Rematch and Forward: Joint Source-Channel Coding for Communications

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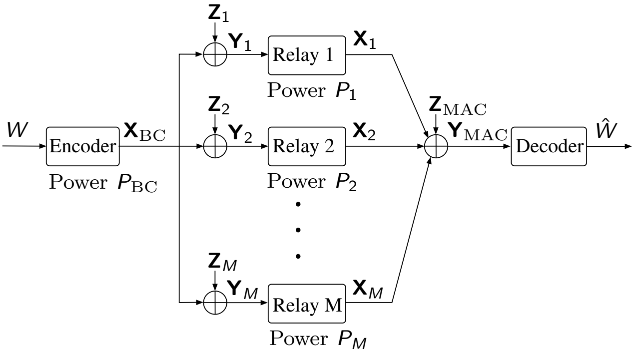
The Parallel Relay Network



- Equal bandwidth ($\rho = 1$) - Schein and Gallager 2000
- Bandwidth expansion/compression factor: $\rho \triangleq \frac{\text{BW}_{\text{BC}}}{\text{BW}_{\text{MAC}}}$.

Definitions

Symmetric Case

$$P_1 = P_2 = \cdots = P_M \triangleq P_{\text{MAC}}$$

Definitions

$$\text{S}_{\text{MAC}} \triangleq \frac{\sum_{m=1}^{M} P_m}{\sigma^2_{Z_{\text{MAC}}}} = \frac{M P_{\text{MAC}}}{\sigma^2_{Z_{\text{MAC}}}}$$

$$\text{S}_{\text{BC}} \triangleq \frac{P_{\text{BC}}}{\sigma^2_{Z_{\text{BC}}}}$$

$$C(\text{S}) \triangleq 1/2 \log(1 + S)$$

For now: Assume equal bandwidths ($\rho = 1$).

Upper Bounds on Capacity

Simple Upper Bounds

- **Noiseless BC:** $C \leq C\left(MS_{\text{MAC}}\right)$
- **Noiseless MAC:** $C \leq C\left(MS_{\text{BC}}\right)$

Improved Upper Bounds

- Schein (Ph.D.) – Other cuts.
- Niesen-Diggavi – Consider several different cuts, *simultaneously*.

Decode-and-Forward

Decode and Forward

Encode the message at the relays and decode it again for the MAC.

$$C_{\mathrm{DF}} = \min\left\{ C\left(MS_{\mathrm{MAC}}\right), C\left(S_{\mathrm{BC}}\right) \right\}$$

Remark

Rate must be low enough, such that each relay can decode reliably.

Compress-and-Forward

Compress and Forward

- Relays digitally compress their analog inputs and transmit them over the MAC.
- Optimal Compression $=$ CEO Approach.

$$C_{\mathrm{CF}} \le C\Big(\mathsf{S}_{\mathrm{BC}} C(\mathsf{S}_{\mathrm{MAC}})\Big)$$

(see Gastpar & Vetterli, 2005)

Remark

Fails to achieve the coherence gain, due to separation.

Amplify-and-Forward

Amplify and Forward

Send the relay inputs up to proper amplification.

$$C_{\mathsf{AF}} = C\left(\frac{M S_{\mathsf{MAC}} S_{\mathsf{BC}}}{S_{\mathsf{MAC}} + S_{\mathsf{BC}} + 1}\right)$$

Remarks

- Accumulates the noise.
- **Gains coherence gains in both sections!**
- Outperforms CF for all SNRs ($\rho = 1$).

Comparsion of Different Strategies

Interpretation of Different Strategies

- **Decode & Forward:** "Channel coding" approach.
- **Compress & Forward:** "Source coding" approach.
- **Amplify & Forward:** "Joint source-channel coding" approach.

Strategy	A & F	D & F	C & F
BC coherence	✓	X	X
MAC coherence	✓	✓	X
avoid noise accumulation	X	✓	X

Colored Problem

General Problem: Noises with general color

$\Downarrow$

Symmetric Case: Noises in each section have the same spectrum

$\Downarrow$

Interesting Case: Unequal Bandwidths

$\|$

Bandwidth Expansion/Compression

Possible Solutions for Bandwidth Mismatch Case

Possible Solutions

- C&F and D&F do not exploit the coherence gains.
- A&F does not exploit full bandwidth.

