Rematch and Forward: Joint Source/Channel Coding for Communications

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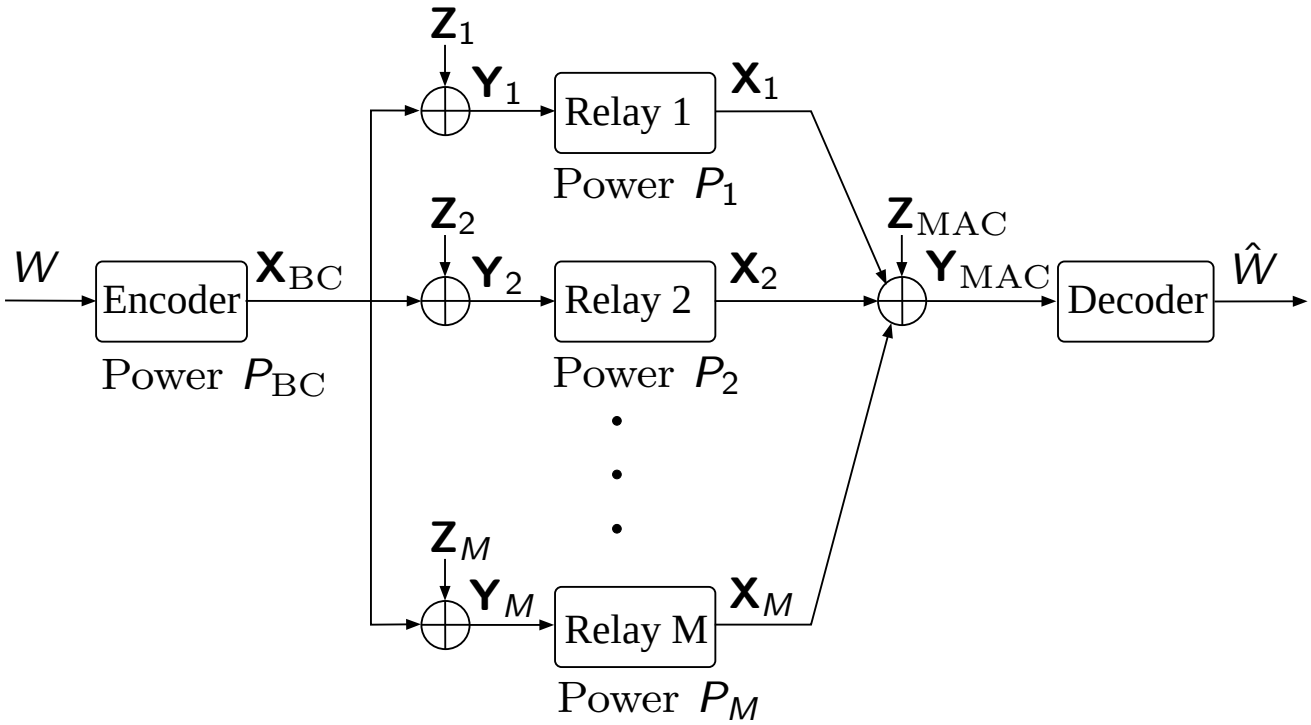
The Parallel Relay Network



- Bandwidth expansion/compression factor: $\rho \triangleq \frac{\mathrm{BW_{BC}}}{\mathrm{BW_{MAC}}}$.
- $\rho = 1$ - Schein Gallager (2000).

Definitions

Symmetric Case

$$P_1 = P_2 = \cdots = P_M \triangleq P_{\text{MAC}}$$

Definitions

$$\mathsf{S}_{\text{MAC}} \triangleq \frac{\sum_{m=1}^{M} P_m}{\sigma^2_{Z_{\text{MAC}}}} = \frac{MP_{\text{MAC}}}{\sigma^2_{Z_{\text{MAC}}}}$$

$$\mathsf{S}_{\text{BC}} \triangleq \frac{P_{\text{BC}}}{\sigma^2_{Z_{\text{BC}}}}$$

Simple Upper Bounds on Capacity

- **Noiseless BC:** $C \leq \frac{1}{2}\log\left(1 + M\mathsf{S}_{\text{MAC}}\right)$
- **Noiseless MAC:** $C \leq \frac{\rho}{2}\log\left(1 + M\mathsf{S}_{\text{BC}}\right)$

Strategies for $\rho = 1$

Strategies

- **Decode & Forward:** Decode the message at the relays and encode it again for the MAC.
- **Compress & Forward:** Relays digitally compress their inputs and transmit them over the MAC.
- **Amplify & Forward:** Send relay inputs adjusted to MAC Power.

