On Robust Dirty Paper Coding

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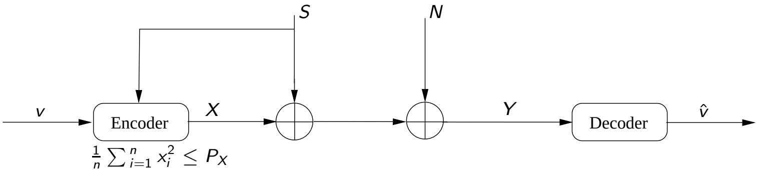
1. Background
   - DP Channel
   - Lattice Strategies for DPC / Tomlinson-Harashima Precoding
   - Compound DP Channel
2. Robust DPC - Smart Rx
   - Finite P_S
   - Infinite P_S
3. Robust DPC - Smart Tx and Rx
4. Concluding Remarks and Further Research

Dirty Paper Channel

$$Y = X + S + N$$



S - interference, known (causally/noncausally) at Tx with average power P_S.

N - AWGN with average power P_N.

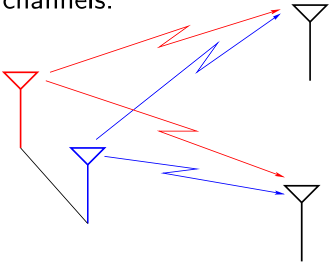
Costa: S known (noncausally) @ Tx $\rightarrow$ as good as no S

$$C = \frac{1}{2}\log(1 + \mathrm{SNR})$$

Applications of Dirty Paper Coding

Model serves as an information-theoretic framework for (known) interference cancellation in:

- ISI channels.
- Information embedding.
- MIMO broadcast channels.



Assumption: Tx knows channel gains.

Lattice Strategies / Tomlinson-Harashima Precoding

Remarks

P_S, for time being, is arbitrarily large.

Transmitter

$$X = [v - \alpha S - U] \bmod \mathcal{A}_\Delta \; ; \; \mathcal{A}_\Delta = \left[-\tfrac{\Delta}{2}, \tfrac{\Delta}{2}\right]$$

Receiver

$$Y' = [\alpha Y + U] \bmod \mathcal{A}_\Delta = [v + N_{\mathrm{eff}}] \bmod \mathcal{A}_\Delta,$$

$$N_{\mathrm{eff}} = (1-\alpha)U + \alpha N.$$

Performances

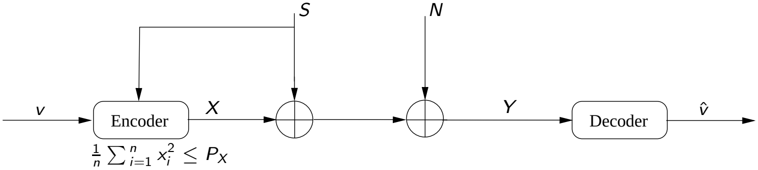
$$\mathrm{SNR}_{\mathrm{eff}} \triangleq \frac{P_X}{P_{N_{\mathrm{eff}}^{\beta}}} \xrightarrow{\alpha_{\mathrm{MMSE}} \triangleq \frac{\mathrm{SNR}}{\mathrm{SNR}+1}} 1 + \mathrm{SNR}.$$

$$R \geq \tfrac{1}{2}\log(\mathrm{SNR}_{\mathrm{eff}}) - \tfrac{1}{2}\log\tfrac{2\pi e}{12} = \tfrac{1}{2}\log(1+\mathrm{SNR}) - \overbrace{\frac{1}{2}\log\frac{2\pi e}{12}}^{\text{Causality Loss}}.$$

Compound Dirty Paper Channel

Dirty paper channel with perfect channel knowledge:

$$Y = X + S + N$$

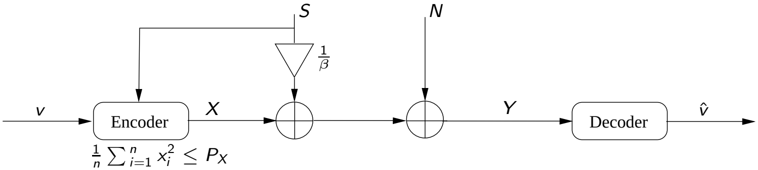


- $\mathrm{SNR} \triangleq \frac{P_X}{P_N}$
- $\mathrm{SIR} \triangleq \frac{P_X}{P_S}$

Compound Dirty Paper Channel

Compound dirty paper channel:

$$Y = X + \frac{S}{\beta} + N$$



- $\beta \in [1-\delta, 1+\delta]$ is known to Rx.
- β constant (non-ergodic) for whole transmission.
- Compound $\rightarrow$ Achievable rate - worst case over all β values.