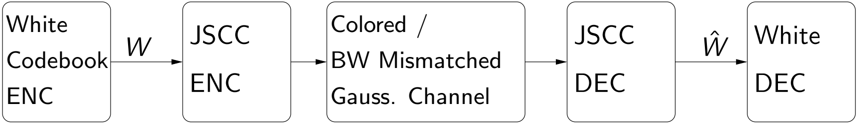
Can we exploit both gains simultaneously?

Yes we can!

$\Downarrow$

Rematch & Forward

Joint Source-Channel Coding for Point-to-Point



- Use white **channel** codebook of arbitrary BW.
- Treat W as a **source** signal.
- Use **joint source-channel** coding to transmit W.
- Treat the reconstruction $\hat{W}$ as output of white channel.

C = R(D) for MMSE distortion

$\Downarrow$

Capacity is Achieved

BW mismatch: Equivalent SNR $\approx$ SNR$^{\rho}$

Rematch & Forward - Idea

Joint Source-Channel Coding Usage

- White codebook of BW = BW_{MAC}.
- The codebook is not matched to the BC section.

$$\Downarrow$$

Use optimal JSCC for the first channel section ($R(D) = C_{\text{BC}}$).

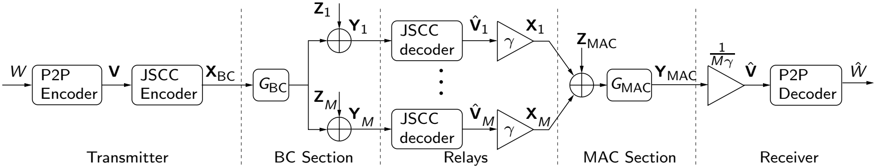
- Reconstruction = Output of white channel with BW_{MAC}.
- Apply A&F to reconstructions.

Conclusion

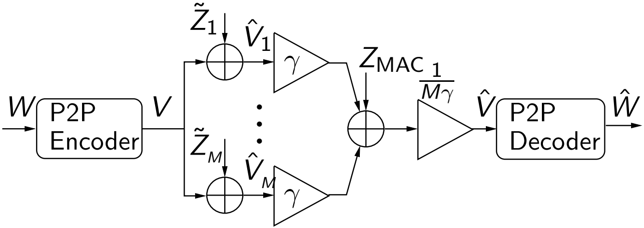
JSCC exploits coherence gains for $BW_{BC} \neq BW_{MAC}$.

Rematch & Forward - Scheme

Scheme:



Equivalent scheme:



Maximally Analog Reconstruction Error

Problem

Not every JSSC scheme achieves full possible coherence.
Errors should be summed non-coherently.

$\Downarrow$

Need analog (codeword independent) JSCC scheme

Definition (Maximally Analog Reconstruction Error JSCC Scheme)

A JSCC scheme for source with BW_{SC} and channel with BW_{CH}, where the unbiased reconstruction error is independent of the source for all $f < \min\{BW_{SC}, BW_{CH}\}$.

Maximally Analog Reconstruction Error JSCC Shemes

BW Mismatch

- Mittal & Phamdo (2002).
- Reznic et al.(2006).

General Colored Case

- Prabhakaran et al.
- Kochman and Zamir.

Hybrid Analog-Digital Schemes

(Mittal & Phamdo, 2002)

BW Expansion ($\rho > 1$)

- Use "excess BW" to digitally transmit quantized source.
- Use source BW to analogically transmit quantization error.
- Reconstruction error = Channel noise in source BW.

BW Compression ($\rho < 1$)

- Quantize excess BW component of the source.
- Transmit by superposition:

Digital code of quantized component
+
Channel BW component

Analog Matching

General Colored (Symmetric) Case

What can we do when noises have arbitrary spectra?