Strategy	A & F	D & F	C & F
BC coherence	✓	X	X
MAC coherence	✓	✓	X
avoid noise accumulation	X	✓	X

Colored Problem

- **General Problem**: Noises with general color.
- **Symmetric Case**: Noises in each section have the same spectrum.
- **Interesting Case**: Unequal Bandwidths

  $\Downarrow$

  Bandwidth Expansion/Compression.

Possible Solutions

Possible Solutions

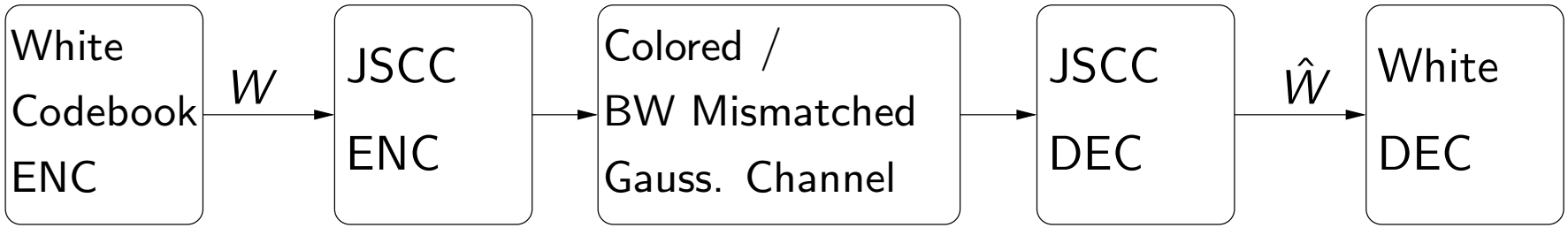
- C&F and D&F do not exploit the coherence gains.
- A&F does not exploit full bandwidth.

Can we exploit both gains simultaneously?

$\Downarrow$

Rematch & **F**orward

Joint Source-Channel Coding for Point-to-Point



- Use white **channel** codebook of arbitrary BW.
- Treat W as a **source** signal.
- Use **joint source-channel** coding to transmit W.
- Treat the reconstruction $\hat{W}$ as output of white channel.

C = R(D) for MMSE distortion

$\Downarrow$

Capacity is Achieved

BW mismatch: Equivalent SNR $\approx$ SNR$^{\rho}$

Rematch & Forward

Joint Source-Channel Coding Usage

- White codebook of BW $=$ $\mathrm{BW}_{\mathrm{MAC}}$.
- The codebook is not matched to the BC section.

$$\Downarrow$$

Use optimal JSCC for the first channel section ($R(D) = C_{\mathrm{BC}}$).

- Reconstruction $=$ Output of white channel with $\mathrm{BW}_{\mathrm{MAC}}$.
- Apply A&F to reconstructions.

Conclusion

JSCC exploits coherence gains for $BW_{BC} \neq BW_{MAC}$.

Maximally Analog Reconstruction Error

Problem

Not every JSC scheme achieves full possible coherence. Errors should be summed non-coherently.

$$\Downarrow$$

Need analog (codeword independent) JSCC scheme

Definition (Maximally Analog Reconstruction Error JSCC Scheme)

A JSCC scheme for source with BW_{SC} and channel with BW_{CH}, where the unbiased reconstruction error is independent of the source for all $f < \min\{BW_{SC}, BW_{CH}\}$.

