

# Honest Signaling in the Cooperative Search

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**Abstract**—We consider a foraging by a group of agents acting in heterogeneous environment, and suggest a new model of cooperative foraging, which implements biological signaling. In the model, the individual foraging follows Brownian walks and the Lévy flights with the varying parameters with respect to the observed states of the environment, and communication between the agents and their aggregation is defined on the basis of the Sir Philip Sidney game, which models the honest communication between animals.

In our simulation, we find that a group of Brownian foragers with signaling behaves similarly to the group of Lévy flyers without signaling, and the resulting cooperative foraging outperforms the known models of foraging without signaling. We argue that it provides a simple yet competitive description of the observed behavior of the foraging animals.

**Index Terms**—Search and foraging, cooperative behavior, autonomous mobile agents, probabilistic local search.

## I. INTRODUCTION

The studies of search and foraging activity of the animals [2], [33], [34], [38] and artificial mobile agents [20], [22], [35] address two general problems. The first problem deals with the models of agents' foraging activity and considers the processes, which govern the motion of the agents, and the second problem addresses the formation of the groups of the agents and the models of swarm dynamics. A working assumption in these studies is that natural foragers act optimally that is “in ways close to (i.e. statistically indistinguishable from) those that maximize their expected fitness, subject to any functional constraints” [26].

The modeling of aggregation of the mobile agents and formation of the animals' groups, known as swarms, flocks, colonies, herds or schools, was initiated the pioneering paper by Reynolds [27], who suggested the basic rules of the simulated flock. Later these rules were detailed and extended in different directions [12], [16], in particular, the methods of aggregation and dynamics of the foraging agents also known as social foraging were suggested [16], [40] and the activity of the swarms of Lévy flyers was considered [1], [28]. In the simplest case, the group is considered as a set of individual particles moving according to the certain stochastic process, and it was demonstrated [9], [23] that the Lévy flyers cover greater territory than the Brownian walkers do.

After the publication of the seminal paper on wondering albatrosses [36] and further arguing the optimality of search

governed by the Lévy flight processes [37], it was widely accepted [10], [15] that while foraging the animals or at least some species “evolved to exploit Lévy flights” [39]. However, at the same time, it was also reported that the motion patterns like the Lévy flights can be explained by the influence of the environmental states [13], [14], [17] or by intermittent strategies, which include Brownian walks and ballistic motion [3], [4], [5].

In this report, we suggest a different approach of cooperative foraging, which does not require specific sensing and intellectual abilities of the foragers. In the model, we assume that the individual foraging follows Brownian walks and the Lévy flights with varying parameters [18], [25], [32] governed by the environmental states or by the target's location probabilities. For definition of communication between the agents and their aggregation for cooperative foraging, we adopt the Sir Philip Sidney game [19], [31], which is widely accepted as a model of honest communication between the animals questing for help from the relatives [6], [7], [8]. In the cases of foraging in homogeneous environment and of individual foraging, the suggested model is reduced to the corresponding models of optimal foraging, in particular, to the widely accepted models of foraging in patchy environment based on the marginal value theorem [11], [24]. A detailed review of such models and general framework of optimal foraging theory are presented by Stephens & Krebs [33].

We demonstrate that the group of environmentally tuned Brownian foragers which include also direct signaling behaves similarly to the group of Lévy flyers without signaling.

## II. INDIVIDUAL FORAGING BY THE LÉVY FLIGHTS WITH VARYING PARAMETERS

The Lévy flight in two dimensions is defined as follows [37], [38]. Assume that at time  $t$  the forager is in the point  $x(t) = (x_1(t), x_2(t))$  and denote by  $\delta(t) > 0$  the length of the step, which the forager will do from its current location  $x(t)$  to the next location  $x(t+1)$ , and by  $\alpha(t)$ ,  $0 \leq \alpha(t) < 2\pi$ , the angle between the current heading of the forager and the direction to the point  $x(t+1)$ . In the ordinary Lévy flight, it is assumed that at each time  $t$  the value  $\delta(t)$  is drawn from the Pareto distribution that is  $p(\delta) = Pr\{\delta(t) = \delta\} \sim \delta^{-\gamma}$ , or, more precisely [17],  $p(\delta) = (\gamma - 1)\delta_{min}^{\gamma-1} \delta^{-\gamma}$ ,  $\delta_{min} \leq \delta < \infty$ , where  $\delta_{min} > 0$  is a minimal step's length, and  $\gamma = \beta + 1$ ,

