Decentralized TOA-based Localization in Non-Synchronized Wireless Networks with Partial, Asymmetric Connectivity

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Abstract—Classical methods for cooperative time-of-arrival (TOA)-based self-localization in wireless sensors networks usually rely either on synchronization between sensors or on two-way TOA (TW-TOA) measurements. However, when the connectivity in the network is limited, accurate synchronization of the nodes may be too costly to attain, and full TW-TOA measurements may not be available either, especially when the connectivity matrix is asymmetric. Centralized processing for the location estimation is also rarely feasible under such conditions. In this paper we propose an algorithm (method and protocol) for decentralized, cooperative TOA-based self-localization, which does not require synchronization and does not involve any “hand-shaking” procedures (of the kind required for TW-TOA measurements) between sensors. The procedure involves a single initial transmission by each sensor, followed by reports of the received TOAs by each sensor, as well as the timing-offsets and positions of relevant neighboring sensors, and announces its estimation results. We demonstrate convergence of the procedure to accurate estimates of the position of nearly all sensors, e.g., in scenarios involving 50 - 200 sensors in an area of 20 by 20 Km with an average 1:3 connectivity.

I. INTRODUCTION

Self localization in wireless sensors networks has attracted considerable research interest over the past decade (see, e.g., [1], [2]). Some of the most prominent methods use time of arrival (TOA) measurements in order to estimate the ranges between sensors (nodes). Each sensor (or some of the sensors) transmits a known, wideband signal, and each (or some) of the other sensors estimate the TOA of that signal at their positions. Assuming that the positions of some of the sensors (sometimes called “anchors”) are known a-priori, the range information can be used in turn to recover the positions of the other sensors. Classically, however, for accurate TOA measurements the sensors have to be synchronized to a common time-base with precision of about the same order as required for the desired positioning accuracy. Such precise synchronization of all sensors is often very difficult (or expensive) to attain. An appealing, commonly used substitute to plain TOA is the concept of Round-trip TOA (RTOA, e.g., [2]) or Two-Way TOA (TW-TOA, e.g., [3], [4]), in which the range measurements between sensors are based on the round-trip delay measurement: a first sensor transmits a signal and waits for a return transmission from a second sensor. The TOA measured by the first sensor (using its own time-base) then provides an estimate of double the range between the two sensors, without need for fine synchronization between them. However, such a scheme entails two shortcomings, especially for large-scale networks:

• A “hand-shaking” procedure is needed for each measured couple, requiring to maintain duplex links, which might not always be available, e.g., due to different transmission powers at different nodes;
• If the sensors do not have processing capabilities for resolving multiple concurrent receptions, each measured couple requires its own pair of transmission and reply, which might result in a relatively large total number of transmissions (up to $2N(N - 1)$ for exhausting all the potential information in a network of $N$ sensors);

To overcome the latter, the number of required transmissions may be reduced, e.g., by using generalized “multi-hop” scheduled transmissions, as suggested in [5], where pre-determined sequences of ordered transmissions within small sub-groups of sensors are used to deduce the mutual ranges. Still, good pairwise connectivity, or connectivity between designated sub-groups, is essential for such schemes, and, moreover, the connectivity matrix should be known in advance (before the network is deployed) for programming the different sensors.

In this work we propose a “free running” scheme for all sensors, which does not require advance knowledge of the network topology. No “hand-shaking” transmission procedure between sensors is needed, and no fine synchronization is required. Each sensor transmits just one coarsely-timed wideband signal (hence only $N$ wide-band transmissions are required in an $N$-sensors network), and the localization estimation is based on the “opportunistic” receptions of any of the other sensors’ transmissions by each sensor. The relative timing offsets of the timebases of all sensors are regarded as additional (nuisance) parameters, which are estimated along with the sensors’ positions - as considered, e.g., in [6], [7]. An essential difference between our proposed method and the more “standard” TW-TOA methods, is that our method enables to exploit timing information entailed in the signals received by a given sensor from other sensors, even if many of these other sensors do not receive the signal transmitted by the given...
sensor (e.g., if its transmission power is weaker).

