Improved Rates and Coding for the MIMO Two-Way Relay Channel

Anatoly Khina, Tel Aviv University

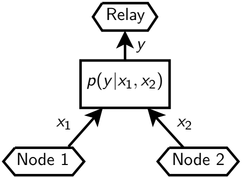
Joint work with:
Yuval Kochman, Hebrew University
Uri Erez, Tel Aviv University

October 29, 2014

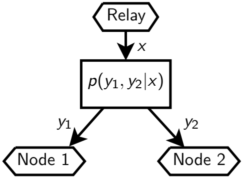
Two-Way Relay Model [Rankov, Wittneben '06][Popovski, Yomo '06]

- Nodes 1 and 2 want to convey messages to each other
- No direct link $\Rightarrow$ have to use relay
- Transmission is divided into two transmission phases:

- **Multiple-access (MAC) phase:**
  Nodes transmit to relay



- **Broadcast (BC) phase:**
  Relay transmits to nodes



Part I

SISO

Model

MAC Phase in the Gaussian Case:

$$y = x_1 + x_2 + z$$

- x_i – input of average power P
- y - channel output
- z - Channel noise $\sim \mathcal{CN}(\mathbf{0}, 1)$
- "Closed loop" (Full channel knowledge everywhere)

Symmetric-rate case

- Equal rates: $R_1 = R_2 \triangleq R$
- C_{common} – Common-message capacity of BC phase

Symmetric Gaussian SISO Case

Cut-Set Upper Bound

- Optimal individual capacity when other user is silent
- $R < R_{\text{CS}} = \min \left\{ \log \left(1 + P \right), C_{\text{common}} \right\}$

Symmetric Gaussian SISO Case

Cut-Set Upper Bound

- Optimal individual capacity when other user is silent
- $R < R_{\text{CS}} = \min\left\{\log\left(1+P\right), C_{\text{common}}\right\}$

Decode & Forward

- Decode both messages at relay
- $R_{\text{DF}} = \min\left\{\frac{1}{2}\log\left(1+2P\right), C_{\text{common}}\right\}$
- $1/2$ pre-log factor $\longleftrightarrow$ extra 2 within log
- Performs poorly at high SNR: half of cut-set upper bound!
- Optimal at low SNR (achieves R_{CS}): $\frac{1}{2}\log(1+2P) \underbrace{\approx}_{P \ll 1} P$

Symmetric Gaussian SISO Case

Amplify & Forward / Naïve Compress & Forward

- $y = x_1 + x_2 + z$
- Gaussian BC: Transmit y as is up to power adjustment: αy
- General BC: Compress noisy sum
- $R_{\text{AF}} = \log\left(1 + \frac{PP_{\text{common}}}{1+2P+P_{\text{common}}}\right)$
- $C_{\text{common}} = \log(1 + P_{\text{common}})$

Symmetric Gaussian SISO Case

Amplify & Forward / Naïve Compress & Forward

- $R_{\text{AF}} = \log\left(1 + \frac{PP_{\text{common}}}{1+2P+P_{\text{common}}}\right)$
- $C_{\text{common}} = \log(1 + P_{\text{common}})$

Compress & Forward

- No need to compromise power with other user!
- Use remote Wyner–Ziv compression [Yamamoto, Itoh '80]
- $R_{\text{CF}} = \log\left(1 + \frac{PP_{\text{common}}}{1+P+P_{\text{common}}}\right)$ [Gunduz, Tuncel, Nayak '10]
- 1 bit of cut-set upper bound for any P, P_{common}
- Becomes optimal for $1 \ll P \ll P_{\text{common}}$

Symmetric Gaussian SISO Case

Partial Decode & Forward [Gunduz, Tuncel, Nayak '10]

- Superimpose layers of DF and CF
- (pure) DF and CF are special cases
- Optimization over power allocation to each layer should be performed

Symmetric Gaussian SISO Case

Structured physical-layer network coding [Narayanan *et al.* '07]

