# Physical-Layer MIMO Relaying

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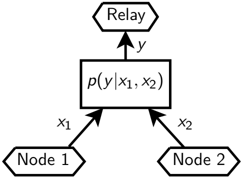
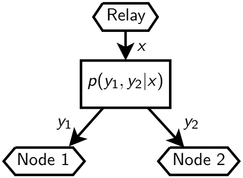
# Two-Way Relay Model

- Nodes 1 and 2 want to convey messages to each other.
- No direct link ⇒ have to use relay.
- Transmission is divided into two transmission phases:

- MAC phase: nodes transmit to relay.




- Broadcast (BC) phase: Relay transmits to nodes.



# Model: MAC Phase in the Gaussian Case

$$y = h_1 x_1 + h_2 x_2 + z$$

- $x_i$ – input of average power $P$.
- $h_i$ - channel gain of user $i$.
- $y$ - channel output.
- $z$ - Channel noise $\sim \mathcal{CN}(\mathbf{0}, 1)$.
- "Closed loop" (Full channel knowledge everywhere).

**Balanced case**

- Equal channel gains: $|h_1| = |h_2| \triangleq |h|$.
- Equal rates: $R_1 = R_2 \triangleq R$.

# Model: Broadcast Phase

- Broadcast (BC) phase can be general.
- Broadcast channel has common-message capacity $C_{\text{common}}$.

# Gaussian SISO Case

## Cut-Set Upper-Bound

$$R_i < R_{CS} = \min\left\{\log\left(1+|h_i|^2 P\right), C_i\right\}$$

$C_i$ – individual capacity of user $i$ in BC phase.

**Balanced case:**

$$R < R_{CS} = \min\left\{\log\left(1+|h|^2 P\right), C_{\text{common}}\right\}$$

## Physical-layer network coding [Narayanan et al. '07]

- Assumes balanced gains: $|h| = |h_1| = |h_2|$, $R = R_1 = R_2$.
- Extends XORing of Network-Coding to Physical-layer.
- Based on nested lattices.
- $R_{PNC} = \min\left\{\log\left(\frac{1}{2}+|h|^2 P\right), C_{\text{common}}\right\}$.

# Gaussian SISO Case

## Unbalanced PNC [Nam, Chung, Lee '08]

- $|h_1| \neq |h_2|, \;\; R_1 \neq R_2$.

$$R_1 \leq \min\left\{\log\left(\frac{|h_1|^2 P}{|h_1|^2 P + |h_2|^2 P} + |h_1|^2 P\right), C_1\right\}$$
$$R_2 \leq \min\left\{\log\left(\frac{|h_2|^2 P}{|h_1|^2 P + |h_2|^2 P} + |h_2|^2 P\right), C_2\right\}$$

- BC phase uses random binning.

# Gaussian SISO Case

## Cut-Set Upper-Bound

$$R < R_{\text{CS}} = \min \left\{ \log\left(1 + |h|^2 P\right), C_{\text{common}} \right\}$$

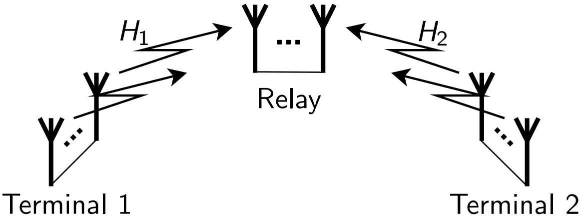
## Decode & Forward

- Decode both messages at relay.
- $R_{\text{DF}} = \min \left\{ \frac{1}{2} \log\left(1 + 2|h|^2 P\right), C_{\text{common}} \right\}$.

## Amplify & Forward / Compress & Forward

- Forward / Compress noisy sum of terminal transmissions.
- $R_{\text{AF}} = \log(1 + \alpha |h|^2 P)$, where $\alpha = \alpha(P|h|^2, C_{\text{common}}) < 1$.

# MIMO




$$\mathbf{y} = H_1\mathbf{x}_1 + H_2\mathbf{x}_2 + \mathbf{z}\,, \qquad i = 1,2$$

- $\mathbf{x}_i$ – $n \times 1$ input vector of power $P$.
- $H_i$ - $n \times n$ channel matrix of user $i$.
- $\mathbf{y}$ - $n \times 1$ output vector.
- $\mathbf{z}$ - Channel noise $\sim \mathcal{CN}(\mathbf{0}, I_n)$.
- "Closed loop" (Full channel knowledge everywhere).

# Model: Assumptions

**Balanced case**

- $R = R_1 = R_2$.
- Equal link capacities $\Rightarrow$ For high SNR: $|H_1| = |H_2|$.

