Rematch and Forward: Joint Source-Channel Coding for Communications

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Equal bandwidth ($\rho = 1$) - Schein and Gallager 2000

Bandwidth expansion/compression factor: $\rho \triangleq \frac{\text{BW}_{BC}}{\text{BW}_{MAC}}$. 

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Symmetric Case

\[ P_1 = P_2 = \cdots = P_M \triangleq P_{MAC} \]

**Definitions**

\[ S_{MAC} \triangleq \frac{\sum_{m=1}^{M} P_m}{\sigma^2 Z_{MAC}} = \frac{MP_{MAC}}{\sigma^2 Z_{MAC}} \]

\[ S_{BC} \triangleq \frac{P_{BC}}{\sigma^2 Z_{BC}} \]

\[ C(S) \triangleq \frac{1}{2} \log(1 + S) \]

For now: Assume equal bandwidths \((\rho = 1)\).
Upper Bounds on Capacity

Simple Upper Bounds

- **Noiseless BC**: $C \leq C(MS_{MAC})$
- **Noiseless MAC**: $C \leq C(MS_{BC})$

Improved Upper Bounds

- Schein (Ph.D.) – Other cuts.
- Niesen-Diggavi – Consider several different cuts, *simultaneously.*
Decode-and-Forward

Encode the message at the relays and decode it again for the MAC.

\[ C_{DF} = \min \left\{ C(MS_{MAC}), C(S_{BC}) \right\} \]

Remark

*Rate must be low enough, such that each relay can decode reliably.*
Compress-and-Forward

Compress and Forward

- Relays digitally compress their analog inputs and transmit them over the MAC.
- Optimal Compression = CEO Approach.

$$C_{CF} \leq C\left(S_{BC} C(S_{MAC})\right)$$

(see Gastpar & Vetterli, 2005)

Remark

*Fails to achieve the coherence gain, due to separation.*
Amplify-and-Forward

Amplify and Forward

Send the relay inputs up to proper amplification.

\[ C_{AF} = C \left( \frac{MS_{MAC}S_{BC}}{S_{MAC} + S_{BC} + 1} \right) \]

Remarks

- Accumulates the noise.
- **Gains coherence gains in both sections!**
- Outperforms CF for all SNRs \((\rho = 1)\).
Comparison of Different Strategies

Interpretation of Different Strategies

- **Decode & Forward**: “Channel coding” approach.
- **Compress & Forward**: “Source coding” approach.
- **Amplify & Forward**: “Joint source-channel coding” approach.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>A &amp; F</th>
<th>D &amp; F</th>
<th>C &amp; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC coherence</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>MAC coherence</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>avoid noise accumulation</td>
<td>X</td>
<td>✓</td>
<td>X</td>
</tr>
</tbody>
</table>
Colored Problem

**General Problem:** Noises with general color

**Symmetric Case:** Noises in each section have the same spectrum

**Interesting Case:** Unequal Bandwidths

Bandwidth Expansion/Compression
Possible Solutions for Bandwidth Mismatch Case

Possible Solutions

- C&F and D&F do not exploit the coherence gains.
- A&F does not exploit full bandwidth.

Can we exploit both gains simultaneously?

Yes we can!

Rematch & Forward
Joint Source-Channel Coding for Point-to-Point

- Use white channel codebook of arbitrary BW.
- Treat $W$ as a source signal.
- Use joint source-channel coding to transmit $W$.
- Treat the reconstruction $\hat{W}$ as output of white channel.

$C = R(D)$ for MMSE distortion

Capacity is Achieved

BW mismatch: Equivalent SNR $\approx \text{SNR}^\rho$
Rematch & Forward - Idea

Joint Source-Channel Coding Usage

- White codebook of $BW = BW_{MAC}$.
- The codebook is not matched to the BC section.
  \[ \downarrow \]
  Use optimal JSCC for the first channel section ($R(D) = C_{BC}$).
- Reconstruction = Output of white channel with $BW_{MAC}$.
- Apply A&F to reconstructions.

Conclusion

JSCC exploits coherence gains for $BW_{BC} \neq BW_{MAC}$. 
Rematch & Forward - Scheme

Scheme:

![Scheme Diagram]

Equivalent scheme:

![Equivalent Scheme Diagram]
Maximally Analog Reconstruction Error

**Problem**

Not every JSSC scheme achieves full possible coherence. Errors should be summed non-coherently.

\[ \Downarrow \]

Need analog (codeword independent) JSCC scheme

**Definition (Maximally Analog Reconstruction Error JSCC Scheme)**

A JSCC scheme for source with \( BW_{SC} \) and channel with \( BW_{CH} \), where the unbiased reconstruction error is independent of the source for all \( f < \min\{BW_{SC}, BW_{CH}\} \).
Maximally Analog Reconstruction Error JSCC Schemes

**BW Mismatch**
- Reznic et al. (2006).

**General Colored Case**
- Prabakaran et al.
- Kochman and Zamir.
Hybrid Analog-Digital Schemes

(Mittal & Phamdo, 2002)

**BW Expansion** ($\rho > 1$)
- Use “excess BW” to digitally transmit quantized source.
- Use source BW to analogically transmit quantization error.
- Reconstruction error = Channel noise in source BW.

**BW Compression** ($\rho < 1$)
- Quantize excess BW component of the source.
- Transmit by superposition:
  - Digital code of quantized component
  - Channel BW component
Analog Matching

**General Colored (Symmetric) Case**

What can we do when noises have arbitrary spectra?

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**Analog Matching**

- Can match *any BW ratio and noise color*.
- Uses time-domain processing.
- Transmits an analog signal modulo-lattice.

\[ \downarrow \]

Achieves maximally analog estimation error.
Performance Example: BW Expansion

For high SNRs:

\[
C_{CF} \leq C \left( S_{BC}^{\rho} \left( \frac{C(S_{MAC})}{\rho} \right)^{\rho} \right)
\]

\[
C_{DF} \approx C \left( \min \left( MS_{MAC}, S_{BC}^{\rho} \right) \right)
\]

\[
C_{AF} \approx C \left( M(S_{MAC} \| S_{BC}) \right)
\]

\[
C_{RF} \approx C \left( M(S_{MAC} \| S_{BC}^{\rho}) \right)
\]

where \( a \| b \triangleq \frac{ab}{a + b} \)

R&F outperforms all other strategies for large enough \( M \).

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Performance Example: BW Expansion (M=1)

\[ \rho = 3, \quad S_{BC} = 10\text{dB} \]
Performance Example: BW Expansion (M=2)

\[ \rho = 3, \quad S_{BC} = 10\text{dB} \]
Performance Example: BW Expansion (M=8)

\[ \rho = 3, \quad S_{BC} = 10\text{dB} \]
Improvement over A&F for $\rho = 1$

- Yellow bands - used for R&F; Cyan bands - used for D&F.
- For R&F: $\rho = \frac{\lambda_{BC}}{\lambda_{MAC}}$.
Improvement over A&F for $\rho = 1$

- R&F-D&F timesharing outperforms any known strategy.
Layered Networks

Layered Network
Not a Layered Network
Rematch and Forward can be applied to "Layered Networks"
Further Research

- Non-symmetric (different noise spectra) case.
- Extension to MIMO channels.
- Usage of R&F for more complex networks.
- Constructing good JSCC schemes for the MAC section.
HAPPY BIRTHDAY!