Incremental Coding over MIMO Channels

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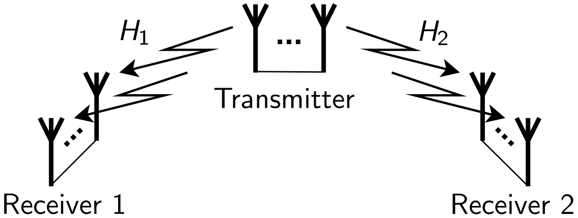
Two-user Gaussian MIMO Broadcast — Channel Model



$$\mathbf{y}_i = H_i \mathbf{x} + \mathbf{z}_i \,, \quad i = 1,2$$

- $\mathbf{x}$ - $N \times 1$ Input vector of power P.
- H_i - $N \times N$ Channel matrix to user i (works for non-square matrices as well).
- $\mathbf{y}_i$ - $N \times 1$ Output vector of user i.
- $\mathbf{z}_i$ - Channel noise $\sim \mathcal{CN}(\mathbf{0}, I_N)$.
- "Closed loop" (full channel knowledge everywhere).

Two-user Gaussian MIMO Broadcast — Goal

Rateless setting

- Transmit the same (common) message to both users.
- Each user aims to decode message after minimal number of channel uses.

- k information bits ("message").
- n_i channel uses needed by user i to decode message.
- Effective rates: $R_i \triangleq \left\lfloor \frac{k}{n_i} \right\rfloor$.

Goal

Approach achievable region of all (R_1, R_2) with **practical scheme**.

Achievable Rate Region

Achievable rate region [Shulman, Ph.D., 2004]

(R_1, R_2) is achievable iff there exists a covariance matrix K with $\text{trace}\{K\} \leq P$ s.t.:

$$\frac{R(H_i, K)}{R_i} + C_i \left[\frac{1}{R_i} - \frac{1}{R_{\bar{i}}} \right]^+ \geq 1 \,, \qquad i = 1, 2 \,,$$

where

$$R(H, K) \triangleq \log \left| I_N + HKH^\dagger \right|$$

and C_i is the point-to-point capacity of user i.

Achievable Rate Region

Interpretation

- Assume $R(H_1, K) \geq R(H_2, K) \quad (\Rightarrow n_1 < n_2)$
- Use K in first $n_1 = k/R(H_1, K)$ channel uses.
- After n_1 channel uses, user 1 is able to decode.
- Use optimal covariance matrix K_2 for user 2.
  $\Rightarrow$ User 2 needs additional

$$n_2 - n_1 = \frac{k - n_1 R(H_2, K)}{C_2}$$

channel uses to decode.

Problem

How to achieve this using *practical* codes?

Practical Schemes for the SISO Case

Different schemes were proposed for the SISO ("scalar") case:

- Superposition of good scalar AWGN codes with appropriate power allocations [Erez, Trott, Wornell '06].
- Punctured Turbo and LDPC codes [Rowitch & Milstein '00; Ha et al. '06].
- Raptor-based codes [Etesami & Shokrollahi '06].

What to do for MIMO?

Can we reduce MIMO to SISO?

How to Extend to MIMO?

- Simultaneous diagonalization of both channel matrices is not possible in general.
- Even if both channels were diagonal, using scalar rateless codes over these channels would not work.

Example

$$H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Different number of channel uses over each SISO sub-channel needs to be used.

  $\Rightarrow$ Not possible to implement simultaneously for all sub-channels...

Equivalent "Augmented" Channel Representation

- Assume, for simplicity, integer ratios: $n_2 = m \cdot n_1$.
- Look at the n_2 channel uses as n_1 uses of the equivalent $mN \times mN$ "augmented" channels (realized by interleaving):

$$\mathcal{H}_1 = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix} H_2 & 0 & \cdots & 0 \\ 0 & H_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_2 \end{bmatrix},$$

- Effective covariance matrix:

$$\mathcal{K} = \begin{bmatrix} K & 0 & \cdots & 0 \\ 0 & K_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_2 \end{bmatrix}.$$

- For the effective matrices: $R(\mathcal{H}_1, \mathcal{K}) = R(\mathcal{H}_2, \mathcal{K})$.

Equivalent "Augmented" Channel Representation

- $R(\mathcal{H}_1, \mathcal{K}) = R(\mathcal{H}_2, \mathcal{K}) \Rightarrow$ "Classical" multicasting (common-message BC) problem

- Any "good" multicasting scheme can be used over the effective channels.

How to construct a good multicasting scheme?

MIMO Multicasting Scheme

Practical schemes:

- Single- and two-stream (Alamouti-based) systems [Lopez, Ph.D., 2002].
- Linear pre- and post-coding [Gohary, Davidson, Luo 2003].

Practical and capacity-achieving scheme:

- Optimal beamforming + joint unitary triangularization [Khina, Kochman, Erez 2010].

MIMO Multicasting Scheme

For now, assume :

- H_1 and H_2 are $N \times N$ full rank.
- $\det(H_1) = \det(H_2)$.
- High SNR.