- High SNR assumption.
- Optimum inputs are white.
- $\log\left|I + \frac{P}{n} H_i H_i^\dagger\right| \approx \log\left|\frac{P}{n} H_i H_i^\dagger\right| = 2\log|H_i| + n\log\frac{P}{n}$ .
- Capacity determined by $|H_i|$ for high SNR.

# Main Idea

## Problem

- How to apply scalar PNC in MIMO case?
- Each antenna sees interference from other streams.

## Idea

- Use **joint matrix decomposition** to reduce to **scalar channels**.
- Use **unitary** transformations at transmitters – preserves power.
- Apply **scalar** PNC over each of these channels.

## Decomposition

- Joint **diagonalization** of both channel matrices **not possible**.
- Joint **triangularization**?

# Triangularization

## Joint triangularization

$$H_1 = U T_1 V_1^\dagger$$
$$H_2 = U T_2 V_2^\dagger$$

where

- $U$, $V_1$, $V_2$ – Unitary.
- $T_1$, $T_2$ – Triangular matrices.

## Application for MAC phase

- **Unitary** $V_i$ is applied at encoder $i$ – preserves power.
- $U$ is applied at relay.
- $T_1$, $T_2$ – Effective channels.

# Triangularization

## Interesting special cases

- Generalized SVD (GSVD).
- Joint Equi-diagonal Triangularization (JET).

## GSVD [Van Loan '76]

$T_1$, $T_2$ have:

- **Non-equal diagonals**
- **Proportional columns**

## JET [Khina, Kochman, Erez '10]

$T_1$, $T_2$ have:

- **Equal diagonals**
- **Non-proportional columns**

# Example of GSVD and JET

$$H_1 = \begin{pmatrix} 0.4757 & 0.1541 \\ 2.2374 & 2.8273 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0.4729 & -3.9719 \\ 3.0858 & -23.8021 \end{pmatrix}$$

$$\det(H_1) = \det(H_2) = 1\,.$$

**GSVD** gives rise to:

$$T_1 = \begin{pmatrix} 1/2 & 0 \\ 3 & 2 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 4 & 0 \\ 24 & 1/4 \end{pmatrix}.$$

**JET** gives rise to:

$$T_1 = \begin{pmatrix} 0.3065 & 0 \\ -1.5843 & 3.2628 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0.3065 & 0 \\ 24.1106 & 3.2628 \end{pmatrix}.$$

# GSVD Scheme vs. JET Scheme

## GSVD-based [Yang et al. '10]

- $n$ diagonal elements $= n$ SISO sub-channels.
- $\text{diag}(T_1) \neq \text{diag}(T_2)$ $\Rightarrow$ **unbalanced** SISO sub-channels. $\Rightarrow$ Use **unbalanced** PNC.
- **Proportional** off-diagonal elements $\Rightarrow$ Subtract sum codewords. (GDFE/VBLAST)
- Relay must decode sum **over the reals** $\Rightarrow$ Can't achieve "the $\frac{1}{2}$".

## JET-based [New]

- $n$ diagonal elements $= n$ SISO sub-channels.
- $\text{diag}(T_1) = \text{diag}(T_2)$ $\Rightarrow$ **balanced** SISO sub-channels. $\Rightarrow$ Use **balanced** PNC.
- Subtract off-diagonal elements "in advance" modulo-lattice. (dirty-paper coding (DPC))
- Relay decodes sum **modulo lattice** $\Rightarrow$ achieves "the $\frac{1}{2}$".

# Example - GSVD-based Scheme

$$T_1 = \begin{pmatrix} 1/2 & 0 \\ 3 & 2 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 4 & 0 \\ 24 & 1/4 \end{pmatrix}.$$

- Use unbalanced PNC over each sub-channel.
- Decode sum-codeword of first sub-channel:
  $2y_1 = x_{1;1} + 8x_{2;1} + z_1 \Rightarrow x_{\text{sum}} = x_{1;1} + 8x_{2;1}$.
- Second sub-channel:
  $$4y_2 = 8x_{1;2} + x_{2;2} + \overbrace{12(x_{1;1} + 8x_{2;1})}^{\text{Interference}} + z_2.$$
- Subtract interference of first sub-channel:
  $12(x_{1;1} + 8x_{2;1}) = 12x_{\text{sum}}$.

# Example - GSVD-based Scheme

## Remarks

- Needs to decode sum **over the reals** $x_{\text{sum}}$ (not modulo lattice).
- BC phase uses random binning scheme (as in [Nam, Chung, Lee '08]).

