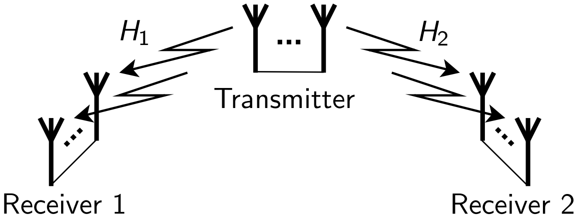
Decomposing the MIMO Broadcast Channel

Anatoly Khina
Joint work with: Yuval Kochman, Uri Erez

September 29th, 2010

Two-User Gaussian MIMO Broadcast



$$\mathbf{y}_i = H_i \mathbf{x} + \mathbf{n}_i \quad i = 1, 2$$

- $\mathbf{x}$ - $N \times 1$ Input vector of power P.
- H_i - $N \times N$ Channel matrix to user i.
- $\mathbf{y}_i$ - $N \times 1$ Output vector of user i.
- $\mathbf{n}_i$ - Channel noise $\sim \mathcal{CN}(\mathbf{0}, I_N)$.
- "Closed loop" (Full channel knowledge everywhere).

Capacity

Common-message capacity

$$\mathcal{C} = \max_{C_X} \min_{i=1,2} \log \left\{ \det \left(I + H_i C_X H_i^{\dagger} \right) \right\}$$

- Optimization over covariance matrices C_X satisfying power constraint: $\mathrm{trace}\left(C_X\right) \leq P$.

White Input / High SNR

$$\mathcal{C}_{\mathrm{WI}} \approx \min_{i=1,2} \log \left\{ \det \left(\frac{P}{N} H_i H_i^{\dagger} \right) \right\}$$

Direct Single-User Implementation

- Joint encoding and decoding of all antenna signals.
- Special codes design.
- Not practical for large antenna arrays.

Requirements of a Practical Scheme

- Single-stream codebooks for constant SNR.
- Linear / successive interference cancellation.

Single-User Practical (Capacity-Achieving) Approaches

- Singular value decomposition (SVD).
- QR decomposition (GDFE / V-BLAST).
- Geometric mean decomposition (GMD) / Uniform channel decomposition (UCD).
- Dirty-paper coding (DPC).

Remark

Both zero-forcing and MMSE (capacity-achieving) solutions exist.

Singular Value Decomposition (SVD)

$$H = Q \Lambda V^{\dagger}$$

- Q, V - Unitary.
- Λ - Diagonal.
- Parallel AWGN SISO channels.
- Diagonal of Λ = SISO channel gains $\rightarrow$ Rates.

Generalization to Broadcast

- Precoding matrix V depends on channel matrix H.
- Not clear how to choose V without losing in rate.
- Rate-allocation problem $(\Lambda_1 \neq \Lambda_2)$.

QR Decomposition (GDFE/V-BLAST)

$$H = QR$$

- Q - Unitary.
- R - Upper-triangular matrix.
- Successive interference cancellation.
- Parallel AWGN SISO channels.
- Diagonal of R - SISO channel gains.

QR Based Scheme

Scheme

- **Channel:** $\mathbf{y} = H\mathbf{x} + \mathbf{n} = QR\mathbf{x} + \mathbf{n}$.
- **Transmitter:** $\mathbf{x}$ - SISO codebooks of rates t.b.d.
- **Receiver:** $\tilde{\mathbf{y}} = Q^\dagger \mathbf{y} = R\mathbf{x} + Q^\dagger \mathbf{n}$.
- $\tilde{\mathbf{n}} = Q^\dagger \mathbf{n} \sim \mathcal{CN}(0, I_N)$

Example for 2×2

$$\begin{aligned} \tilde{y}_1 &= [R]_{11} x_1 + \overbrace{[R]_{12} x_2}^{\text{Interference}} + \tilde{n}_1 \\ \tilde{y}_2 &= [R]_{21} \cdot 0 + [R]_{22} x_2 + \tilde{n}_2 \end{aligned}$$

QR Based Scheme

Generalization of QR based solution to Broadcast

- R depends on H.
- For two channel matrices (H_1 and H_2) the diagonals of R_1 and R_2 are different = different SISO channel gains.
- Successive decoding requirement limits to working at rates:

$$\mathcal{R}_j = \log\left(\frac{P}{N}\left|\min\left\{[R_1]_{jj},[R_2]_{jj}\right\}\right|^2\right)$$

- Can we have equal diagonals?
  Yes we can!

