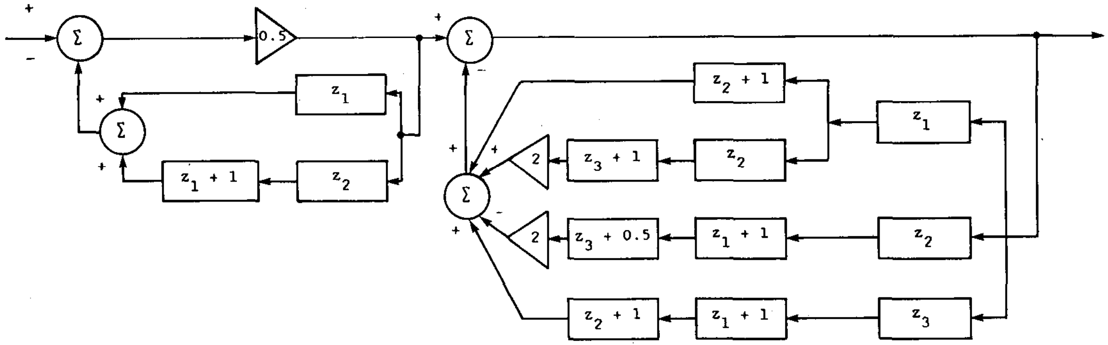

Fig. 2.

After carrying out all the steps of the proposed algorithm, we arrive in the implementation of the denominator given in Fig. 2.

IV. Comments

A general decomposition theorem of a rational function of m variables, in terms of functions of one variable only, leads to implementations of m-dimensional filters with great modularity and parallelism. Many different implementations can be achieved, depending on the choice of the matrices $\boldsymbol{R}_i$. These matrices can be chosen to achieve filter realizability, easy stability testing or optimum filter implementation. Such considerations have been already reported in the 2-D case [4], [5]. Their extension to the $m-D$ case is now in progress.

References

[1] T. S. Huang, W. F. Schreiber, and O. I. Tretiak, "Image processing," *Proc. IEEE*, vol. 29, pp. 1586–1609, 1971.

[2] R. Eising, "Separability of 2-D transfer matrices," *IEEE Trans. Automat. Contr.* vol. AC-24, pp. 508–510, 1979.

[3] M. Morf, B. C. Levy, and S. Y. Kung, "2-D polynomial matrices, factorization and comprimeness," *Proc. IEEE*, vol. 65, pp. 861–872, 1977.

[4] A. N. Venetsanopoulos and B. G. Mertzios, "General decomposition of two-dimensional filters," *Proc. 6th European Conf. Circuit Theory and Design (ECCTD'83)*, (Stuttgart, FRG), pp. 444–446, Sept. 6–8, 1983.

[5] B. G. Mertzios and A. N. Venetsanopoulos, "Noise properties of two-dimensional IIR digital filters implemented by the decomposition theorem," in *Proc. 2nd European Signal Processing Conf.*, (EUSIPCO'83), (*Erlangen, FRG*), *Sept.* 12–16, 1983.

A Stability New Test for Linear Discrete Systems in a Table Form

Y. Bistritz

Abstract—**A new stability testing table is presented for discrete linear system. The table is similar to the Routh table in size, amount of computation as well as other features. The new table requires for a discrete system polynomials about half the number of entries and arithmetic operations that is needed in the previous Jury–Marden criterion in tabular form.**

I. The Stability Criterion

Assume a real polynomial of degree n

$$D(z)=d_0z^n+d_1z^{n-1}+\cdots+d_n, \qquad d_0>0. \tag{1}$$

Define its reciprocated polynomial $D^*(z)$ as $z^nD(z^{-1})$, that is $D^*(z)$ denotes the polynomial

$$D^*(z)=d_nz^n+d_{n-1}z^{n-1}+\cdots+d_0. \tag{2}$$

The polynomial $D(z)$ can be written as the sum

$$D(z)=\frac{1}{2}S(z)+\frac{1}{2}A(z). \tag{3}$$

In the above $S(z)$ and $A(z)$ are the symmetric and antisymmetric polynomials, respectively, defined by