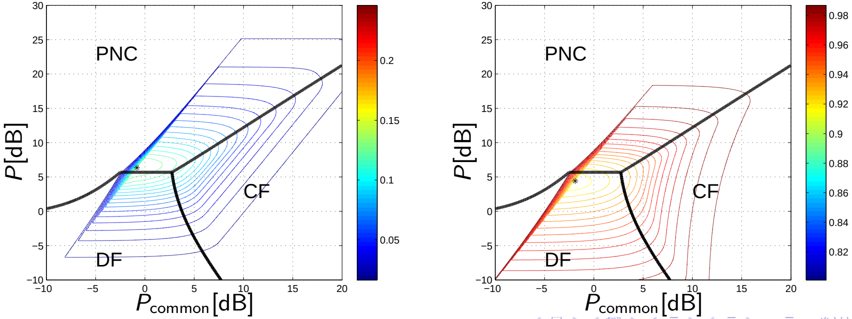
- Extends XORing of Network-Coding to Physical-layer:
  - Use the same lattice for both users: $\mathbf{x}_1, \mathbf{x}_2 \in \Lambda$
  - Relay decodes sum-message from: $[\mathbf{y}] = [\overbrace{\mathbf{x}_1 + \mathbf{x}_2}^{\mathbf{x}_+} + \mathbf{z}] \mod \Lambda$
  - Broadcast $\mathbf{x}_+$ to both users
  - User i recovers $\hat{\mathbf{x}}_{\bar{i}} = [\hat{\mathbf{x}}_+ - \mathbf{x}_i] \mod \Lambda$
- $R_{\text{PNC}} = \min\left\{\log\left(\frac{1}{2} + P\right), C_{\text{common}}\right\}$
- 1 bit of cut-set upper bound for any P, C_{common}
- Optimal at high SNR: $P \gg 1$

Asynchronization between users

Sensitive to synchronization, in contrast to all previous strategies.

Symmetric Gaussian SISO Case

- Each strategy performs better for some values of (P, P_{common})
- Time-sharing improves over "pure" strategies
- At most 0.2625 bits of cut-set UB
  (compared to the 1 bit gap of PNC and CF)
- At least 78.78% of cut-set UB



Model: MAC Phase in the Gaussian Case

$$y = h_1 x_1 + h_2 x_2 + z$$

- x_i – input of average power P
- h_i - channel gain of user i
- y - channel output
- z - Channel noise $\sim \mathcal{CN}(\mathbf{0}, 1)$
- "Closed loop" (Full channel knowledge everywhere)

Model: Broadcast Phase

- Broadcast (BC) phase can be general
- We characterize it by its "side-information rate region":

$$\mathcal{C}_{\text{BC}} = \text{cl conv} \left\{ \begin{matrix} R_1 \leq I\left(X; Y_2 | X_2\right) \\ R_2 \leq I\left(X; Y_1 | X_1\right) \end{matrix} \right\}$$

- Gaussian BC: No tension between maximal R_1 and R_2 in $\mathcal{C}_{\text{BC}}$
  $\Rightarrow$ Define C_1^{BC} and C_2^{BC}

Symmetric-rate case

- Equal rates: $R_1 = R_2 \triangleq R$
- $C_1^{\text{BC}} \underbrace{=}_{\text{w.l.o.g.}} C_2^{\text{BC}} = C_{\text{common}}$
- C_{common} – Common-message capacity (*without* side info.)

Gaussian SISO Case: Asymmetric Setting

Decode & Forward and Compress & Forward

$$R_i^{\text{DF}} \le \min\left\{\log\left(1+|h_i|^2 P\right), C_i^{\text{BC}}\right\}$$

$$R_1^{\text{DF}} + R_2^{\text{DF}} \le \log\left(1+\left[|h_1|^2+|h_2|^2\right]P\right)$$

- CF and DF work using modulo-lattice strategies
- CF (and pDF) hard to evaluate in asymmetric case
- CF perform poorly due to asymmetry in side info.

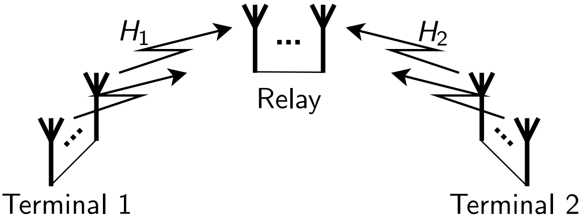
Asymmetric PNC [Nam, Chung, Lee '08]

$$R_i \le \min\left\{\log\left(\frac{|h_i|^2}{|h_1|^2+|h_2|^2}+|h_i|^2 P\right), C_i^{\text{BC}}\right\}$$

- 3 levels of nesting of lattice codes

Part II

MIMO



$$\mathbf{y} = H_1\mathbf{x}_1 + H_2\mathbf{x}_2 + \mathbf{z}\,, \qquad i = 1,2$$

- $\mathbf{x}_i$ – $n \times 1$ input vector of power P
- H_i - $n \times n$ channel matrix of user i
- $\mathbf{y}$ - $n \times 1$ output vector
- $\mathbf{z}$ - Channel noise $\sim \mathcal{CN}(\mathbf{0}, I_n)$
- "Closed loop" (Full channel knowledge everywhere)

Model: Assumptions

Symmetric-rate case

- $R = R_1 = R_2$
- Equal link capacities $\Rightarrow$ For high SNR: $|H_1| = |H_2|$

- High SNR assumption
- Optimum inputs are white
- $\log\left|I + \frac{P}{n} H_i H_i^\dagger\right| \approx \log\left|\frac{P}{n} H_i H_i^\dagger\right| = 2\log|H_i| + n\log\frac{P}{n}$
- Capacity determined by $|H_i|$ for high SNR

Goals

Theoretical

Generalize scalar strategies to MIMO case.