Joint Equi-Diagonal Decomposition

Theorem

- H_1 and H_2 - $N \times N$ non-singular matrices.
- $det(H_1) = det(H_2)$
- H_1 and H_2 can be jointly decomposed as:

$$H_1 = Q_1 R_1 V^\dagger$$
$$H_2 = Q_2 R_2 V^\dagger$$

- Q_1, Q_2, V - unitary.
- R_1 and R_2 are upper-triangular with **equal** diagonals.

For $det(H_1) > det(H_2)$:

$$H_1 = \sqrt[N]{\det(H_1)} Q_1 R_1 V^\dagger$$
$$H_2 = \sqrt[N]{\det(H_2)} Q_2 R_2 V^\dagger$$

Proof Idea

- Constructive proof based on the QR and GMD.

Geometric Mean Decomposition (GMD)

- Jiang, Li and Hagger, 2005.

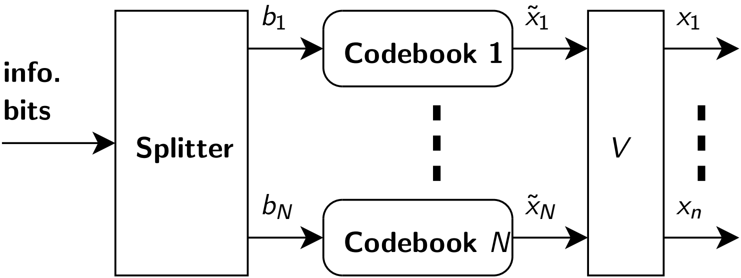
$$H = QRV^{\dagger}$$

- Q, V - Unitary.
- R - Has **constant** diagonal.

- Constant diagonal $\rightarrow$ Constant **ratio** between diagonals.

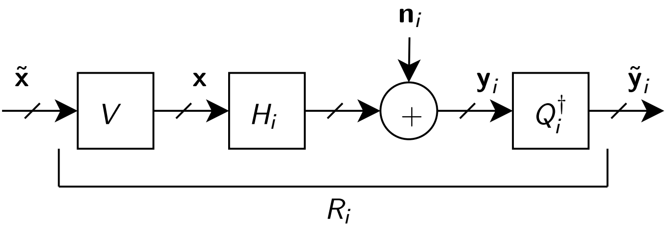
MIMO Broadcast Scheme

Encoder:



MIMO Broadcast Scheme

Effective Channel:

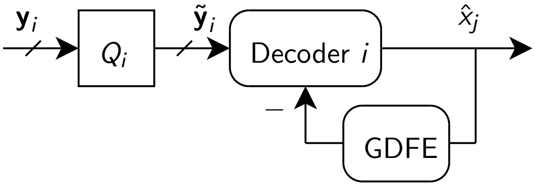


$$\tilde{\mathbf{y}}_i = R_i \tilde{\mathbf{x}}_i + \tilde{\mathbf{n}}_i$$

$$\tilde{\mathbf{n}}_i = Q^\dagger \mathbf{n} \sim \mathcal{CN}(0, I)$$

MIMO Broadcast Scheme

Decoder:



Optimal MMSE (Capacity-Achieving) Scheme

- Holds for channel matrices of any dimension and rank (not equal between H_1 and H_2).
- Based upon an extension of the decomposition to non-square matrices.
- Similar to the extension of V-BLAST from zero-forcing to MMSE.
- Any number of codebooks $\geq$ number of Tx antennas.

Summary

- The problem of transmitting the same message to two users was considered.
- A new joint-matrix decomposition was introduced.
- A low-complexity scheme which uses standard scalar codebooks, linear operations and successive interference cancellation was proposed.
- This technique is applicable in other network problems, e.g., rateless coding.