Compound Channel Case

Remark

α is used at both ends (Tx and Rx).

Transmitter Knows β

The problem reduces to classical DPC case ($\beta \equiv 1$).

Transmitter Does NOT know β

What is the best strategy?

- To work with α_{MMSE} at both ends?
- To work with α_{MMSE} at Tx and $\alpha_{\mathrm{MMSE}}/\beta$ - at Rx?
- Other α_{T}, α_{R} selections?

Naïve Approach $\longrightarrow$ Work as Before

Transmitter

$$X = [v - \alpha S - U] \bmod \mathcal{A}_\Delta$$

Receiver

$$Y = [\alpha Y + U] \bmod \mathcal{A}_\Delta = [v + N_{\text{eff}}^{\beta}] \bmod \mathcal{A}_\Delta,$$

$$N_{\text{eff}}^{\beta} = (1-\alpha)U + \alpha\frac{(\beta-1)}{\beta}S + \alpha N.$$

Remark

$$P_S \to \infty \Rightarrow R = 0$$

What can we do?

Smart Receiver

Distinguish between α_{R} (α @ Rx) and α_{T} (α @ Tx)

Transmitter

$$X = [v - \alpha_{\mathrm{T}} S - U] \bmod \mathcal{A}_{\Delta}.$$

Receiver

$$Y = [\alpha_{\mathrm{R}} Y + U] \bmod \mathcal{A}_{\Delta} = [v + N_{\mathrm{eff}}^{\beta}] \bmod \mathcal{A}_{\Delta},$$

$$N_{\mathrm{eff}}^{\beta} = (1 - \alpha_{\mathrm{R}})U + (\alpha_{\mathrm{T}}\beta - \alpha_{\mathrm{R}})\frac{S}{\beta} + \alpha_{\mathrm{R}} N.$$

Remarks

- $\alpha_{\mathrm{R}} = \alpha_{\mathrm{T}}$ - **Bad!**
- *For time being, we optimize w.r.t.* $\mathrm{SNR}_{\mathrm{eff}}^{\mathrm{wc}}$.

MMSE Estimation

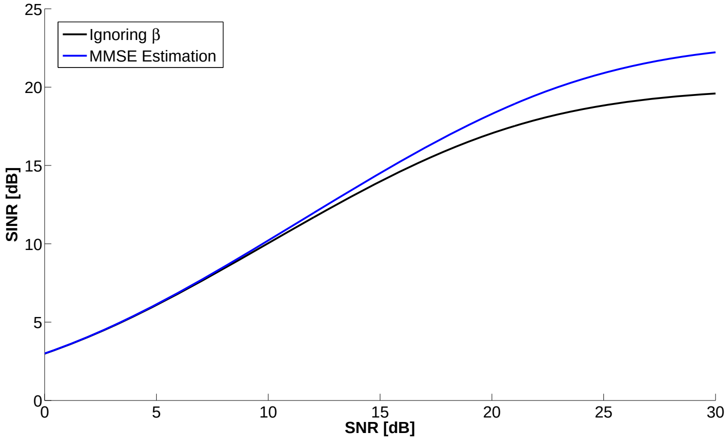
$$\alpha_{\mathsf{R}}{}^{\mathrm{MMSE}} = \frac{1 + \frac{\alpha_{\mathsf{T}}{}^{\mathrm{MMSE}}\beta}{\mathsf{SIR}}}{1 + \frac{1}{\mathsf{SIR}} + \frac{1}{\mathsf{SNR}}}$$

$$\Rightarrow \mathsf{SNR}_{\mathrm{eff}}^{\mathsf{wc}} = (1+\mathsf{SNR})\underbrace{\frac{1 + \frac{1}{\mathsf{SNR}} + \frac{1}{\mathsf{SIR}}}{\left(1 + \frac{1}{\mathsf{SIR}} + \frac{1}{\mathsf{SNR}}\right) + \frac{\mathsf{SNR}}{\mathsf{SIR}}(1-\beta)^2}}_{\text{compound loss}}$$