Based on the measured TOAs of the received signals at each sensor from some of the other sensors, the positions estimation can either take a centralized or a decentralized approach. In a centralized approach all estimated TOAs are sent to a fusion center for estimation of all the positions. We shall briefly discuss this option, but would then turn to the more realistic decentralized approach, in which each sensor computes its own position based on information announced by all other sensors, within a sequential (recursive) framework. Such estimation methods (in different network configurations) have been considered, e.g., in [8], [9], and usually operate by starting out with the known anchor nodes and gradually adding new anchor nodes as the positions of additional nodes is resolved. Our proposed decentralized approach essentially uses “soft” anchors: Rather than declare some nodes as “anchors” and others as unresolved, we attribute an uncertainty (covariance) matrix to the estimated (or known) position of each node, which in turn attributes proper weights to that information in the estimation process. In so doing, we enable nodes to use the surrounding information for refining their own estimated positions, even if none of their neighbors’ positions is known a-priori to within absolute precision. As the process evolves, the recursive estimation procedure inherently reduces the recalculated uncertainties (covariance matrices), until convergence is attained - as we shall demonstrate.

II. PROBLEM FORMULATION

Consider a wireless network consisting of N sensors (nodes), which are arbitrarily positioned at \( p_1, \ldots, p_N \in \mathbb{R}^2 \). We assume that each sensor has its own transmission power, so that signals transmitted by sensor \( n \) can be received up to range \( r_n \) from that sensor. For convenience let us denote a connectivity indicators matrix \( G \in \{0,1\}^{N \times N} \), such that \( G_{m,n} = 1 \) iff \( \| p_m - p_n \| \leq r_n, m \neq n \), indicating that sensor \( m \) can receive the signals from sensor \( n \) (\( \| \cdot \| \) denotes the Euclidean norm). Note that since different sensors may have different transmission powers, \( G \) is not necessarily symmetric.

We further assume that the sensors are coarsely synchronized to a common timebase, namely the precision of their mutual synchronization is far too poor for supporting accurate absolute TOA measurements, but is good enough to support the timing for a scheduled transmission protocol (to be described shortly). We denote by \( \tau_n \) the time-offset of each sensor’s time origin from the “true” (or “absolute”) time-origin. Without loss of generality we may assume one of these offsets, say \( \tau_1 \), to be zero: Under such an assumption, the timing offsets of all other sensors would be relative to the timing offset of sensor 1, rather than being “absolute”.

The measurements are obtained by means of transmissions, receptions and TOA estimation by all sensors, to the extent supported by the network topology, as follows. Each sensor, being aware of its serial number \( n \in [1,N] \), transmits on its turn a pre-defined short wide-band signal \( s(t) \) at time \( nT \) (where \( T \) is large relative to the possible synchronization errors and relative to the length of \( s(t) \)), as measured along its own timebase - namely, with offset \( \tau_n \). The received signal at each sensor is the sum of delayed and attenuated signals received from some of the other sensors. Each sensor correlates its sampled \((NT\text{-long})\) received signal with \( s(t) \), thereby obtaining several peaks at lags corresponding to the TOAs of the transmissions from its fellow sensors. Subtracting the nominal transmission times \((nT)\) from the obtained peak locations, each sensor obtains estimates of the respective time-delays, further offset by its own timing offset (in addition to their transmitters’ timing offsets). We denote by \( \hat{\tau}_{m,n} \) the estimated time-delay of the \( n \)-th sensor’s transmission as received by the \( m \)-th sensor (these measurements are only available for \((m,n)\) pairs for which \( G_{m,n} = 1 \)), therefore:

\[
\hat{\tau}_{m,n} = \tau_n + \frac{1}{c} \| p_m - p_n \| - \tau_m + v_{m,n} \quad \forall m,n \in [1,N],
\]

where \( c \) denotes the propagation speed and \( v_{m,n} \) denote zero-mean estimation-errors. See figure 1 for an illustration.

The goal is to use the set of all available measurements \( \{ \hat{\tau}_{m,n} \} \), together with additional prior information regarding the positions of anchors, in order to estimate the positions of all \( N \) sensors.