Practical: "Black box" approach

- Transform MIMO links into SISO links
- Use scalar strategies over SISO links

Cut-set Upper Bound

- $R < R_{\text{CS}} = \min\left\{C_1, C_2, C_{\text{common}}\right\}$
- $C_i \triangleq \max_{K_i} \min\left\{\log\left|I + H_i K_i H_i^\dagger\right|\right\}$
- C_i – Optimal individual capacity when other user is silent

Decode & Foward

$$C_{\text{DF}} = \min\{C_{\text{MAC}}, C_{\text{common}}\}$$

where

$$C_{\text{MAC}} = \max_{K_1, K_2} \min \left\{ \log \left| I + H_1 K_1 H_1^\dagger \right|, \right.$$
$$\log \left| I + H_2 K_2 H_2^\dagger \right|,$$
$$\left. \frac{1}{2} \log \left| I + H_1 K_1 H_1^\dagger + H_2 K_2 H_2^\dagger \right| \right\}$$

Practical scheme

DF can be achieved via V-BLAST:

- Apply QR decomposition to "total channel matrix" $\begin{bmatrix} H_1 K_1^{1/2} & H_2 K_2^{1/2} \end{bmatrix}$ (or to its "augmented" version for MMSE variant)
- Use scalar coding over resulting SISO links

Compress & Forward

- Information-theoretic expression can be derived
- Hard to evaluate in MIMO case
- Suboptimal scalar approaches have been proposed [Lin *et al.* '13][Kamoun *et al.* '13]

Structured Physical-layer Network Coding (PNC)

Problem

- How to apply scalar PNC in MIMO case?
- Each antenna sees interference from other streams

Idea

- Use **joint matrix decomposition** to reduce **MIMO channel** to **SISO channels**
- Use **unitary** transformations at transmitters – preserves power
- Apply **scalar** PNC over resulting SISO channels

Decomposition

- Joint **diagonalization** of both channel matrices **not possible**
- Joint **triangularization**?

Triangularization

Joint triangularization

$$H_1 K_1^{1/2} = U T_1 V_1^\dagger$$
$$H_2 K_2^{1/2} = U T_2 V_2^\dagger$$

where

- U, V_1, V_2 – Unitary
- T_1, T_2 – Triangular matrices

Application for MAC phase

- **Unitary** V_i is applied at encoder i – preserves power
- **Unitary** U is applied at relay
- T_1, T_2 – Effective triangular channels

Triangularization

Interesting special cases

- Generalized Singular Value Decomposition (GSVD)
- Joint Equi-diagonal Triangularization (JET)

Triangularization: Special Cases

$$H_1 = \begin{pmatrix} 0.4757 & 0.1541 \\ 2.2374 & 2.8273 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0.4729 & -3.9719 \\ 3.0858 & -23.8021 \end{pmatrix}$$

$$\det(H_1) = \det(H_2) = 1\,.$$

GSVD [Van Loan '76]

T_1, T_2 have:

- **Non-equal diagonals**
- **Proportional columns**

$$T_1 = \begin{pmatrix} 1/2 & 0 \\ 3 & 2 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} 4 & 0 \\ 24 & 1/4 \end{pmatrix}$$

JET [Kh., Kochman, Erez '12]

T_1, T_2 have:

- **Equal diagonals**
- **Non-proportional columns**

$$T_1 = \begin{pmatrix} 0.3065 & 0 \\ -1.5843 & 3.2628 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} 0.3065 & 0 \\ 24.1106 & 3.2628 \end{pmatrix}$$

GSVD Scheme vs. JET Scheme

GSVD-based [Yang *et al.* '10]

- n diagonal elements
  $= n$ SISO sub-channels
- $\text{diag}(T_1) \neq \text{diag}(T_2)$
  $\Rightarrow$ **asymmetric** SISO sub-channels
  $\Rightarrow$ Use **asymmetric** PNC
- **Proportional** off-diagonal elements
  $\Rightarrow$ Subtract sum codewords (GDFE/VBLAST)
- Relay decode sum **over the reals**
  $\Rightarrow$ Can't achieve "the $\frac{1}{2}$"