Analog Matching

- Can match *any BW ratio and noise color.*
- Uses time-domain processing.
- Transmits an analog signal modulo-lattice.

$\Downarrow$

Achieves maximally analog estimation error.

Performance Example: BW Expansion

For high SNRs:

$$C_{\text{CF}} \leq C\left(S_{\text{BC}}^{\rho} \left(\frac{C(S_{\text{MAC}})}{\rho} \right)^{\rho} \right)$$

$$C_{\text{DF}} \cong C\left(\min\left(M S_{\text{MAC}}, S_{\text{BC}}^{\rho} \right) \right)$$

$$C_{\text{AF}} \cong C\left(M \left(S_{\text{MAC}} \| S_{\text{BC}} \right) \right)$$

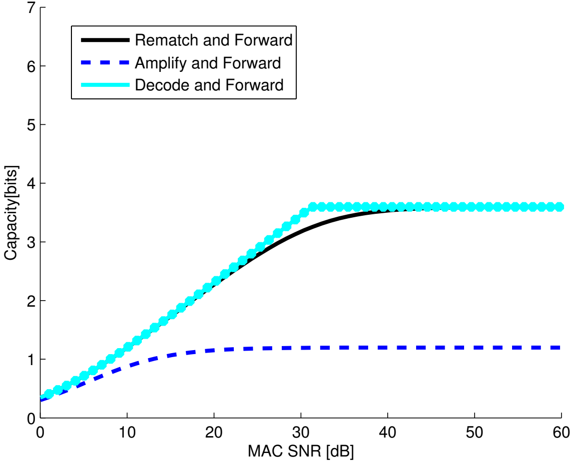
$$C_{\text{RF}} \cong C\left(M \left(S_{\text{MAC}} \| S_{\text{BC}}^{\rho} \right) \right)$$

where $a \| b \triangleq \frac{ab}{a+b}$

R&F outperforms all other strategies for large enough M.

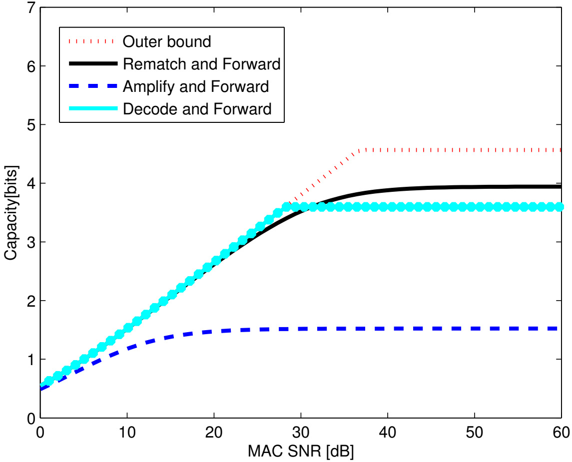
Performance Example: BW Expansion (M=1)

$$\rho = 3, \quad S_{BC} = 10\text{dB}$$



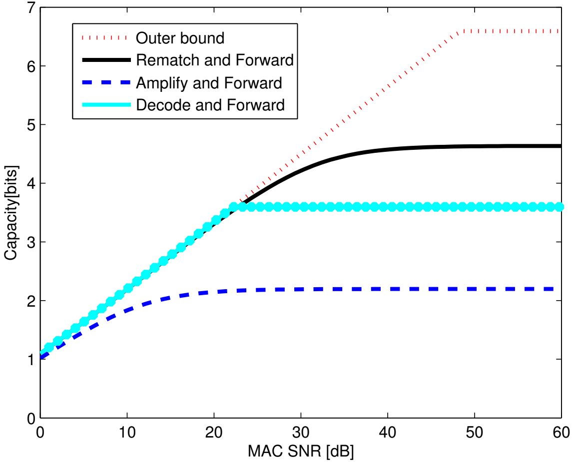
Performance Example: BW Expansion (M=2)