where  $0 < \beta \leq 2$  is a stability or Lévy index; and the angle  $\alpha(t)$  is drawn from the uniform distribution on the interval  $[0, 2\pi)$ . For real foragers, both natural and artificial, the step length  $\delta$  has also a finite upper bound; so it is assumed that  $\delta_{min} \leq \delta \leq \delta_{max}$ , where the length  $\delta_{max} < \infty$  is specified with respect to the abilities of the considered forager. In the paper on wandering albatrosses [36] and further studies of Lévy flights search [15], [37] it is assumed that the parameter  $\gamma$  is constant and the angles  $\alpha(t)$  are arbitrary. That corresponds to the nondestructive foraging, in which the forager does not destroy the target (or the destroyed target is renewing at the same location like grass, vegetables and fruits) and revisits the previously visited locations [37], [38] and does not consider the environmental states [3], [4], [5].

In our model, in contrast, we consider the Lévy flights with varying parameter  $\gamma$  and assume that both  $\gamma$  and the angles  $\alpha(t)$  depend on the environmental states [18], [25], [32]. It implies that the forager is able to perceive the state of the environment in its location  $x(t)$  and the distant environmental states, and allows implementing the destructive search, in which the forager is considered as a predator that eats the detected target and avoids revisiting the already visited locations [37], [38]. In the terms of the target's location probabilities it means that the forger has errorless detection abilities and zeroes the target's location probabilities in the visited locations. If the forager implements the nondestructive search in homogeneous environment, then the suggested model is directly reduced to the ordinary Lévy flights search [10], [15], [26], [37].

The foraging process according to the suggested model is outlined as follows.

1. Being in the point  $x(t)$  at time  $t$ , the forager observes the state  $\epsilon(x) \geq 0$  of the environment and decides whether to continue foraging or not. In the case of destructive foraging, the forager zeroes the value  $\epsilon(x)$  ("eats" the prey) but stores the original value  $\epsilon(x)$  in its memory up to obtaining the parameter  $\gamma(\epsilon)$  (see the next line) and then forgets it. In the case of nondestructive foraging the value  $\epsilon(x)$  is remained without changes and is used in the calculation of the parameter  $\gamma(\epsilon)$ . If the foraging continues, the forager conducts the following actions.

2. The forager defines the value of the parameter  $\gamma(\epsilon)$  inversely proportional to the value  $\epsilon(x)$ , e.g.  $\gamma \sim 2(1 - \epsilon/\epsilon_{max})$ , where  $\epsilon_{max}$  is a maximal value of the environmental state, which the forager is able to perceive and distinguish by its sensors.

3. Using the value  $\gamma(\epsilon)$ , the forager draws the length of the next step  $\delta(t)$ ,  $\delta_{min} \leq \delta(t) \leq \delta_{max}$ , from the Pareto distribution  $p(\delta) \sim \delta^{-\gamma(\epsilon)}$ . In the suggested model, we assume that  $\delta_{min} = 0$  and  $\delta_{max}$  is equivalent to the maximal distance, at which the forager is able to perceive environmental states e.g. by smelling or direct vision.

4. To define the direction of the next step, the forager screens the environment around at the radius of the obtained step  $\delta(t)$  and specifies the direction that is the  $\alpha(t)$  that

corresponds to the maximal value of the environment at the distance  $\delta(t)$  from the forager.

5. Having both step length  $\delta(t)$  and the angle  $\alpha(t)$  the forager flies to the next point  $x(t+1)$  and the process continues from line 1.

The suggested procedure does not assume long-term internal memory of the forager and requires only simple calculations that can be processed by analog gates [21], [22], [29], [30]. Certainly, if  $\gamma(\epsilon)$  is constant and  $\alpha(t)$  is random, the presented procedure is reduced to ordinary Lévy flight, and if  $\delta(t)$  is constant and  $\alpha(t)$  is random, then the procedure represents a basic nondestructive Brownian motion, in which the forager is allowed to revisit the already considered points.