Maximally Analog Reconstruction Error JSCC Shemes

BW Mismatch

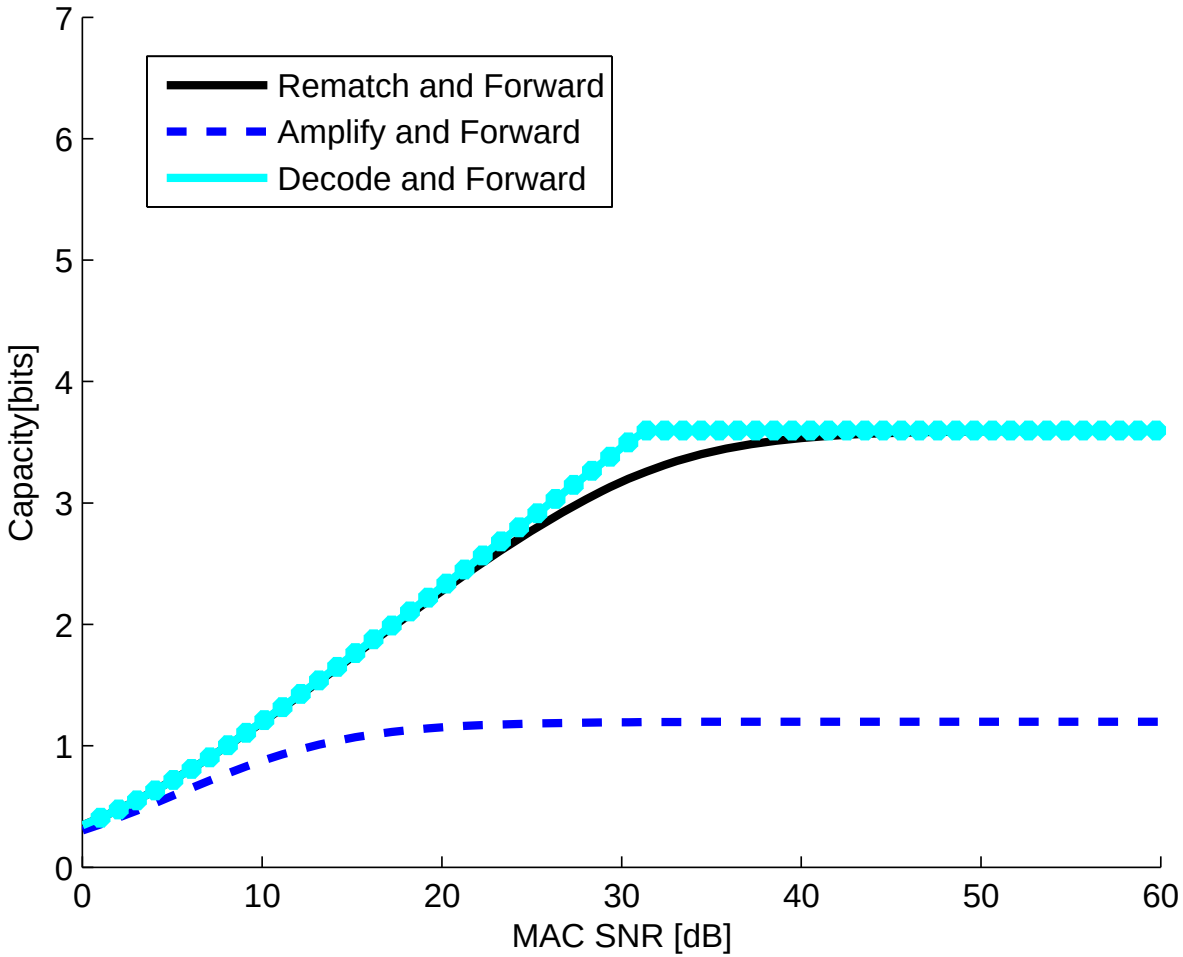
- Mittal & Phamdo (2002).
- Reznic et al.(2006).

General Colored Case

- Prabhakaran et al.
- Kochman and Zamir.