JET-based [Kh. *et al.* '11]

- n diagonal elements
  $= n$ SISO sub-channels
- $\text{diag}(T_1) = \text{diag}(T_2)$
  $\Rightarrow$ **symmetric** SISO sub-channels
  $\Rightarrow$ Use **symmetric** PNC
- Subtract off-diagonal elements "in advance" modulo-lattice (dirty-paper coding (DPC))
- Relay decodes sum **modulo lattice**
  $\Rightarrow$ achieves "the $\frac{1}{2}$"

Example – GSVD-based Scheme

$$T_1 = \begin{pmatrix} 1/2 & 0 \\ 3 & 2 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 4 & 0 \\ 24 & 1/4 \end{pmatrix}$$

- Use asymmetric PNC over each sub-channel
- Decode sum-codeword of first sub-channel:
  $2y_1 = x_{1;1} + 8x_{2;1} + z_1 \Rightarrow x_+ = x_{1;1} + 8x_{2;1}$
- Second sub-channel:
  $$4y_2 = 8x_{1;2} + x_{2;2} + \overbrace{12\underbrace{(x_{1;1} + 8x_{2;1})}_{x_+}}^{\text{Interference}} + z_2$$
- Subtract interference of first sub-channel:
  $12(x_{1;1} + 8x_{2;1}) = 12x_+$

GSVD-based Scheme

- Decodes sum **over the reals** x_+ (not modulo lattice)
- BC phase uses random binning scheme (as in [Nam, Chung, Lee '08])
- Achievable rate:

$$R_{\mathrm{PNC}}^{\mathrm{GSVD}} = \min\left\{ R_{\mathrm{PNC},1}^{\mathrm{GSVD}} ,\ R_{\mathrm{PNC},2}^{\mathrm{GSVD}} ,\ C_{\mathrm{common}} \right\}$$

$$R_{\mathrm{PNC},i}^{\mathrm{GSVD}} = \sum_{j=1}^{n} \left[\log \left(\left| d_{i,j}^{\mathrm{GSVD}} \right|^2 \right) \right]^+$$

GSVD-based Scheme

- Decodes sum **over the reals** x_+ (not modulo lattice)
- BC phase uses random binning scheme (as in [Nam, Chung, Lee '08])
- Achievable rate:

$$R_{\text{PNC}}^{\text{GSVD}} = \min\left\{ R_{\text{PNC},1}^{\text{GSVD}}\,,\; R_{\text{PNC},2}^{\text{GSVD}}\,,\; C_{\text{common}} \right\}$$

$$R_{\text{PNC},i}^{\text{GSVD}} = \sum_{j=1}^{n} \left[\log\left(\left| d_{i,j}^{\text{GSVD}} \right|^2 \right) \right]^+$$

Theorem [New]: Improved GSVD-based Scheme; using [Nazer '12]

$$R_{\text{PNC},i}^{\text{GSVD}} = \sum_{j=1}^{n} \left[\log\left(\frac{\left| d_{i,j}^{\text{GSVD}} \right|^2}{\left| d_{1,j}^{\text{GSVD}} \right|^2 + \left| d_{2,j}^{\text{GSVD}} \right|^2} + \left| d_{i,j}^{\text{GSVD}} \right|^2 \right) \right]^+$$

Example – JET-based Scheme

$$T_1 = \begin{pmatrix} 0.3065 & 0 \\ -1.5843 & 3.2628 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0.3065 & 0 \\ 24.1106 & 3.2628 \end{pmatrix}$$

- Use symmetric PNC over each sub-channel
- Decode sum-codeword (modulo lattice) of first sub-channel
- Second sub-channel:

$$\frac{y_2}{3.2628} = (x_{1;2} + x_{2;2}) + \overbrace{\left(\frac{-1.5843}{3.2628} x_{1;1} + \frac{24.1106}{3.2628} x_{2;1}\right)}^{\text{Interference}} + \frac{z_2}{3.2628}$$

- Use doubly dirty-paper coding (Philosof *et al.* '07) for second sub-channel: $x_{1;2} = \left[\tilde{x}_{1;2} - \frac{-1.5843}{3.2628} x_{1;1}\right] \mod \Lambda$

$$x_{2;2} = \left[\tilde{x}_{2;2} - \frac{24.1106}{3.2628} x_{2;1}\right] \mod \Lambda$$

