The Confidential MIMO Broadcast Capacity

A Simple Derivation

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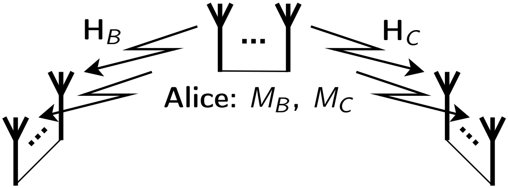
ISIT 2015
Hong Kong
June 18, 2015

Model: Confidential Gaussian MIMO Broadcast



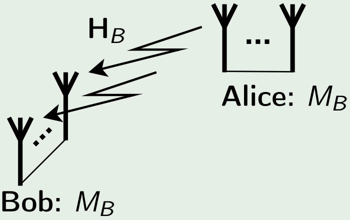
Bob: M_B, $\cancel{M_C}$ **Charlie:** $\cancel{M_B}$, M_C

$$\mathbf{y}_B = \mathbf{H}_B\mathbf{x}_A + \mathbf{z}_B \qquad \mathbf{y}_C = \mathbf{H}_C\mathbf{x}_A + \mathbf{z}_C$$

- $\mathbf{x}_A$ – $N_A \times 1$ input vector
- $\mathbf{y}_B$, $\mathbf{y}_C$ – $N_B \times 1$, $N_C \times 1$ received vectors
- $\mathbf{H}_B$, $\mathbf{H}_C$ – $N_B \times N_A$, $N_C \times N_A$ channel matrices
- $\mathbf{z}_B \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_B})$, $\mathbf{z}_C \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_C})$ – noise vectors
- "Closed loop" (full channel knowledge everywhere)

Outline of Talk (Special Cases)

MIMO Without Secrecy (No Charlie)



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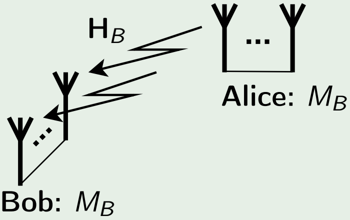
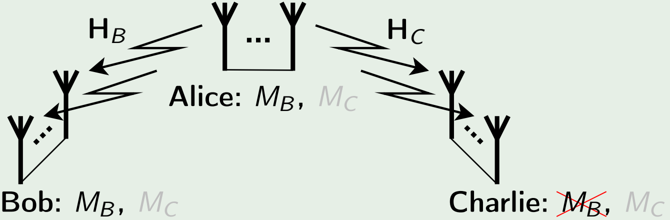
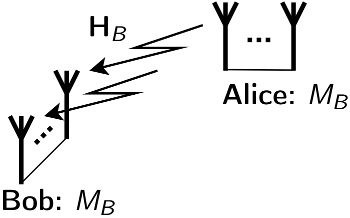


MIMO Wiretap (Maximize Rate to Bob)



MIMO Without Secrecy (No Charlie)

Practical Scheme via Matrix Decompositions



MIMO Without Secrecy (No Charlie)

Total power constraint: Capacity

$$C\left(\mathbf{H}_B\right) = \max_{\mathbf{K}:\, \mathrm{tr}\{\mathbf{K}\} \le P} \overbrace{\log\left|I + \mathbf{H}_B \mathbf{K} \mathbf{H}_B\right|}^{I(\mathbf{X}_A;\mathbf{Y}_B)}$$

- $X \sim$ Gaussian
- Maximization over all admissible covariance matrices $\mathbf{K}$
- Explicit expression for optimal $\mathbf{K}$ via SVD [Telatar '99]

Input covariance constraint: Capacity

- Replace $\mathrm{tr}\left\{\mathbf{K}\right\} \le P$ with $\mathbf{K} \preceq \bar{\mathbf{K}}$
- Use all of the “available covariance” $\bar{\mathbf{K}}$

V-BLAST Scheme: QR Decomposition Based Scheme

Zero-forcing V-BLAST [Foschini '96][Wolniansky et al. '98]

