & \cdots & * \\ 0 & d_2 & * & \cdots & * \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{n-1} & * \\ 0 & 0 & \cdots & 0 & d_n \end{pmatrix}}_{R_i} V^\dagger$$

- H_i – $n \times n$ non-singular
- $|\det(H_1)| = |\det(H_2)|$
- Q_1, Q_2, V – unitary
- $\text{diag}(R_1) = \text{diag}(R_2)$

JET for More Users

Can be accomplished for more users via time-extensions

Multicast is (Almost) Everywhere...



Multicast is (Almost) Everywhere...