$$S(z)=D(z)+D^*(z) \tag{4}$$

$$A(z)=D(z)-D^*(z) \tag{5}$$

where a general polynomial

$$P(z)=\sum_{i=0}^{n}p_iz^i \tag{6}$$

is called symmetric if

$$P^*(z)=P(z) \quad p_i=p_{n-i}, \qquad i=0,1,\cdots \tag{7}$$

and it is called antisymmetric if

$$P^*(z)=-P(z) \quad p_i=-p_{n-i}, \qquad i=0,1,\cdots. \tag{8}$$

Necessary and sufficient conditions for the polynomial $D(z)$ to have all its zeros inside the unit circle are imposed on the following table:

$$\begin{array}{cccccccccccccccc}
a_0 & & a_1 & & a_2 & \cdots & & & \cdots & (a_{n-2}) & & & (a_{n-1}) & & (a_n) \\
& b_0 & & b_1 & & b_2 & \cdots & \cdots & (b_{n-3}) & & (b_{n-2}) & & & (b_{n-1}) & \\
& & c_0 & & c_1 & \cdots & & & \cdots & (c_{n-1}) & & & (c_{n-2}) & & \\
& & & d_0 & & d_1 & \cdots & \cdots & (d_{n-4}) & & (d_{n-3}) & & & & \\
& & & & e_0 & \cdots & & & \cdots & (e_{n-4}) & & & & & \\
& & & & & \ddots & & & \iddots & & & & & & \\
& & & & & & u_0 & & (u_1) & & & & & & \\
& & & & & & & v_0 & & & & & & &
\end{array} \tag{9}$$

Manuscript received September 30, 1983.
The author is with the Department of Electrical Engineering, School of Engineering, Tel Aviv University, Ramat Aviv 69978, Israel.

that is constructed as follows:

(i) The first two rows are

$$a_0z^n + a_1z^{n-1} + \cdots + a_n = \begin{cases} A(z), & n=2m+1 \\ S(z), & n=2m \end{cases} \tag{10}$$

$$b_0z^{n-1} + b_1z^{n-2} + \cdots + b_{n-1} = \begin{cases} S(z)/(z+1), & n=2m+1 \\ A(z)/(z+1), & n=2m \end{cases} \tag{11}$$

(ii) The next $(n-1)$ rows are constructed by the rule

$$c_k = -\begin{vmatrix} a_0 & a_{k+1} \\ b_0 & b_{k+1}-b_k \end{vmatrix}\frac{1}{b_0} = a_{k+1} + \left(\frac{a_0}{b_0}\right)(b_k - b_{k+1})$$

$$d_k = -\begin{vmatrix} b_0 & b_{k+1} \\ c_0 & c_{k+1}-c_k \end{vmatrix}\frac{1}{c_0} = b_{k+1} + \left(\frac{b_0}{c_0}\right)(c_k - c_{k+1}) \tag{12}$$

and so on.

The necessary and sufficient conditions for $D(z)$ to have all its zeros inside the unit circle $|z|=1$ are

(1) All first entries in all rows are positive

$$a_0>0, \quad b_0>0, \quad \cdots, \quad v_0>0. \tag{13}$$

(2) The following sums defined on the first and every second row thereafter are all positive

$$\sigma_0 = a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + \cdots + a_n > 0$$
$$\sigma_2 = c_0 - c_1 + c_2 - c_3 + \cdots + c_{n-2} > 0$$
$$\sigma_4 = e_0 - e_1 + \cdots e_{n-4} > 0$$
$$\vdots$$
$$\sigma_{2m} > 0 \quad (\sigma_{2m} = u_0 - u_1 \text{ for } n=2m+1, \ \sigma_{2m} = v_0 \text{ for } n=2m). \tag{14}$$

The proof of the underlining necessary and sufficient conditions for this stability criterion is given in [2]. Other aspects and extensions of the method are presented in [3] and [4]. It is noted that the completion of the table is a necessary condition for stability [4]. Therefore singular conditions (division by a vanishing entry in (12)) indicate unstable polynomial. Following are some remarks on the computational aspects of the method.