High SNR Limit

- $\alpha_{\mathsf{T}} \to 1$.
- $\alpha_{\mathsf{R}}{}^{\mathrm{MMSE}} = \frac{1+\frac{\beta}{\mathsf{SIR}}}{1+\frac{1}{\mathsf{SIR}}} = \frac{\mathsf{SIR}+\beta}{\mathsf{SIR}+1}$.
- Optimized $\alpha_{\mathsf{R}} \Rightarrow \mathsf{SNR}_{\mathrm{eff}}^{\mathsf{wc}} = \frac{1+\mathsf{SIR}}{(1-\beta)^2}$.
- Non-Optimized $\alpha_{\mathsf{R}} \Rightarrow \mathsf{SNR}_{\mathrm{eff}}^{\mathsf{wc}} = \frac{\mathsf{SIR}}{(1-\beta)^2}$.

Example: SIR = 1



Remark

Even for SIR = 1, the gain is 3dB, in the limit of high SNR.

Moral

Conclusions

- *In the high SNR regime, the scheme is interference limited due to β.*
- *Ignoring β at Rx is suboptimal and could lead to significant rate losses.*
- *For large S, Rx partially compensates for ignorance of Tx.*

max $\text{SNR}_{\text{eff}}^{\text{wc}}$ criterion $\neq$ max MI criterion,

since the effective noise is not Gaussian.

Random Dirty Paper Strategies

- So far - “Smart Rx”.
- Tx does not know β, but is aware of its ignorance...
- Can Tx do better?

How about Guessing β?

- α_{T} varies from symbol to symbol.
- Common Randomness: for any α_{T}, Rx uses optimal α_{R} as previously.
- Increases (worsens) the MSE.
- Improves the mutual-information.

Motivation

Remark

Concentrate on the case of $P_N = 0$.

Example ($\mathcal{B} = 2$)

- $\beta \in \{1 \pm \delta\}$
- $\alpha_\mathsf{T} \equiv 1$ minimizes the MSE but achieves a *finite rate*.
- $P(\alpha_\mathsf{T} = 1 - \delta) = P(\alpha_\mathsf{T} = 1 + \delta) = 0.5$
  obtains a *larger MSE*, but achieves *infinite rate*.

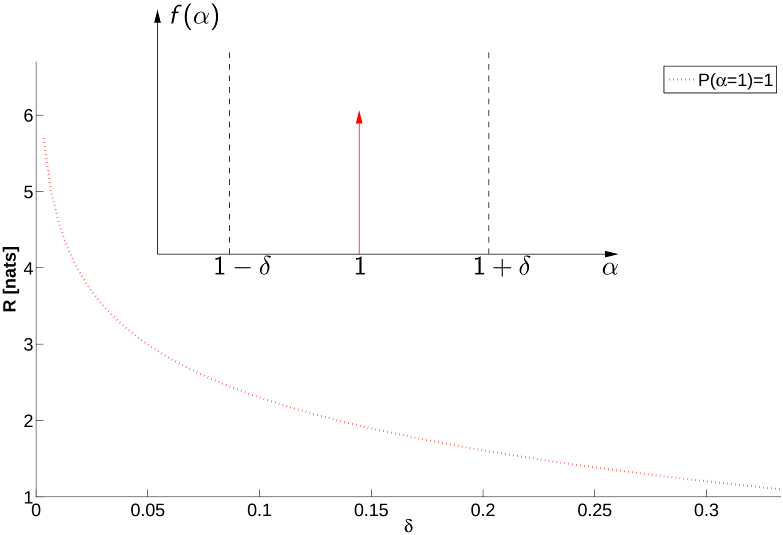
Back to our case of interest...