III. CENTRALIZED AND DECENTRALIZED ESTIMATION

Each sensor is characterized by three unknown parameters - its time-offset and 2D position, which we concatenate into vectors \( \theta_1 = [\tau_1 \ p_{11} \ p_{12}]^T \in \mathbb{R}^3 \) for \( n = 1, \ldots, N \), which may be further concatenated into a vector of all unknown parameters \( \theta = [\theta_1^T \ \cdots \ \theta_N^T]^T \in \mathbb{R}^{3N} \). Defining the function

\[
h(\theta_m, \theta_n) = \tau_n + \frac{1}{c} \| p_m - p_n \| - \tau_m,
\]

and recalling (1), we may concatenate all measurements as

\[
\begin{bmatrix}
\hat{\tau}_{1,2} \\
\hat{\tau}_{1,3} \\
\vdots \\
\hat{\tau}_{N,N-1}
\end{bmatrix} =
\begin{bmatrix}
h(\theta_1, \theta_2) \\
h(\theta_1, \theta_3) \\
\vdots \\
h(\theta_N, \theta_{N-1})
\end{bmatrix} +
\begin{bmatrix}
v_{1,2} \\
v_{1,3} \\
\vdots \\
v_{N,N-1}
\end{bmatrix}.
\]

Note that the length \( \hat{N} \) of the concatenated measurements vector \( \hat{t} \) is not known in advance; its maximal possible length is \( N(N-1) \), but since not all sensors receive all the transmissions, its eventual length would normally be much smaller, given by the number of 1-s in \( G \), namely \( \hat{N} = 1^T G \).

A plausible approach (at least in a centralized estimation framework) would be to seek the Least-Squares (LS) solution, minimizing \( C_{\text{LS}}(\theta) = \| \tau - h(\theta) \|^2 \) w.r.t. \( \theta \), but obviously this would be an ill-posed problem (admitting infinitely many solutions), since it does not involve any additional information regarding the positions of anchor sensors or regarding the arbitrarily-set zero time-offset of the first sensor. An apparently straightforward remedy would be to plug these known parameters into the equation (3), thereby reducing the number of unknown parameters and hoping to gain uniqueness of the global minimum. However, we propose to take a more general approach, which would prove instrumental later on,
especially for the decentralized framework. Rather than regard the prior information on the anchors and on \( \tau_1 \) as unknown parameters, we would assume that additional “fictitious” measurements are available, directly providing the values of all unknown parameters, but with varying levels of uncertainty: the uncertainty of these position “measurements” would be arbitrarily large for most sensors, but negligibly small for the anchors; the uncertainty of the time-offset “measurements” would be very large for almost all sensors, but negligibly small for the first sensor. We would then seek a Weighted LS (WLS) solution, which would attribute proper relative weights to all the available measurements, “true” and “fictitious”. The uncertainty levels would be expressed by the variances (or covariances) of the fictitious measurements’ errors, and the weighting matrix for the WLS solution would be the inverse of the overall covariance matrix. In fact, this approach can be regarded as a regularization strategy with a Bayesian flavor.

The additional, “fictitious” measurements and their associated variances / covariances would therefore be the following:

- For the timing offsets \( \tau_n \) - the fictitious measurements are denoted \( \tau_n \) and are all taken as zeros. Their associated variances are denoted \( \sigma_{\tau_n}^2 \). The variance for the first sensor is taken to be zero (or a negligibly small positive number, to avoid singularity issues), whereas the variances for the other sensors are taken to be of the (squared) order of the assumed synchronization errors.

- For the positions \( p_n \) - the fictitious measurements are denoted \( p_n \). For anchor nodes we take \( p_n \) to be their true, known positions, whereas for the other nodes we take \( p_n \) to be random positions drawn around the center of the area of deployment of the sensors. The associated covariance matrices are denoted \( C_n \) and are taken to be of the form \( C_n = \sigma_{p_n}^2 I \), where \( I \) denotes the \( 2 \times 2 \) Identity matrix and where \( \sigma_{p_n} \) is small (say, a few centimeters) for the anchor nodes, but large relative to the deployment area for all other nodes. Note that using this strategy we may also allow “soft anchors” - nodes for which some a-priori position information is available, but with some level of uncertainty (as expressed by their covariance matrices).