Joint decompositions (same unitary matrix on the right)

- Joint unitary diagonalization not possible.
- Joint unitary triangularization is always possible.
- Diagonals of triangular matrices correspond to gains of sub-channels.
- We want diagonals of triangular matrices to be equal.

MIMO Multicasting Scheme

Joint equi-diagonal triangularization [Khina, Kochman, Erez 2010]

$$H_1 = U_1 T_1 V^\dagger \quad ,$$
$$H_2 = U_2 T_2 V^\dagger \quad ,$$

such that

- U_1, U_2, V are unitary.
- T_1, T_2 are triangular with *equal diagonals*.

MIMO multicast scheme

- Apply V at Tx and U_i and Rx-i $\Rightarrow$ Effective channels T_i
  $\Rightarrow$ n parallel sub-channels of *equal gains* $\text{diag}(T_1) = \text{diag}(T_2)$.
- Use good AWGN *SISO* codebooks over sub-channels.
- Decode using successive interference cancellation (as in GDFE/V-BLAST).

MIMO Multicasting Scheme

Problem

In our case the channel matrices are:

$$\mathcal{H}_1 = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix} H_2 & 0 & \cdots & 0 \\ 0 & H_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_2 \end{bmatrix},$$

$\Rightarrow$ **full rank and high SNR assumptions do not hold!**

Optimal scheme for general SNR and any channel matrices

Decompose

$$A_i \triangleq \begin{pmatrix} H_i K^{1/2} \\ I_N \end{pmatrix}$$

- K – Optimal covariance matrix.
- Diagonals of triangular matrices T_i correspond to SINRs.

Application to SISO

$$y = \alpha x + z\,,$$

where $\alpha \in \{\alpha_1, \alpha_2\}$ corresponds to SNR.

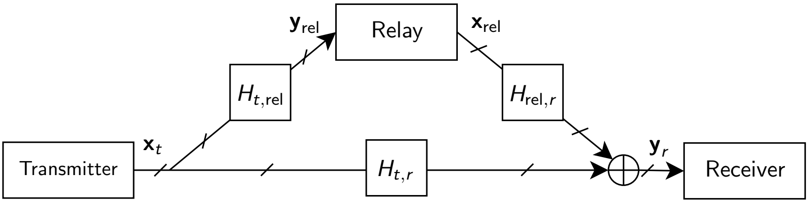
- For example, assume $n_2 = 2 \cdot n_1$.

  $\Rightarrow \log\left(1 + |\alpha_1|^2\right) = 2\log\left(1 + |\alpha_2|^2\right)$.
- Effective channel matrices:

$$\mathcal{H}_1 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{H}_2 = \begin{bmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{bmatrix}.$$

- Resulting scheme coincides with SISO rateless scheme of [Erez, Trott, Wornell 2006].
- Can be applied for any two SNRs using an appropriate number of non-zero diagonal elements.

Half-duplex Relay



- **Half-duplex:** Relay can receive or transmit but **not both**.

Rateless relay

- Implementation of Decode-and-Forward [Cover, El Gamal '79].
- Proposed first for "open-loop" [Mitran, Ochiai, Tarokh '05].
- Relay decodes the message first, before Rx does.
- Relay then transmits coherently with Tx until Rx decodes too.

Half-duplex Relay

- n_2 – Whole transmission duration until Rx is able to decode.
- n_1 – Transmission duration until relay is able to decode.
- Assume again for simplicity: $n_2 = m \cdot n_1$

Equivalent matrix representation for SISO case:

$$\mathcal{H}_1 = \begin{bmatrix} \sqrt{P_1} h_{t,\mathrm{rel}} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\mathcal{H}_2 = \begin{bmatrix} \sqrt{P_1} h_{t,r} & 0 & \cdots & 0 \\ 0 & \begin{matrix}\sqrt{P_2} h_{t,r} \\ +\sqrt{P_{\mathrm{rel}}} h_{\mathrm{rel},r}\end{matrix} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \begin{matrix}\sqrt{P_2} h_{t,r} \\ +\sqrt{P_{\mathrm{rel}}} h_{\mathrm{rel},r}\end{matrix} \end{bmatrix}$$

Half-duplex Relay

- MIMO rateless coding technique is useful even for SISO relay.
- For MIMO case, replace diagonal elements in $\mathcal{H}_i$ with corresponding channel matrices.
- Use additional unitary matrix at relay to achieve coherence.
- Apply MIMO rateless coding technique.

Summary

- Technique for constructing practical rateless codes for SISO and MIMO channels.
- Uses "off-the-shelf" scalar AWGN codes, linear transformations and successive decoding.
- Allows designing practical transmission schemes for the half-duplex relay channel.
- Special case of "**Network Modulation**":
  Joint decomposition of channel matrices in MIMO network problems.
- Can be generalized for more than 2 users
  [Khina, Hitron, Erez '2011].

Supplementary