$\beta \in [1-\delta, 1+\delta]$

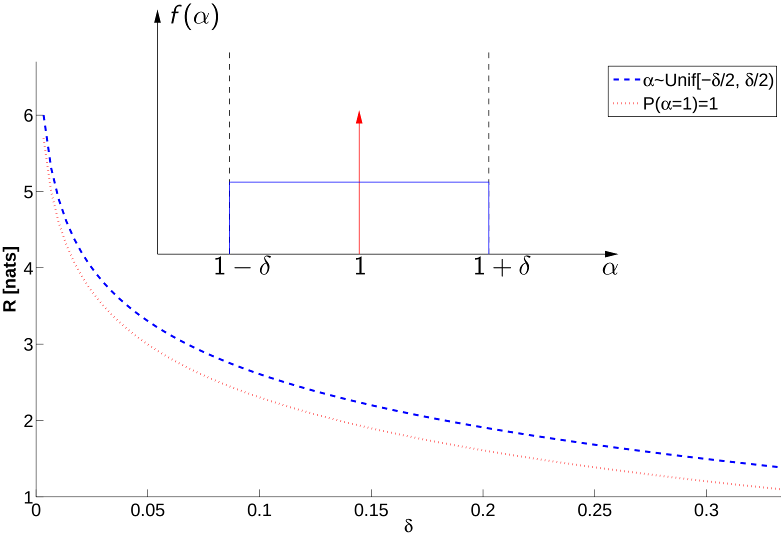
- Optimal distribution of $\alpha_T \rightarrow$ numerical optimization.
- Lower bounds by specific choices of distribution of α.
- **Derived upper bound**

$$\log(1+\delta) - \log(\delta) + 1.$$

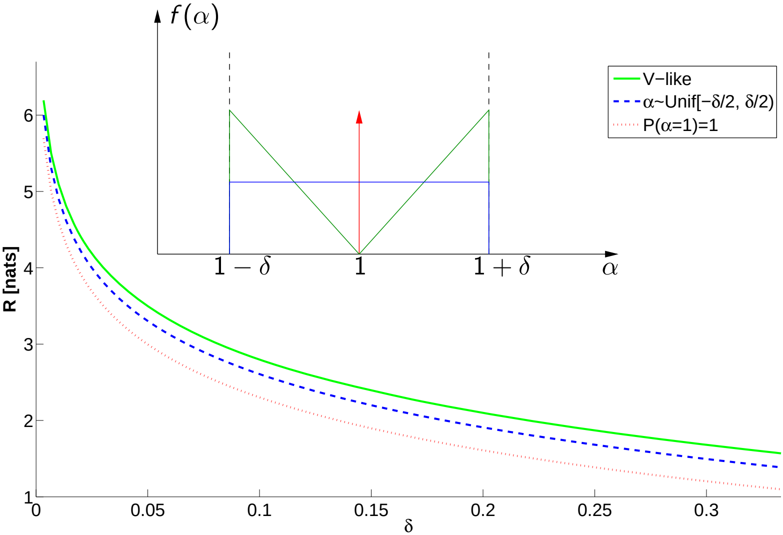
Robust DPC Performances



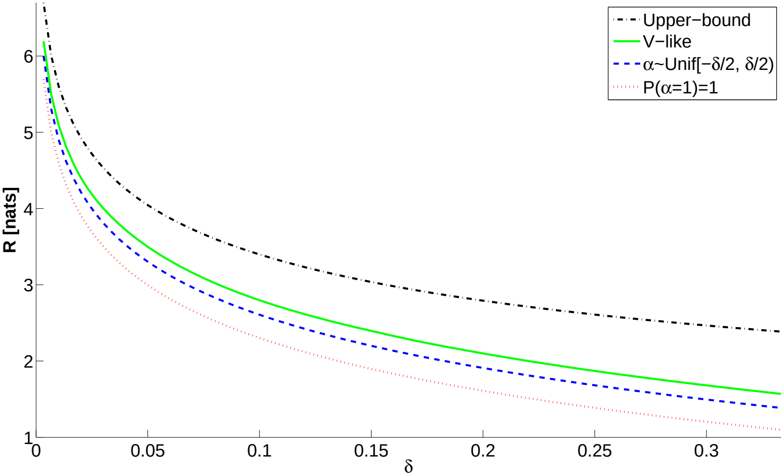
Robust DPC Performances



Robust DPC Performances



Upper Bound



Open Question: How tight is the Upper Bound?

Further Research

- Multidimensional Lattices attain smaller gain than in classical DP channel.
- Finding the Optimal α_{T} distribution for the robust DPC scheme.
- Quantifying gain at finite-SNR DPC.
- Finding the capacity of the compound DP channel.
- Capacity of the "Fast-Fading" DP channel.

Noisy Performances for SNR = 17dB