For convenience of the exposition later on, we define augmented fictitious measurements vectors and associated error-covariance matrices,

\[
\begin{bmatrix}
\tau_n \\
p_n
\end{bmatrix} \in \mathbb{R}^3, \quad \begin{bmatrix}
\sigma_{\tau_n}^2 & 0 \\
0 & \sigma_{p_n}^2
\end{bmatrix} \in \mathbb{R}^{3 \times 3}. \tag{4}
\]

Note that the structure of \( C_n \) reflects a reasonable assumption that the timing errors are uncorrelated with the presumed position errors. This assumption holds for the initial fictitious measurements, but would not hold later on, in the recursive process proposed for our decentralized estimation scheme - where these covariance matrices would eventually take full general forms.

For determining the weight matrices we also need the variances of the TOA estimation errors \( \tau_{m,n} \). Although it is possible to assume different variances for different measurements, we take a simplifying approach in here and assume that they are all equal, denoting their value as \( \sigma_{\tau_n}^2 \).

To facilitate the exposition, we begin by describing the centralized estimation approach. Evidently, such a scheme requires a link from all sensors to a fusion center - an unrealistic assumption in some scenarios. We then proceed to describe the more realistic decentralized approach, in which the sensors collaborate (opportunistically) in a recursive procedure (using only the available connectivity) to estimate their own positions.

### A. Centralized estimation

We assume that the entire measurements vector \( t \in \mathbb{R}^N \) is reported to the fusion center. The “fictitious” measurements can also be concatenated into one long vector \( \vec{\theta} \triangleq \begin{bmatrix} \vec{\theta}_1^T & \cdots & \vec{\theta}_N^T \end{bmatrix}^T \in \mathbb{R}^{3N} \), so that the entire measurements (true and fictitious) model can be expressed as

\[
\begin{bmatrix}
t \\
\vec{\theta}
\end{bmatrix} = \begin{bmatrix} h(\theta) \\
v \end{bmatrix} + e \quad \in \mathbb{R}^{N+3N}, \tag{5}
\]

where \( h(\theta) \) and \( v \) have been defined in (3) and where \( e \) is the fictitious measurements’ errors vector. Under the reasonable assumption that the two errors vectors \( v \) and \( e \)
are uncorrelated, their joint covariance matrix takes a block-diagonal form, where the upper-left $N \times N$ block is a scaled Identity matrix (scaled by $\sigma^2$). Under the additional reasonable assumption that the fictitious measurements errors of different sensors are uncorrelated, the lower-right $3N \times 3N$ block is another block-diagonal matrix, consisting of $C_1, ..., C_N$ as its blocks. Therefore, an optimally-weighted LS solution would be obtained by minimizing (w.r.t. $\theta$)

$$C_{WLS}(\theta) \triangleq \sum_{m,n}(\hat{t}_{m,n} - h(\theta_m, \theta_n))^2 + \sum_{n=1}^{N}(\hat{\theta}_n - \theta_n)^T C^{-1}_n(\hat{\theta}_n - \theta_n).$$

This nonlinear WLS problem can be solved by using Gauss-Newton iterations (see, e.g., [10]), using the “fictitious measurements” $\bar{\theta}$ as an initial guess for $\theta$, and noting that the required derivatives of $h(\theta_m, \theta_n)$ are given by

$$\frac{\partial h(\theta_m, \theta_n)}{\partial \theta_m} = -\frac{1}{c} \frac{(\hat{p}_m - p_m)^T}{\|\hat{p}_m - p_m\|}$$

$$\frac{\partial h(\theta_m, \theta_n)}{\partial \theta_n} = \frac{1}{c} \frac{(\hat{p}_m - p_m)^T}{\|\hat{p}_m - p_m\|}.$$  

(7)