# Example - JET-based Scheme

$$T_1 = \begin{pmatrix} 0.3065 & 0 \\ -1.5843 & 3.2628 \end{pmatrix} , \quad T_2 = \begin{pmatrix} 0.3065 & 0 \\ 24.1106 & 3.2628 \end{pmatrix} .$$

- Use balanced PNC over each sub-channel.
- Decode sum-codeword (modulo lattice) of first sub-channel.
- Second sub-channel:

$$\frac{y_2}{3.2628} = (x_{1;2} + x_{2;2}) + \overbrace{\left( \frac{-1.5843}{3.2628} x_{1;1} + \frac{24.1106}{3.2628} x_{2;1} \right)}^{\text{Interference}} + \frac{z_2}{3.2628} .$$

- Use doubly dirty-paper coding (Philosof et al. 2007) for second sub-channel: $x_{1;2} = \left[ \tilde{x}_{1;2} - \frac{-1.5843}{3.2628} x_{1;1} \right] \mod \Lambda$

  $x_{2;2} = \left[ \tilde{x}_{2;2} - \frac{24.1106}{3.2628} x_{2;1} \right] \mod \Lambda$
- Decode modulo lattice:
  $y_2' = \left[ \frac{y_2}{3.2628} \right] \mod \Lambda = \left[ \tilde{x}_{1;2} + \tilde{x}_{2;2} + \frac{z_2}{3.2628} \right] \mod \Lambda$

# Example - JET-based Scheme

## Remarks

- Standard scalar balanced PNC strategy over each sub-channel is used.
- MMSE version of each sub-channel can be used – achieves "the $\frac{1}{2}$" over each sub-channel.
- Any common-message scheme for BC phase.

# GSVD Scheme vs. JET Scheme

## High SNR asymptotic optimality

**Both schemes** achieve optimum for $P \to \infty$:

$$R \leq \min\left\{ \log \overbrace{\left| \frac{P}{n} H_1 H_1^\dagger \right|}^{= \left| \frac{P}{n} H_2 H_2^\dagger \right|}, C_{\text{common}} \right\} = \min\left\{ \log \left| \frac{P}{n} T_1 T_1^\dagger \right|, C_{\text{common}} \right\}$$

$$= \min\left\{ \log \prod_{j=1}^{n} \frac{P}{n} \left| t_{1;jj} \right|^2, C_{\text{common}} \right\}$$

# GSVD Scheme vs. JET Scheme –Achieving the "$\frac{1}{2}$"

## Achievable rate using GSVD specialized to balanced case

$$R = \min\left\{\log\prod_{j=1}^{n}\frac{P}{n}\left|t_{1;jj}\right|^2, C_{\text{common}}\right\}$$

## Achievable rate using JET [New]

$$R = \min\left\{\log\prod_{j=1}^{n}\left(\frac{1}{2}+\frac{P}{n}\Big|\overbrace{t_{1;jj}}^{=t_{2;jj}}\Big|^2\right), C_{\text{common}}\right\}$$

- Strictly better than GSVD-based scheme.
- Standard lattice coding (modulo lattice decoding at relay; no need of binning at BC stage)

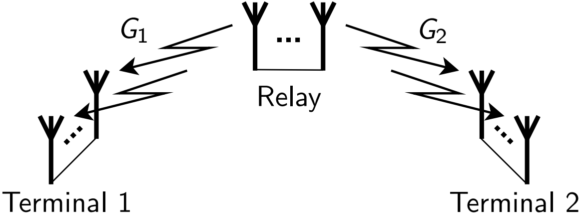
# "The $\frac{1}{2}$" – MMSE Variant

- We saw how to achieve "the $\frac{1}{2}$" for each **scalar** channel.
- Can we achieve **matrix** variant of "the $\frac{1}{2}$":

$$R \leq \min \left\{ \min_i \max_{C_{\mathbf{x}_i}} \log \left| \frac{1}{2} I + H_i C_{\mathbf{x}_i} H_i^\dagger \right|, C_{\text{common}} \right\} \quad \textbf{?}$$

- Currently under research (similar to BC treatment for general SNR [Khina, Kochman, Erez 2010]).

# Gaussian BC Case




$$\mathbf{y}_i = G_i \mathbf{x} + \mathbf{z}_i \,, \qquad i = 1, 2$$

**Similar approach to MAC approach [Khina, Kochman, Erez 2010]**

- Apply the JET to $G_1$ and $G_2$
  (for augmented matrices if SNR not high).
- Achieves triangular matrices with equal diagonals.
- Use GDFE/VBLAST (instead of DPC).
- Optimal for any matrices and any SNR!

# Generalizations

- MIMO Gaussian BC link can be treated in a similar manner (optimal for any SNRs [Khina, Kochman, Erez '10]).
- Better PNC-based scheme may be constructed for general SNR using JET.
- Time-sharing between balanced PNC and DF gives better results for intermediate SNR.
- Special case of "**Network Modulation**":
  Joint decomposition of channel matrices for MIMO network problems.
- Generalization for more than 2 matrices/users exists (last talk in this session).