- $\mathbf{H}_B = \mathbf{Q}_B \mathbf{T}_B$
- $\mathbf{Q}_B$ – unitary; $\mathbf{T}_B$ – triangular
- Bob applies $\mathbf{Q}_B^\dagger$ (no SP is required by Alice)
- $\mathbf{T}_B = \begin{pmatrix} b_1 & * & * & \cdots & * \\ 0 & b_2 & * & \cdots & * \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{N_A-1} & * \\ 0 & 0 & \cdots & 0 & b_{N_A} \end{pmatrix} \Rightarrow \begin{aligned} y_1^{\text{eff}} &= b_1 \tilde{x}_1 + z_1 \\ y_2^{\text{eff}} &= b_2 \tilde{x}_2 + z_2 \\ &\vdots \\ y_{N_A}^{\text{eff}} &= b_{N_A} \tilde{x}_{N_A} + z_{N_A} \end{aligned}$
- Off-diagonal elements are canceled via either:
  - Successive interference cancellation (SIC)
  - Dirty-paper coding (DPC)

V-BLAST Scheme: QR Decomposition Based Scheme

MMSE-VBLAST for a given covariance $\mathbf{K}$ [Hassibi '00]

- $\begin{bmatrix} \mathbf{H}_B \mathbf{K}^{1/2} \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \mathbf{T}_B$
- $\mathbf{Q}_B$ – unitary; $\tilde{\mathbf{Q}}_B$ – $N_B \times N_A$ submatrix of $\mathbf{Q}_B$
- Bob applies $\tilde{\mathbf{Q}}_B^\dagger$ (no SP is required by Alice)
- $\tilde{\mathbf{Q}}_B^\dagger$ contains Wiener-filtering ("FFE")
- Effective noise has channel noise and "ISI" components
- Effective SNRs satisfy: $b_i^2 = 1 + \mathrm{SNR}_i$

$$\log(b_i^2) = \log(1 + \mathrm{SNR}_i) = I(\tilde{x}_i; \mathbf{y}_B | \tilde{x}_{i+1}^{N_A})$$

- Off-diagonal elements above diagonal canceled via SIC or DPC

V-BLAST Scheme: QR Decomposition Based Scheme

- For square invertible $\mathbf{H}$, ZF-VBLAST achieves:

$$R = \log \left| \mathbf{H}_B \mathbf{H}_B^\dagger \right|$$

$$\left(\text{Using } \mathbf{K} \text{ at the transmitter achieves: } R = \log \left| \mathbf{H}_B \mathbf{K} \mathbf{H}_B^\dagger \right| \right)$$

- MMSE-VBLAST achieves:

$$R = \sum_{i=1}^{N_A} \log b_i^2 = \log \left| \mathbf{I}_{N_B} + \mathbf{H}_B \mathbf{K} \mathbf{H}_B^\dagger \right|$$

Precoded V-BLAST: Alice Applies Unitary $\mathbf{V}_A$

MMSE-VBLAST with precoding for a given covariance $\mathbf{K}$

- $\begin{bmatrix} \mathbf{H}_B \mathbf{K}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \mathbf{T}_B$
- $\mathbf{V}_A$ can be used to design diagonal values $\Leftrightarrow$ design SNRs

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SVD-scheme as MMSE-VBLAST (QR)

Choosing $\mathbf{V}_A$ of the SVD of $\mathbf{H}_B \mathbf{K}^{1/2}$ $\Rightarrow$ SVD scheme
(no SIC/DPC needed)

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Geometric-mean decomposition [Jiang et al. '05]/ QRS [Zhang et al. '05]

- $\mathbf{V}_A$ is choosing s.t. all diagonal values (all SNRs) are equal
- The same codebook can be used over all subchannels
- No need for bit-loading

MIMO Wiretap (Maximize Rate to Bob)

Gaussian MIMO Wiretap: Maximize rate to Bob

Total power constraint [Khisti–Wornell '10][Oggier–Hassibi '11]

$$C_S\left(\mathbf{H}_B, \mathbf{H}_C\right) = \max_{\mathbf{K}:\, \mathrm{tr}\{\mathbf{K}\} \le P} \left[\overbrace{\log\left|\mathbf{I} + \mathbf{H}_B \mathbf{K} \mathbf{H}_B^\dagger\right|}^{I(\mathbf{X}_A;\mathbf{Y}_B)} - \overbrace{\log\left|\mathbf{I} + \mathbf{H}_C \mathbf{K} \mathbf{H}_C^\dagger\right|}^{I(\mathbf{X}_A;\mathbf{Y}_C)} \right]$$