B. Decentralized estimation

The decentralized estimation scenario is much more realistic (and obviously more challenging) when the network allows only partial connectivity. We propose an iterative-recursive procedure, interlacing Gauss-Newton iterations with updates of the sensors’ estimated positions. Before describing the procedure, let us define the following for ease of reference. We denote by $\mathcal{T}_n$, the set of all sensors whose transmissions can be received by sensor $m$, namely $\mathcal{T}_m = \{n | G_{m,n} = 1\}$. We denote by $\mathcal{R}_m$ the set of all sensors capable of receiving transmissions from sensor $m$, namely $\mathcal{R}_m = \{n | G_{n,m} = 1\}$ (these are the sensors within range $r_m$ from sensor $m$). Our proposed procedure takes the following form:

1) Apply the TOA measurements phase described above. At the conclusion of this phase each sensor $n$ has its set of estimated TOAs of the signals transmitted by all sensors which it can receive, namely $\{\hat{t}_{n,m}\}_{m \in \mathcal{T}_n}$.

2) In a similarly scheduled (one-by-one) pattern, each sensor $n$ announces its set of estimated TOAs, together with the identities (serial numbers) of the respective sensors in $\mathcal{T}_n$. This information is received by all sensors in $\mathcal{R}_n$. Each of these sensors notes the announced TOA of its own signal at sensor $n$, provided that it is included in the announced list, namely provided that the “listening” sensor belongs not only to $\mathcal{R}_n$ but also to $\mathcal{T}_n$. Thus, following these two initial stages, each sensor $n$ has its own set of estimated TOAs of its received signals from $\mathcal{T}_n$: $\{\hat{t}_{n,m}\}_{m \in \mathcal{T}_n}$, as well as the set of TOAs of its own transmitted signals as estimated by sensors in $\mathcal{T}_n \cap \mathcal{R}_n$: $\{\hat{t}_{m,n}\}_{m \in \mathcal{T}_n \cap \mathcal{R}_n}$. In addition, each sensor $n$ has its own “fictitious measurement” (or “initial estimate”) $\hat{\theta}_n$, and its associated covariance $C_n$. As we shall see, the fictitious measurements will turn into educated estimates in the course of the following recursive estimation procedure.

3) Each sensor $n$ announces (in a pre-defined scheduled pattern) its fictitious measurement $\hat{\theta}_n$ with its associated covariance $C_n$. This information is received and noted by all sensors in $\mathcal{R}_n$. Each sensor can then solve a WLS problem with respect to its relevant unknowns, which are first and foremost $\theta_n$, and, in addition, all (“nuisance parameters”) $\{\theta_m\}_{m \in \mathcal{R}_n}$. The respective WLS criteria (for $n = 1, ..., N$) take the form

$$C_{WLS_n}(\theta_n, \{\theta_m\}_{m \in \mathcal{R}_n}) \triangleq \sum_{m \in \mathcal{R}_n} \frac{1}{\sigma^2} (\hat{t}_{m,n} - h(\theta_m, \theta_n))^2 + \sum_{m \in \mathcal{R}_n \cap \mathcal{T}_n} \frac{1}{\sigma^2} (\hat{t}_{m,n} - h(\theta_m, \theta_n))^2$$

$$+ \sum_{m \in \mathcal{T}_n} (\hat{\theta}_m - \theta_m)^T C^{-1}_m (\hat{\theta}_m - \theta_m) + (\hat{\theta}_n - \theta_n)^T C^{-1}_n (\hat{\theta}_n - \theta_n).$$

(8)

The solution of each of these non-linear WLS problems can again be attained using Gauss-Newton iterations (with the derivatives specified in (7)), but we propose to apply only a single iteration by each sensor, for which the initial guess would be the fictitious measurements:

4) Each sensor $n$ applies one Gauss-Newton iteration for minimizing (8), with $\hat{\theta}_n$, $\{\hat{\theta}_m\}_{m \in \mathcal{T}_n}$, as an initial guess. Following this single iteration, each sensor obtains a slightly updated estimate of its own position (as well as of the positions of some of the other sensors). Rather than proceed with additional Gauss-Newton iteration, it is more important at that point to share the updated estimates (as new, improved “fictitious measurements”) with the relevant sensors. Our procedure therefore proceeds as:

5) Each sensor uses the result of the single iteration to update its own fictitious measurement $\hat{\theta}_n$ and possibly (see below) also its associated covariance $C_b$.

The system then returns to Step 3, for all sensors to announce and obtain updated information regarding the estimated positions of their relevant neighbors, with which to proceed recursively until the process converges.