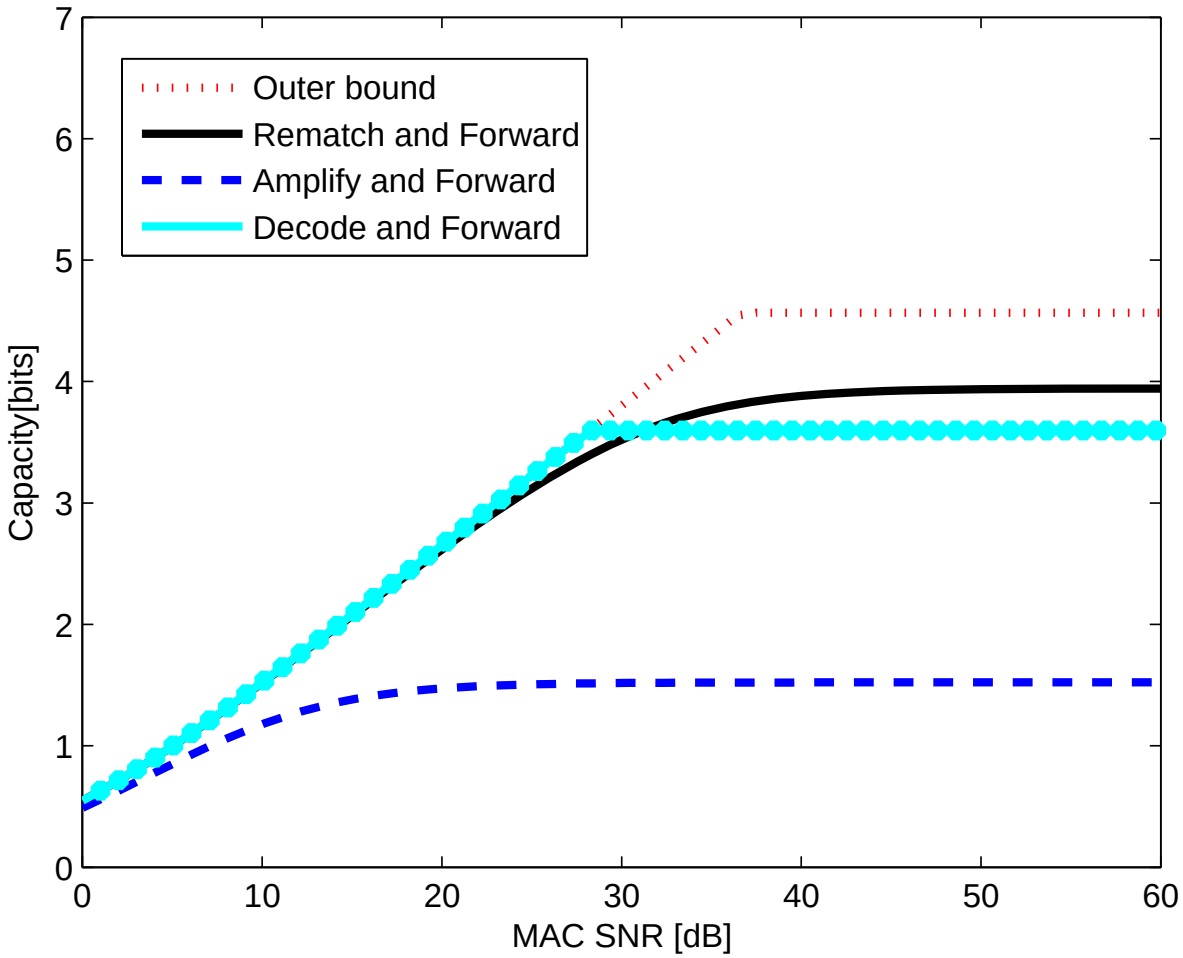
Performance Example: BW Expansion (M=1)