Remark 1:

The odd number rows $1,3,5\cdots$ of the table have the symmetry of the first row (are antisymmetric for $n=2m+1$ and symmetric for $n=2m$). The even number rows $2,4,6,\cdots$ have the symmetry of the second row (are symmetric for $n=2m+1$ and antisymmetric for $n=2m$). By symmetry and antisymmetry of a row we mean that its right half entries are "mirror" and "anti-mirror" (i.e., minus) reflections of the left half entries in accordance to the patterns (7) and (8), respectively. The important consequences is that the entries put in parentheses in the presentation (9) of the table need not be actually calculated. One can even completely drop out the right-half part of the table once a familiarity with its structure has been gained.

Remark 2:

The new table involves a number of entries (in view of the former remark) that is exactly equal to the number of entries of a Routh table for a polynomial of the same degree. Other evident points of closeness to the Routh table are the determinant rule (12) of construction and the conditions in (13). The number of multiplicative arithmetics involved is less if the second right hand sides of (12) are used (noting also that $a_0/b_0, b_0/c_0, \cdots$ are calculated once for all the entries of the row). This number (given by $M \equiv 0.25n^2+n-1$ for n even or by $M-0.25$ for n odd) is exactly equal to the number of multiplications required for the Routh table and is half of the corresponding number for the Jury–Marden table (It is assumed in these comparisons that also in the mentioned two other tables a similar admissible compact common row factor construction rule is adopted). The number of additive operations is higher than in the table of Routh by one extra additive operation per entry. However, the total number of additive operations required for the new table ($A \equiv 0.5n^2+1.5n+2$ for n even, $A-1$ for n odd) is still comparable with the corresponding number for the table of Jury. This is so because the relative extra additive operations per entry are favorably compensated by the relative half factor in the total number of entries.

Remark 3:

Additional useful computational hints are the following two;

—The entries of a row in the table may be multiplied by a common positive real number without affecting the results. The property may be convenient for hand computation.

—The elimination of a $(z+1)$ factor from a polynomial (6) required in (11) for setting the second row involves the following simple additive arithmetic (performed sufficely for half length only).

$$P(z)/(z+1) = \sum_{i=0}^{n-1} q_i z^{n-1-i}, \qquad q_0 = p_0, \quad q_i = p_i - q_{i-1}$$

II. Illustrative Examples

Consider the polynomial

(1) $D(z) = z^4 - 1.368z^3 + 0.4126z^2 + 0.08z + 0.0025$.

By (4) and (5),

$$S(z) = 1.0021z^4 - 1.288z^3 + 0.8252z^2 - 1.288z + 1.0025$$
$$A(z) = 0.9975z^4 - 1.448z^3 + 1.448z - 0.9975$$
$$= (z+1)(0.9975z^3 - 2.4455z^2 + 2.4455z - 0.9975).$$

The table is constructed using (10)–(12), (the numbers in parentheses are completed by symmetry, see Remark 1)

$\sigma_0 = 5.4062$	1.0025		−1.288		0.8252		(−1.288)		(1.0025)
		0.9975		−2.4455		(2.4455)		(−0.9975)	
$\sigma_2 = 8.4348$			2.1723		−4.0903		(2.1723)		
				0.4303		(−0.4303)			
$\sigma_4 = 0.2542$					0.2542				

Conditions (13) and (14) are satisfied. Therefore the polynomial has all its zeros inside the unit circle.

(2) Consider next the polynomial

$$D(z) = 16.5z^3 - 15.6z^2 - 16.4z + 13.5$$

now we have

$$A(z) = 3z^3 + 0.8z^2 - 0.8z - z$$

$$S(z) = 30z^3 - 32z^2 - 32z + 30 = (z+1)(30z^2 - 62z + 30).$$

The table is

$\sigma_0 = 4.4$	3		0.8		(-0.8)		(-3)
		30		-62		(30)	
$\sigma_2 = 20$			10		(-10)		
				-2			

The -2 term in the last row violates condition (13). Therefore, the polynomial does not have all its zeros inside the unit circle.

III. Conclusion

The criterion presented in this paper is useful to test the stability of discrete systems whose characteristic polynomial is known. The new methodology presents a significant reduction in size and computation relative to the Jury–Marden table [1]. The table has many features in common with the Routh table which is used to test the stability of continuous systems. Some properties of similarity are evident from this paper, additional properties are discussed in [3]. The method can be extended to obtain information also on the location of eigenvalues of unstable discrete systems [4].