Important ingredients in the recursive process are the covariance matrices of the “fictitious measurements”, which determine the weighting matrices in the WLS criteria. For example, the very small covariance attributed to the position parameters of the anchors results in attributing very large weights to deviations of their position estimates from their nominal values, thereby “locking” these estimates to these values. As the process evolves, the level of confidence in the estimated positions of additional nodes (eventually of all nodes) gradually increases, and nodes whose positions are “nearly resolved” should earn higher weights. This can and should be reflected in updating the covariance matrices $C_n$, which can be recalculated from the WLS model: under a small-errors assumption, $C_n$ is given by the $\theta_n$-related $3 \times 3$
block of the matrix \((H_n^T W_n H_n)^{-1}\), where \(H_n\) is the locally-linearized form of the WLS equations for sensor \(n\), and \(W_n\) is the respective weight matrix (see, e.g., [10]). However, at least in the beginning of the process, when most of the “fictitious measurements” are totally unreliable and the errors are large, reduction of their presumed covariance matrices might be misleading, and was seen to tend to slow down and even “stall” the convergence process too early, before approaching the true solution. Our experience (with our tested scenarios) shows that updating the covariance matrices once in every five iterations (or so) achieves good convergence.

IV. SIMULATION RESULTS

We simulated \(N\) sensors, four of which were anchor nodes placed at the four corners of a \(20[Km] \times 20[Km]\) square, and the rest uniformly and independently deployed within the square. The transmission ranges \(r_1, \ldots, r_4\) of the anchor sensors were all set to \(20[Km]\), whereas the ranges \(r_5, \ldots, r_N\) of each of the other sensors were uniformly and independently drawn between \(5[Km] \) and \(10[Km]\). This yields (empirically) an average connectivity level \(\bar{N}/(N(N-1)) \approx 32\%\). The synchronization offsets \(\tau_1, \ldots, \tau_N\) of all sensors were independently drawn as \(\tau_n \sim \mathcal{N}(0, \sigma^2_n)\) with \(\sigma_n = 1[Km]/c \approx 3.3[pS]\) \(\forall n\). The TOA estimation errors \(\epsilon_{mn} = \epsilon_{nm}\) were drawn as \(\epsilon_{mn} \sim \mathcal{N}(0, \sigma^2_m)\) with \(\sigma_m = 30[m]/c \approx 100[pS]\).

The initial guesses \(p_{11}, \ldots, p_{1N}\) of the anchor nodes were set to their true positions; for all other sensors they were drawn uniformly and independently within the middle \(4[Km] \times 4[Km]\) square. We applied the centralized and decentralized algorithms with the following parameters: Synchronization error variances: \(\sigma_1 = 1[cm]/c \approx 33[pS]\), all other \(\sigma_n\) were set to their true value of \(1[Km]/c \approx 3.3[\mu S]\); Position error variances: \(\overline{\sigma}_1, \ldots, \overline{\sigma}_4 = 1[cm]\) for the anchor nodes, \(\overline{\sigma}_n = 35[Km]\) for all other nodes. Stopping criteria were based on comparing the estimation updates to a threshold. A collaborative stopping scheme was implemented for the decentralized algorithm (description omitted due to space limitations).

Figure 2 illustrates snapshots of the true and estimated positions by the decentralized algorithm, axes in [Km]. Initial guess + state after 1, 4, 8 iterations.

Table 1: \(M\): average number of iterations until convergence; \(N_\sigma\): number of successful trials out of 100 (A successful trial is counted when the mean error distance \(\left\|\tilde{p}_n - p_n\right\|\) is smaller than \(100[m]\)); \(E\): mean error distance (in [m]), averaged over the successful trials; \(P\): average percentage of error distances below 100 meters, averaged over the successful trials.

V. CONCLUSION

We proposed a TOA-based collaborative, recursive estimation scheme, which does not require fine synchronization, and can be applied to ad-hoc networks with partial and possibly non-symmetric connectivity. The opportunistic nature of the approach allows pre-programming of all sensors, with no need for prior knowledge of the eventual network topology.

REFERENCES