$$\rho = 3, \quad S_{BC} = 10\text{dB}$$



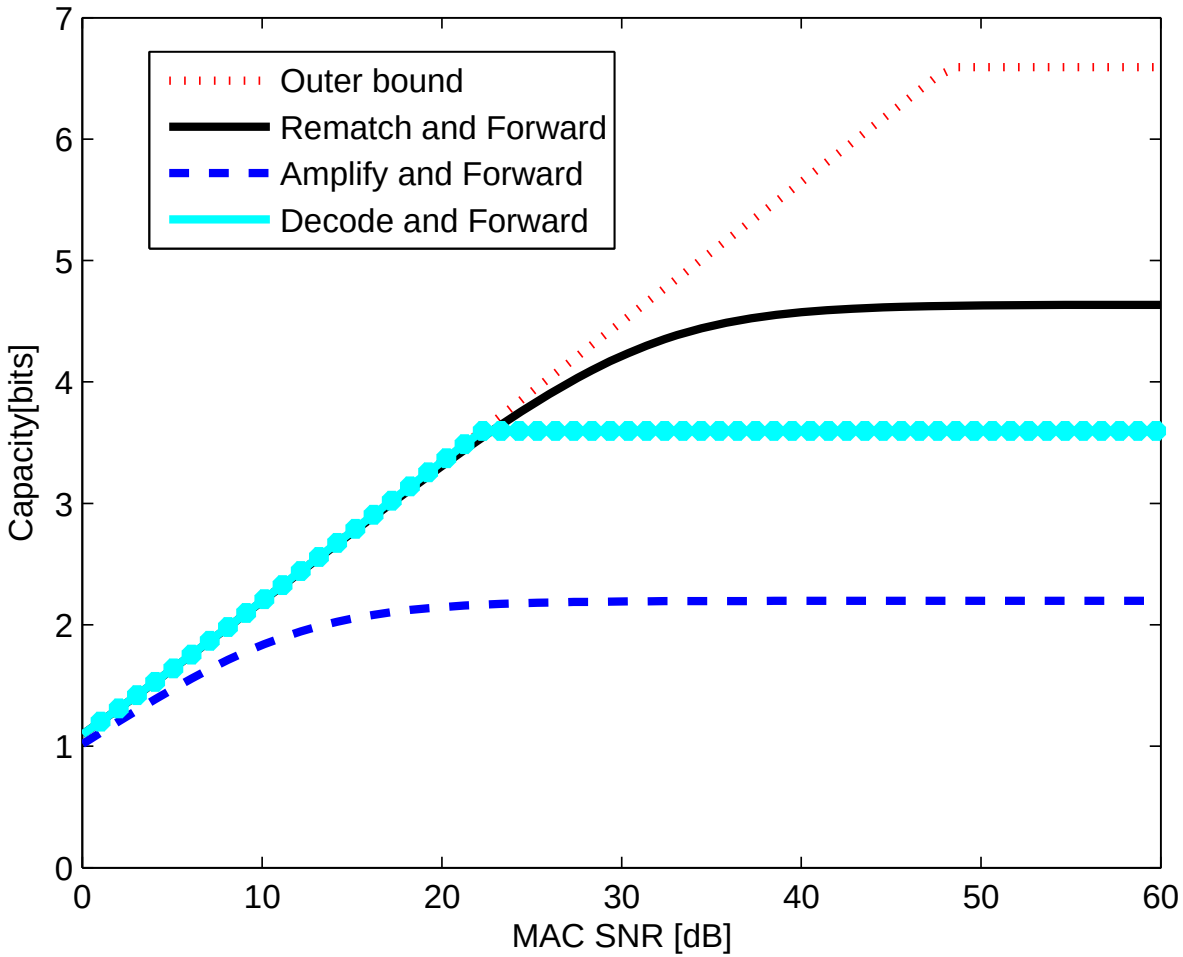
Performance Example: BW Expansion (M=2)

$$\rho = 3, \quad S_{BC} = 10\text{dB}$$



Performance Example: BW Expansion (M=8)