References

[1] E. I. Jury, *Theory and application of the z transform method.* New York: Wiley, 1964.

[2] Y. Bistritz, "A new unit circle stability criterion," in *Proc. 1983 Int. Symp. Mathematical Theory of Networks and Systems*, MTNS-83 June 20–24, 1983, Beer-Sheva, Israel.

[3] ——, "Z domain continued fraction expansions for stable discrete systems polynomials," *IEEE Trans. Circuits Syst.*, (submitted for publication).

[4] ——, "Zero locations with respect to the unit circle of discrete-time linear system polynomials" Tel Aviv University, Faculty of Engineering, report in preparation.

Pole and Zero Determination of Arbitrary Linear Digital Networks

HON-KEUNG KWAN

Abstract —**A method for poles and zeros determination between any two nodes of a linear time-invariant single-input single-output/multi-input multi-output one-dimensional arbitrary digital network is described. It could be used for frequency domain analysis. Computational results show that this method is computationally accurate and efficient, and requires small computer execution storage.**

I. Introduction

Difficulties arise in the determination of the poles and zeros between any two nodes of a matrix represented [1] digital network by the eigenvalue approach [2]. This is particularly so for a digital network having a large number of nodes as compared with its number of poles and zeros. Numerical roundoff errors in the direct computation of eigenvalues from two large matrices render the problem of deciding which pole(s) and zero(s) will cancel and which will not difficult. Moreover, the problem can become numerically sensitive and prone to errors.

Manuscript received July 26, 1983.

The author is with the Department of Electrical Engineering, University of Hong Kong, Pokfulam Road, Hong Kong.

Starting with a matrix represented arbitrary digital network, a simplified eigenvalue method has been proposed [3] with considerable savings in both computer execution time and storage as compared to the direct computation. However, the above two difficulties still remain to a certain extent.

Applying the state-space approach of analog networks [4] to digital networks, accurate and efficient determination of the poles and zeros between any two nodes of a single-input single-output/multi-input multi-output arbitrary digital network can be obtained. In this approach, the number of poles and the number of zeros computed will be respectively equal to, and not greater than, the number of delays in a network. For a canonic digital network, the precise number of poles and zeros are computed. For a noncanonic digital network, the problem of a small number of extra pole(s) and zero(s) cancellation is negligible. Moreover, for a digital network, the sizes of the two matrices, from which the poles and zeros between any two nodes are computed, are small as compared to those of the direct computation and the simplified eigenvalue method. Therefore, numerical roundoff errors that may conceivably arise in the present method are negligible.

II. Transfer Function Matrix

A 1-D arbitrary digital network of N delay nodes and M signal nodes can be uniquely represented [5] in the time-domain as

$$y_d(n) = Sy_d(n-1) + Ty_c(n-1) \tag{1}$$

$$y_c(n) = Uy_d(n) + Vy_c(n) + x_c(n) \tag{2}$$

where $y_d(n)$ is a $N \times 1$ vector of signal values at delay nodes: $y_c(n)$ and $x_c(n)$ are respectively $M \times 1$ vectors of signal values and inputs at signal nodes; S, T, U, and V are real matrices of appropriate dimensions.

Taking the z-transform of (1) and (2) and rearranging, we obtain

$$(zI - E)Y_d(z) = FX_c(z) \tag{3}$$

$$Y_c(z) = GY_d(z) + HX_c(z) \tag{4}$$

where

$$E = S + T(I' - V)^{-1}U \tag{5}$$

$$F = T(I' - V)^{-1} \tag{6}$$

$$G = (I' - V)^{-1}U \tag{7}$$

$$H = (I' - V)^{-1}. \tag{8}$$

I and I' are identity matrices of dimensions $N \times N$ and $M \times M$, respectively.

From (3) and (4), the transfer function matrix, $T(z)$, of a M-input M-output network can be expressed as

$$T(z) = \frac{Y_c(z)}{X_c(z)} \tag{9}$$

$$= H + G(zI - E)^{-1}F \tag{10}$$

$$= \frac{H\det(zI - E) + G\operatorname{adj}(zI - E)F}{\det(zI - E)} \tag{11}$$

$$= \frac{P(z)}{q(z)} \tag{12}$$