Notice that the suggested on-line procedure results in the trajectory, which has the same form as trajectories of foraging by patches based on the marginal value theorem [11], [24], which provides an off-line solution of the problem of foraging under general assumption that the forager acts as economical as possible.

### III. HONEST SIGNALING BETWEEN THE AGENTS AND ACTIVITY OF THE GROUP

For aggregation of the foragers acting according to the suggested above procedure (or any other procedure specifying an individual foraging), we adopt the model of honest communication between the animals based on the discrete Sir Philip Sidney game [19], [31]. Originally, this is a one stage game between two related players, in which one player, the signaler, decides whether to send a costly request for a resource, which will increase the sender's fitness, and the second player, the donor, decides whether to provide the resource to the sender, and so to decrease the own fitness, or not. If the signaler is needy for the resource, then its fitness without the resource is  $(1 - a)$ , and if signaler is not needy for the resource, then its fitness without the resource is  $(1 - b)$ . If the donor decides to provide the resource to the signaler, then its fitness becomes  $(1 - d)$ . In addition, it is assumed that the signaler and the donor are related with relatedness value  $k$ ; the cost of sending the signal is denoted by  $c$ . For all these values it is assumed that  $0 \leq a, b, c, d, k \leq 1$ .

In our model, we implement the special case in which the signaler signals only when it is needy, and the donor donates only if the signal received. The Nash equilibrium in such a game is provided by the following inequalities [6]:  $a \geq c + kd \geq b$  and  $a \geq d/k \geq b$ , which we adopt for making decisions about signaling and donating. The variables appearing in the inequalities are interpreted as follows. During the search, the forager screens the environment around it and obtains the sum of the values  $\epsilon(x')$  over all points  $x'$  such that the distance between  $x'$  and forager location  $x$  is less than or equal to a screening radius  $r_{screen}$ . This sum is a quantity of prey around the search and it is interpreted as  $a$  that is a value of the signaler's necessity for help; we assume that  $a$  is proportional to the ratio of the indicated sum to the value  $\pi r_{check}^2$ , where  $r_{check}$  is the radius of the checked area. The value of non-necessity is  $b \ll 1$  that is the signaler sends the

signal only in the case that it needs the help. Notice that for individual foraging the agent does not need to screen the environment and does this for signaling only. Thus, the signaling cost  $c$  is proportional to the screened area that is  $c \sim \pi r_{screen}^2 / s_{max}$ , where  $s_{max}$  is a maximal screened area that is a volume of the domain. If the donor receives the signal and decides to help to the signaler, it loses the possibility to encounter the prey around itself. Then, the value  $d$  is calculated similar to the value  $a$  as a sum of the values  $\epsilon(y')$  around the donors location  $y$ . Finally, the reliveness  $k$  between the signaler and the donor is specified proportionally to distance between the signaler and the donor that is  $k \sim |x - y| / D_{max}$ , where  $D_{max}$  is a maximal signaling distance between the agents. In real situations, we assume that  $D_{max} \geq \delta_{max}$  that is the maximal distance, at which the foragers is able to sense the environmental state; also it is assumed that the signals are sent for the agents, which are close enough but not too close to the signaler. The scheme, which illustrates the introduced parameters, is shown in Fig. 1.

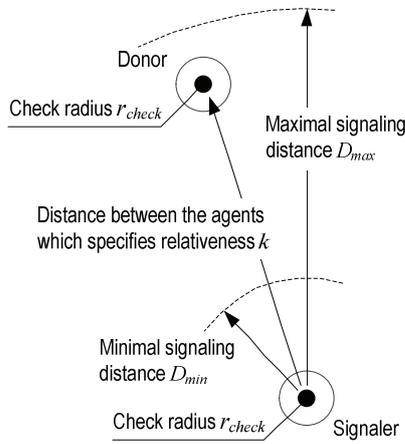


Fig. 1. Parameters of the honest signaling in the probabilistic search.

If the donor receives the signal and decides to leave its current location and to donate to the signaler, it walks or flies to the signaler's location, and then acts as a signaler that is it screens the environment for prey and signals to donors if need.