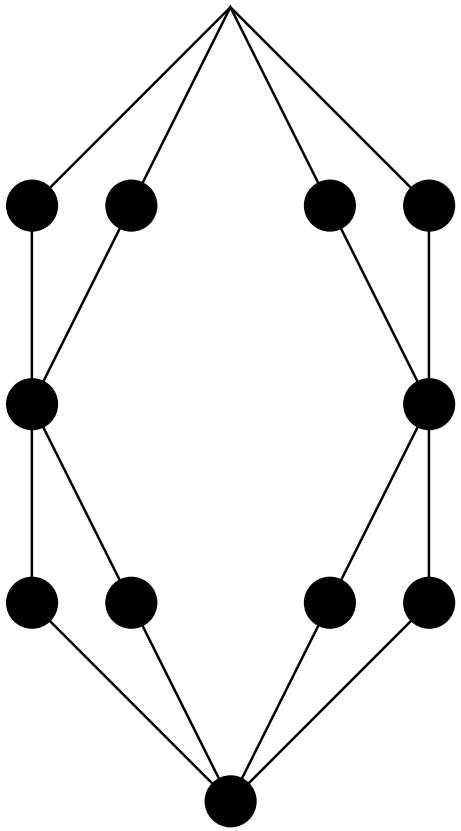
$$\rho = 3, \quad S_{BC} = 10\text{dB}$$



- Best known strategy: timesharing between R&F and D&F.

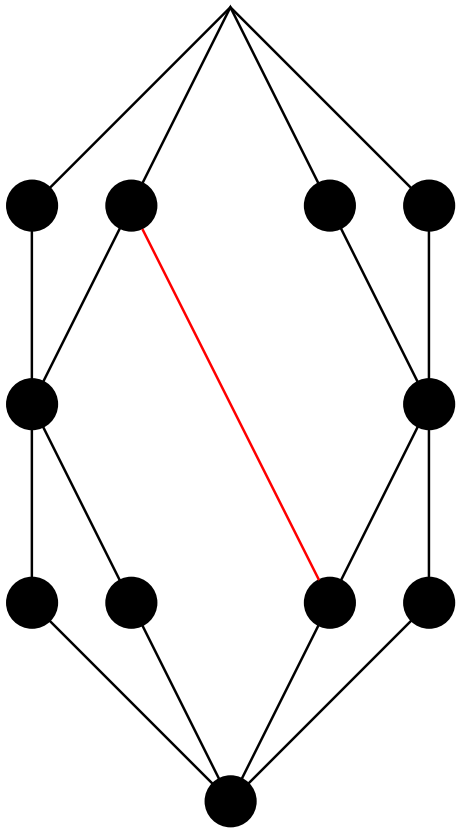
Layered Networks

Layered Network



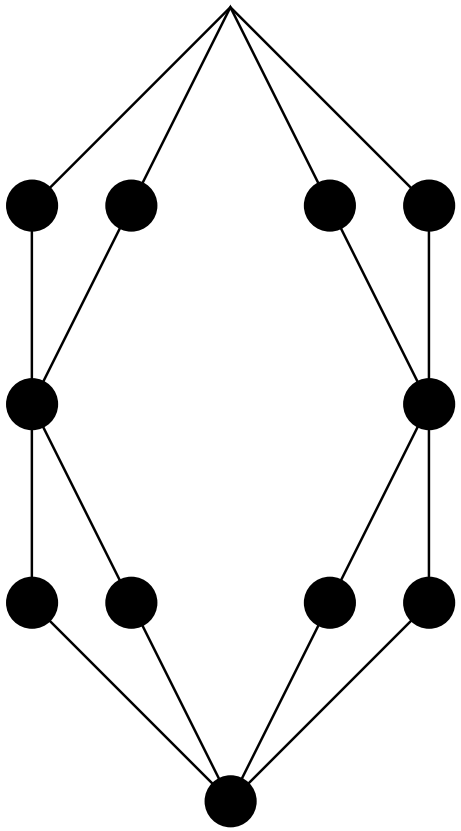
Layered Networks

Not a Layered Network



Layered Networks

- Rematch and Forward can be applied to "Layered Networks".



Further Research

- Non-symmetric (different noise spectra) case.
- Extension to MIMO channels.
- Usage of R&F for more complex networks.
- Constructing good JSCC schemes for the MAC section.