- $X \sim$ Gaussian
- Maximization over all admissible covariance matrices $\mathbf{K}$
- No explicit expression for optimal $\mathbf{K}$

Gaussian MIMO Wiretap: Maximize rate to Bob

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- $X \sim$ Gaussian
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- No explicit expression for optimal $\mathbf{K}$

Input covariance constraint [Liu–Shamai '09]

- Replace $\mathrm{tr}\{\mathbf{K}\} \le P$ with $\mathbf{K} \preceq \bar{\mathbf{K}}$
- Explicit exp. for optimal $\mathbf{K}$! [Bustin–Liu–Poor–Shamai '09]

Scheme for General SNR [Kh., Kochman, Khisti ISIT'14]

$$\begin{bmatrix} \mathbf{H}_B \mathbf{K}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \overbrace{\begin{pmatrix} b_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & b_{N_A} \end{pmatrix}}^{\mathbf{T}_B}, \quad b_i^2 = 1 + \mathrm{SNR}_i^B$$

$$\begin{bmatrix} \mathbf{H}_C \mathbf{K}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_C \overbrace{\begin{pmatrix} c_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & c_{N_A} \end{pmatrix}}^{\mathbf{T}_C}, \quad c_i^2 = 1 + \mathrm{SNR}_i^E$$

- Use good SISO wiretap codes for SNR-pairs $(b_i^2 - 1, c_i^2 - 1)$
- $\mathbf{V}_A$ of Charlie's SVD $\Rightarrow$ Easy secrecy analysis + strong secrecy
- $\mathbf{V}_A$ of Bob's SVD $\Rightarrow$ No need for V-BLAST
- $\mathrm{diag}\{\mathbf{T}_B\}, \mathrm{diag}\{\mathbf{T}_C\}$ are const. $\Rightarrow$ Same code over all channels

Scheme for General SNR [Kh., Kochman, Khisti ISIT'14]

$$\begin{bmatrix} \mathbf{H}_B \mathbf{K}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \overbrace{\begin{pmatrix} b_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & b_{N_A} \end{pmatrix}}^{\mathbf{T}_B}, \quad b_i^2 = 1 + \mathrm{SNR}_i^B$$

$$\begin{bmatrix} \mathbf{H}_C \mathbf{K}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_C \overbrace{\begin{pmatrix} c_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & c_{N_A} \end{pmatrix}}^{\mathbf{T}_C}, \quad c_i^2 = 1 + \mathrm{SNR}_i^E$$

- Use good SISO wiretap codes for SNR-pairs $(b_i^2 - 1, c_i^2 - 1)$

Genie-aided secrecy-proof

- Charlie tries to recover messages sequentially (from last to first)
- For the recovery of message i all previous messages are revealed

Dirty-Paper Coding Variant of Scheme [New]

Alice:

- Cancels out off-diagonal elements of Bob via DPC
- Uses good SISO DPC wiretap codes for SNR-pairs $(b_i^2 - 1, c_i^2 - 1)$

Bob: Recovers using DPC decoder (as in MIMO without secrecy)

Genie-aided secrecy-proof

- Similar to secrecy analysis for SIC scheme
- Charlie tries to recover messages sequentially
- Messages this time are Costa-like auxiliaries (U_i)
- For the recovery of message i previous auxiliaries are revealed (from which the signals X_i can be constructed)

Restatement of Optimal K Expression of Bustin *et al.* via GSVD

MIMO Wiretap Capacity under Covariance Constraint $\mathbf{K} \preceq \bar{\mathbf{K}}$