#### IV. SIMULATION RESULTS

The actions of the suggested algorithm of search with signaling were simulated for different distributions of prey and different parameters of signaling. The results of these simulations demonstrate that, in spite of different quantities of eaten prey and trajectories length, the relation between the eat rates in considered search processes is preserved. Below, we present the results of simulated individual and collective search by ten agents. The domain is of the size  $n = 100 \times 100$  with impermeable bounds. Similar to previous examples, parameters of signaling and sensing are specified as follows: minimal and maximal distances are  $\delta_{min} = 1$  and  $\delta_{max} = 12.5$ , respectively, the length of the step is  $\delta = 3$ , the screening and checking radiuses are  $r_{screen} = 1$  and  $r_{check} = 5$ , and minimal

and maximal signaling distances are  $D_{min} = 10$  and  $D_{max} = 25$ , respectively. The trajectories of the agents in different search processes are illustrated in Fig. 2 (individual search) and Fig. 3 (collective search).

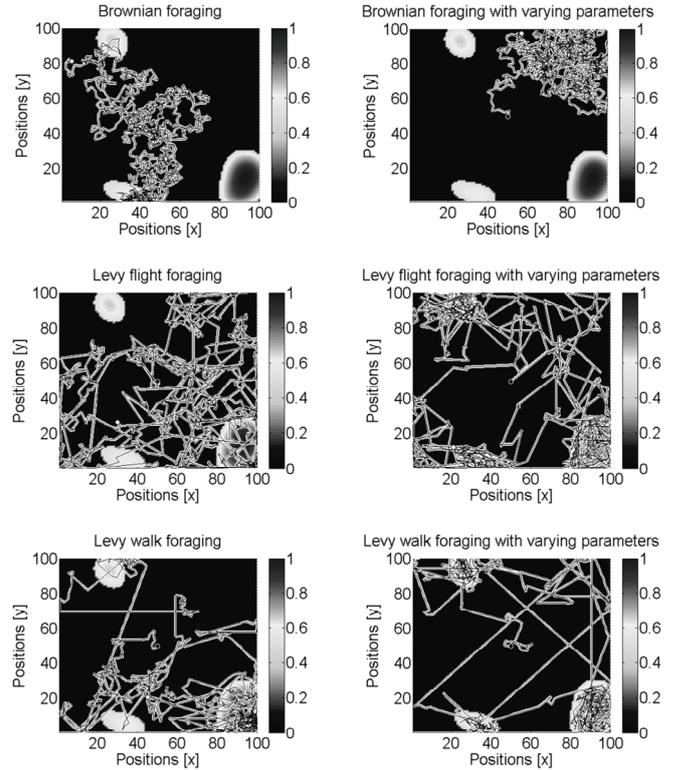


Fig. 2. Trajectories of the individual search according to different processes with constant and with varying parameters.

It is seen that single agents that implement varying parameters, tend to stay in the regions of higher prey probabilities, however, if the agent has not arrived a prey rich region it continues searching with constant parameters. In the case of collective search, the agents mostly follow the basic processes with varying parameters, and tend to arrive to regions high probabilities due to the signals received from the neighboring agents.

Numerical results of simulations for three different prey distributions are summarized in TABLE I. Each trial included 1000 sessions, and for each session the prey distribution was generated randomly. In the Table, CP stands for constant parameters, VP – for varying parameters, NS means the search without signaling and S – the search with signaling. In the table, maximal values for each distribution are denoted by bold.