$$\rho = 3, \quad S_{BC} = 10\text{dB}$$

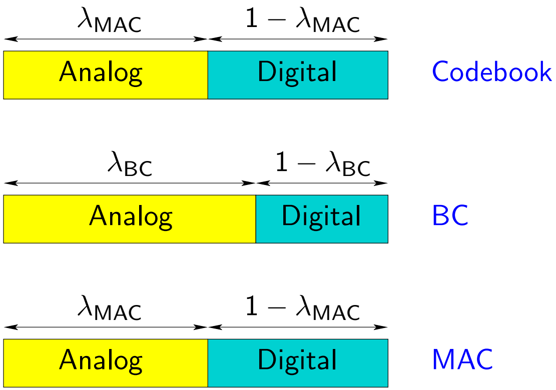


Performance Example: BW Expansion (M=8)

$$\rho = 3, \quad S_{BC} = 10\text{dB}$$

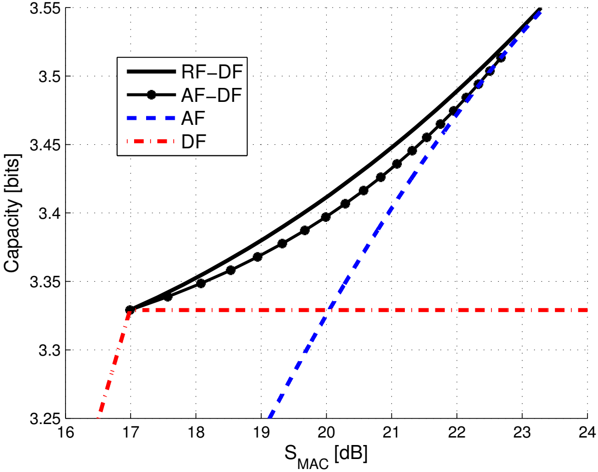


Improvement over A&F for $\rho = 1$



- Yellow bands - used for R&F; Cyan bands - used for D&F.
- For R&F: $\rho = \frac{\lambda_{\text{BC}}}{\lambda_{\text{MAC}}}$.

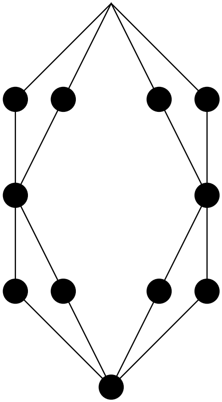
Improvement over A&F for $\rho = 1$



- R&F-D&F timesharing outperforms any known strategy.

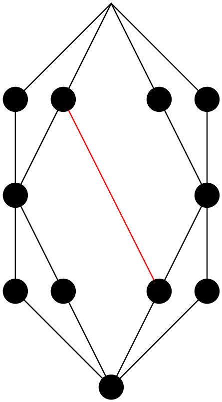
Layered Networks

Layered Network



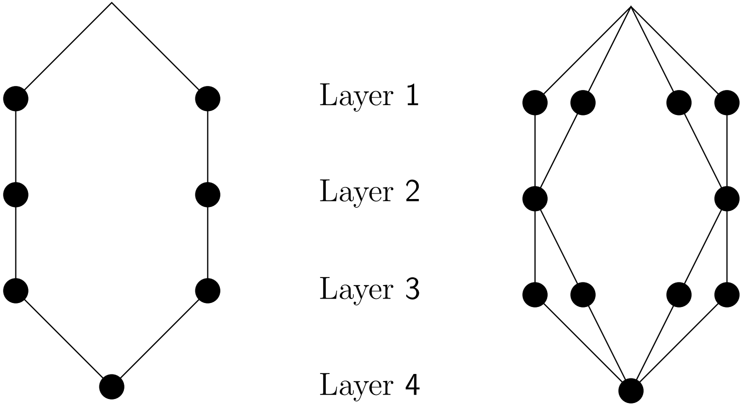
Layered Networks

Not a Layered Network



Layered Networks

- Rematch and Forward can be applied to "Layered Networks".



Further Research

- Non-symmetric (different noise spectra) case.
- Extension to MIMO channels.
- Usage of R&F for more complex networks.
- Constructing good JSCC schemes for the MAC section.

HAPPY BIRTHDAY!