- Decode modulo lattice:
  $y_2' = \left[\frac{y_2}{3.2628}\right] \mod \Lambda = \left[\tilde{x}_{1;2} + \tilde{x}_{2;2} + \frac{z_2}{3.2628}\right] \mod \Lambda$

JET-based Scheme

- Standard scalar symmetric PNC over each sub-channel is used
- MMSE version of each sub-channel can be used – achieves "the $\frac{1}{2}$" over each sub-channel
- Any common-message scheme for BC phase
- Achievable rate:

$$R_{\mathrm{PNC}}^{\mathrm{JET}} = \min\left\{ R_{\mathrm{PNC},1}^{\mathrm{JET}} \,,\; R_{\mathrm{PNC},2}^{\mathrm{JET}} \,,\; C_{\mathrm{common}} \right\}$$

$$R_{\mathrm{PNC},i}^{\mathrm{JET}} = \sum_{j=1}^{n} \left[\log\left(\frac{1}{2} + \left| d_{i,j}^{\mathrm{JET}} \right|^2 \right) \right]^{+}$$

PNC: GSVD Scheme vs. JET Scheme

High SNR asymptotic optimality

Both schemes achieve optimum for $P \to \infty$

General SNR

The $\frac{d_i^2}{d_1^2+d_2^2}$ gain is more significant in symmetric case

$\Rightarrow$ JET-based scheme performs better in most cases

Combining PNC with CF, DF: GSVD Scheme

- Optimization of even "pure" CF is hard in asymmetric cases
- "One-layered" CF in asymmetric case:
  Suboptimal within CF class ☹
- DF and PNC must preceed CF in decoding

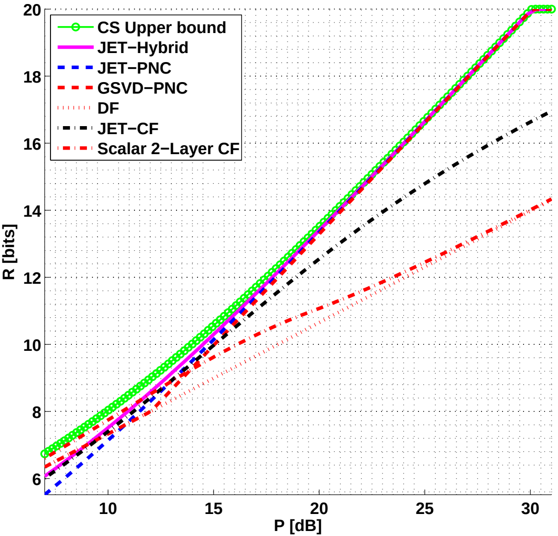
  $\Downarrow$

  Problem using best time-shared scheme over each SISO link
- Asymmetric links impair performance, its analysis, and constructing a practical scheme for CF, pDF, and even PNC
- PNC rate deteriorates when additional interference is present

Combining PNC with CF, DF: JET Scheme

- "One-layered" CF in symmetric case: Optimal within CF class!
- Combining JET-based PNC with CF and DF is simple:
  - Apply JET to channel matrices ⇒ parallel symmetric channel
  - Apply optimal SISO strategy over resulting channel: Time-sharing between CF/DF/PNC
- CF and PNC work better in symmetric case!
- Achieves the same rate when additional interference is present

$$H_1 = \begin{pmatrix} 1/4 & 0 \\ 0 & 4 \end{pmatrix} \qquad H_2 = \begin{pmatrix} 4 & 0 \\ 0 & 1/4 \end{pmatrix}$$



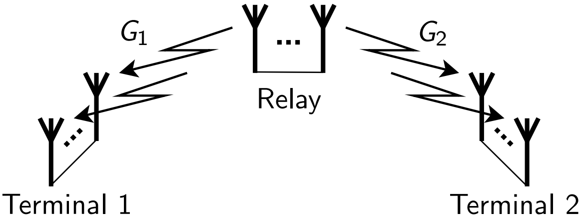
General SNR in MIMO Setting

- No claim on bounded gap from cut-set upper bound! (Except in high SNR)
- This is in contrast to SISO setting!

Open problem

- How to make MMSE variants for PNC/CF?
- For DF: MMSE V-BLAST does the job

Gaussian BC Case



$$\mathbf{y}_i = G_i \mathbf{x} + \mathbf{z}_i\,, \qquad i = 1, 2$$

Similar approach to MAC approach [Kh., Kochman, Erez 2010]

- Apply the JET to G_1 and G_2
  (for augmented matrices if SNR not high).
- Achieves triangular matrices with equal diagonals.
- Use GDFE/VBLAST (instead of DPC).
- Optimal for any matrices and any SNR!