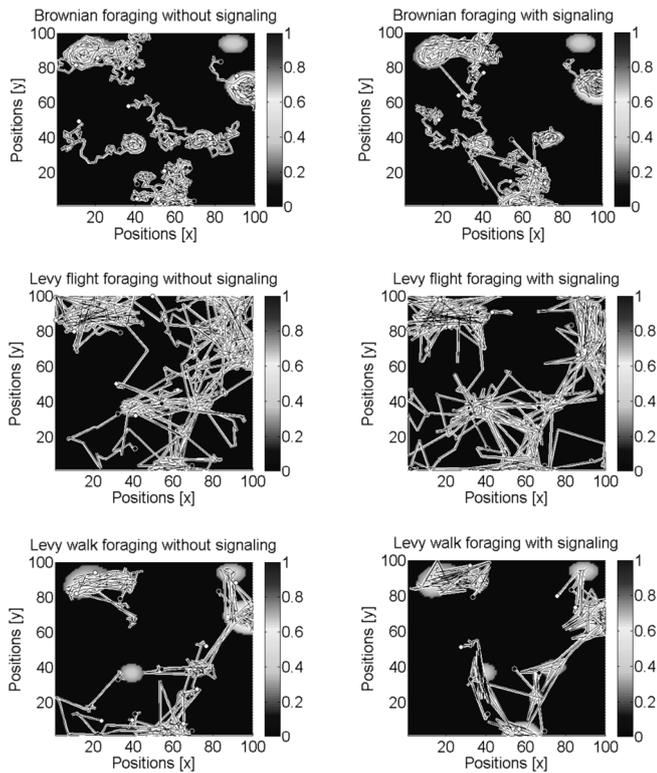


Fig. 3. Trajectories of the collective search by ten agents acting according to different processes with varying parameters and without and with signaling.

TABLE I. QUANTITIES OF EATEN PREY AND EAT RATES

	Brown walk		Lévy flight		Lévy walk	
	Eaten prey	Eat rate	Eaten prey	Eat rate	Eaten prey	Eat rate
The prey distributed over a fourth part of the domain						
NS/CP	107.00	0.037	120.72	0.019	116.16	0.047
S/CP	160.98	0.055	187.84	0.026	127.58	0.048
NS/VP	<b>262.83</b>	<b>0.090</b>	398.13	<b>0.062</b>	<b>355.16</b>	<b>0.144</b>
S/VP	253.66	0.086	<b>404.70</b>	0.057	310.36	0.117
The prey distributed over a half of the points in the domain						
NS/CP	226.71	0.077	257.47	0.041	256.35	0.104
S/CP	320.17	0.109	361.82	0.048	247.42	0.091
NS/VP	<b>591.11</b>	<b>0.201</b>	<b>849.63</b>	<b>0.129</b>	<b>668.94</b>	<b>0.272</b>
S/VP	567.29	0.192	800.79	0.102	510.45	0.191
The prey distributed over all points of the domain						
NS/CP	341.34	0.116	393.07	0.063	372.71	0.151
S/CP	431.01	0.146	482.56	0.062	330.07	0.120
NS/VP	<b>830.67</b>	<b>0.280</b>	<b>944.95</b>	<b>0.143</b>	<b>758.35</b>	<b>0.309</b>
S/VP	730.20	0.246	887.05	0.112	600.29	0.224

As seen by simulations, best results measured by both total eaten prey and by eat rate were reached for the agents moving according to the processes with varying parameters (agents with sensing abilities). If signaling is added, than the search becomes less effective, but still strongly outperforms the foraging with constant parameters without signaling. The only exception is the foraging by Lévy flight over the domain, which is large empty regions, where sensing and bio-signaling results in slightly larger quantity of eaten prey; however, statistically this difference is insignificant. Signaling leads to larger quantities of eaten prey when the agents have no sensing abilities. If most of the regions do not contain prey, then the signaling results in greater eat rates, but if the domain is rich enough with prey, then the greater eat rate for the Lévy walk foraging is provided by the motion with constant parameters and with signaling.

In addition, notice that in some cases it was observed that the Brownian foraging outperforms the Lévy walk foraging for both total eaten prey and the eat rates. Nevertheless, in the Brownian foraging the search often results in small or even zero eaten prey, while in the Lévy walk foraging such results are rather rare.

## V. CONCLUSION

In the report, we suggested a new model of cooperative foraging with varying parameters and honest signaling based on the Sir Philip Sidney game, which is widely known as a model of honest communication between the animals.

An application of the suggested model to the group of the simplest Brownian foragers demonstrates that such agents act similarly or even equivalent to the Lévy flight foragers, which is widely accepted as an adequate model of natural foragers.

The simulations' results show that, informally speaking, the signaling can disturb the effective activity of the smart enough agents (Lévy flyers or walkers with sensing), but can increase the effectiveness of the activity of less smart agents (Brownian walkers). However, the correct proportion of the agent's smartness and the level of signaling are still not clear enough.

The obtained results clarify the individual behavior of the animals acting in groups and the role of communication, and can form a basis for biomimetic navigation of artificial mobile agents.

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