- Apply $\mathbf{V}_A$ of the *Generalized Singular Value Decomposition* (GSVD) to the "MMSE–VBLAST" matrices:

$$\begin{bmatrix} \mathbf{H}_B \bar{\mathbf{K}}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \begin{pmatrix} b_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & b_{N_A} \end{pmatrix}$$

$$\begin{bmatrix} \mathbf{H}_C \bar{\mathbf{K}}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_C \begin{pmatrix} c_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & c_{N_A} \end{pmatrix}$$

- $C_S\left(\mathbf{H}_B, \mathbf{H}_C\right) = \sum_{i=1}^{N_A} \left[\log \frac{b_i}{c_i}\right]^+$

- $\mathbf{K}_{\text{opt}} = \bar{\mathbf{K}}^{1/2} \mathbf{V}_A \mathbf{I}_B \mathbf{V}_A^\dagger \bar{\mathbf{K}}^{1/2\dagger}$

- $\mathbf{I}_B$ – Diagonal matrix with 1 whenever $b_i > c_i$ and 0 otherwise

Wiretap under Cov. Constraint: Alternative Proof Outline

- W.l.o.g., $\mathbf{K} \preceq \bar{\mathbf{K}}$ can be written as $\mathbf{K} = \bar{\mathbf{K}}^{1/2}\mathbf{V}_A\mathbf{D}\mathbf{V}_A^\dagger\bar{\mathbf{K}}^{\dagger/2}$
- $\mathbf{D}$ is non-negative diagonal with $0 \le D_{ii} \le 1$
- For any $\mathbf{V}_A$: $I(\mathbf{H}_B, \mathbf{K}) - I(\mathbf{H}_C, \mathbf{K}) = \sum_{i=1}^{N} \log \frac{b_i^2}{c_i^2}$
- Optimal $\mathbf{D}$ for a given $\mathbf{V}_A$: truncation

$$C_S(\mathbf{H}_B, \mathbf{H}_C, \bar{\mathbf{K}}) = \max_{\mathbf{V}_A} \sum_{i=1}^{N} \left[\log \frac{b_i^2}{c_i^2}\right]^+$$

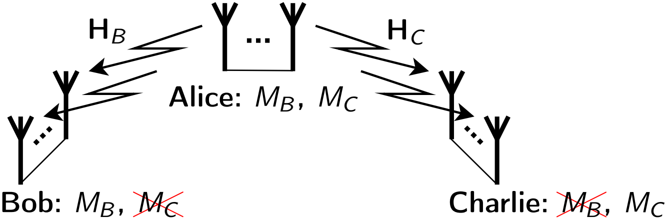
- By multiplicative majorization of joint unitary triangularization [Khina, Kochman, Erez SP'12], $\mathbf{V}_A$ of the GSVD is optimal

Optimal covariance matrix

$$\mathbf{K}_{\text{opt}} = \bar{\mathbf{K}}^{1/2}\mathbf{V}_A\mathbf{I}_B\mathbf{V}_A^\dagger\bar{\mathbf{K}}^{1/2\dagger}$$

- $\mathbf{I}_B$ – Diagonal matrix with 1 whenever $b_i > c_i$ and 0 otherwise

Finally... Confidential MIMO Broadcast



Confidential MIMO BC Capacity [Liu–Liu–Poor–Shamai '10]

Total power constraint

Using Lemma 1 of [Weingarten–Steinberg–Shamai '06]:

$$\mathcal{C}\left(\mathbf{H}_B, \mathbf{H}_C, P\right) = \bigcup_{\mathbf{K}:\, \mathrm{tr}\{\mathbf{K}\}=P} \Big(C_S\left(\mathbf{H}_B, \mathbf{H}_C, \mathbf{K}\right), C_S\left(\mathbf{H}_C, \mathbf{H}_B, \mathbf{K}\right) \Big)$$

$\Rightarrow$ Suffices to determine capacity region under covariance constraint

Covariance constraint

- No tension between users
- Both users achieve optimal wiretap capacities **simultaneously**!
- Rectangular capacity region

Confidential MIMO BC Capacity [Liu–Liu–Poor–Shamai '10]

Method of proof for covariance constraint

Converse: Trivial (both users achieve optimality simultaneously)

Achievable:

- Development of new achievable for MIMO wiretap channel that uses "artificial noise"
- Showing that new techniques achieves capacity (via *channel enhancement* technique)
- Double random-binning schemes and "secret dirty-paper coding"
- Alice $\rightarrow$ Bob: Uses "standard" random binning (standard technique)
- Alice $\rightarrow$ Charlie: Uses random binning with artificial noise (new technique)

Confidential MIMO BC Capacity [Liu–Liu–Poor–Shamai '10]

Connection to [Bustin–Liu–Poor–Shamai '09]

- Generalization of the techniques of Bustin *et al.*
- Capacity region can be represented via "generalized eigenvalues"

Confidential MIMO BC Capacity: New Derivation

- Simple derivation: Easy consequence of MIMO Wiretap treatment
- Uses standard MIMO wiretap capacity expression (no need for expression with "artificial noise")
- Uses standard (Costa) dirty-paper coding
- Explains why dirty-paper coding is required (for at least one of the users)
- Connection to the *generalized singular values* is natural
- Allows construction of a practical scheme

Capacity-Achieving Confidential MIMO Broadcast

$$\begin{bmatrix} \mathbf{H}_B \bar{\mathbf{K}}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \begin{pmatrix} b_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & b_{N_A} \end{pmatrix}$$

$$\begin{bmatrix} \mathbf{H}_C \bar{\mathbf{K}}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_C \begin{pmatrix} c_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & c_{N_A} \end{pmatrix}$$

- $\mathbf{V}_A$ of the GSVD
- Choosing directions of $b_i > c_i$ is optimal for Bob
- **But...** Choosing directions of $b_i < c_i$ is optimal for Charlie!

$\Rightarrow$ **Allocate $b_i > c_i$ to Bob**
Allocate $b_i < c_i$ to Charlie

Capacity-Achieving Confidential MIMO Broadcast

$$\begin{bmatrix} \mathbf{H}_B \bar{\mathbf{K}}^{1/2} \mathbf{V}_A \\ \mathbf{I}_{N_A} \end{bmatrix} = \mathbf{Q}_B \begin{pmatrix} b_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & b_{N_A} \end{pmatrix}$$

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Scheme

Charlie: Uses SIC or DPC over elements satisfying $c_i > b_i$

Bob: Uses DPC scheme over elements satisfying $b_i > c_i$

- Alice pre-cancels interference also from Charlie's signals
- Bob can gain no info. of Charlie's message from DPC

Capacity-Achieving Confidential MIMO Broadcast

Decoupling the Modulation

- Alice can apply additional unitary block-matrix operations:

$$\tilde{\mathbf{T}}_B = \overbrace{\begin{pmatrix} \mathbf{Q}_{B;B}^\dagger & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{B;C}^\dagger \end{pmatrix}}^{\text{Bob}} \begin{pmatrix} \mathbf{B}_B & * \\ \mathbf{0} & \mathbf{B}_C \end{pmatrix} \overbrace{\begin{pmatrix} \mathbf{V}_B & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_C \end{pmatrix}}^{\text{Alice}}$$

$$\tilde{\mathbf{T}}_C = \overbrace{\begin{pmatrix} \mathbf{Q}_{C;B}^\dagger & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{C;C}^\dagger \end{pmatrix}}^{\text{Charlie}} \begin{pmatrix} \mathbf{C}_B & * \\ \mathbf{0} & \mathbf{C}_C \end{pmatrix} \overbrace{\begin{pmatrix} \mathbf{V}_B & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_C \end{pmatrix}}^{\text{Alice}}$$

- SVD of Charlie's "block" $\mathbf{C}_C$ allows him to avoid SIC or DPC
- Other decompositions with desired properties can be applied (SVD of $\mathbf{B}_C$, 2-GMD of $(\mathbf{B}_B, \mathbf{B}_C)$, 2-GMD of $(\mathbf{C}_B, \mathbf{C}_C)$, etc.)

Summary

- Alternative derivation of the result of Bustin *et al.* for MIMO WTC via matrix analysis tools
- Simple derivation of capacity region of confidential MIMO BC
- Connection to generalized singular values is immediate
- Explicitly shows why DPC by at least one user is required