How Good is Bargained Routing?

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Game Theory in Networking

- **Non-Cooperative Game:**
  - Each player attempts to maximize its utility.
  - Players cannot communicate and/or reach a binding agreement.
  - The main solution concept: [Nash Equilibrium](#)

**Nash Equilibrium:**
A set of actions (strategy choices), one per player, where no player can unilaterally improve its performance by changing its strategy.
Game Theory in Networking

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  - Each player attempts to maximize its utility.
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  - The main solution concept: **Nash Equilibrium**
  - Figure of merit: **Price of Anarchy (PoA)**
    [Koutsoupias, Papadimitriou, 99]

The PoA quantifies the degradation of system performance due to
1. Lack of cooperation
2. Selfishness of users
Game Theory in Networking

- **Non-Cooperative Game:**
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  - The main solution concept: **Nash Equilibrium**
  - Figure of merit: **Price of Anarchy (PoA)**

- **Cooperative Game:**
  - Each player attempts to maximize its utility.
  - Players can communicate and try to reach a binding agreement.
  - A main solution concept: **Nash Bargaining Solution**
Nash Bargaining Scheme

- Informally, it’s an unique agreement scheme that fulfills the following conditions (Axioms):

  - **Individual Rationality**: Each player gets at least as much as in case of disagreement.
  - **Pareto Efficiency**: Nobody could be happier without hurting somebody else.
  - **Symmetry**: If the players are indistinguishable, the agreement should not discriminate between them.
  - **Invariance to Equivalent Payoff Representations**
  - **Independence of Irrelevant Alternatives**

*Each Bargaining Scheme has a disagreement point.*

*In our case: the NEP*
Game Theory in Networking

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  - Each player attempts to maximize its utility.
  - Players can communicate and try to reach a binding agreement.
  - A main solution concept: Nash Bargaining Solution

Revisit the agenda by standardizing a widely used operating point.

New Figure of merit: Price of Selfishness
We introduce a new concept: 
(The figure of merit for a cooperative game)

**Definition: Price of Selfishness (PoS)**

The Price of Selfishness is the ratio of the system performance at a NBS to the optimal system performance.

\[
PoS = \frac{J_{NBS}}{J_{opt}} \leq PoA = \frac{J_{NEP}}{J_{opt}}
\]
Price of Selfishness

The figure of merit for a cooperative game

\[
PoS = \frac{J_{NBS}}{J_{opt}} \leq PoA = \frac{J_{NEP}}{J_{opt}}
\]

How much smaller than the PoA??
Description of Model

- Set of users = \{1, 2, ..., N\}
- Each user has a demand: \( r^i \)
- \( R = \sum_i r^i \)
- Set of links = \{1, 2, ..., L\}
- Each user can split its demand among the links
Description of Model

- $f_l^i$ denotes the demand that player $i$ routes to link $l$.
- $f_i = \sum_i f_l^i$ is the total load of link $l$.
- $J_l^i$ : cost of user $i$ for sending part of its demand to link $l$.
- $J_l^i = J_l^i(f_l^i, f_l)$
  - Differentiable in its arguments
  - Monotonously increasing in its arguments
  - Convex in $f_l^i$
Description of Model

\[ J^i = \sum_l J^i_l \] is the total cost of user i for flowing its demand \( r^i \) according to its routing strategy under the current choice of routing strategies of the other players.

\[ J_{\text{System}} = \sum_{i=\{\text{players}\}} \sum_{l=\{\text{links}\}} J^i_l \]

Proven in [Orda, Rom, Shimkin,93]: This model has a unique NEP
1. Homogeneous:
   • All players have the same goals (the same cost per unit of flow to minimize)
   • e.g. they all attempt to minimize their delay

2. Heterogeneous
   • Players have different goals
   • e.g. one minimizes packet loss, the other delay.

Homogeneous players: \[ J_i^i = f_i^i \cdot T_i(f_i) \]

Heterogeneous players: \[ J_i^i = J_i^i(f_i^i, f_i) \]
Homogeneous Costs

All players have the **same goals** (the same cost per unit of flow to minimize)
- e.g. they all attempt to minimize their delay

- $J^i_l = f^i_l \cdot T_l(f_l)$
- $T_l(f_l)$ is positive, strictly increasing, convex, continuous differentiable
- $T_l(f_l) \leq T_n(f_n)$ if and only if $T_l'(f_l) \leq T_n'(f_n)$
  (e.g., M/M/1 model)
Theorem:
For homogeneous players (i.e. same costs per unit of flow on the links, the PoA can be unbounded, but the Price of Selfishness = 1.

\[ \text{PoS} = \frac{J_{NBS}}{J_{opt}} = 1 \]

Thus the Nash Bargaining Solution provides optimal system performance.
Homogeneous Costs

can get to the optimum of the system
Homogeneous Costs

All users see:

- **Green** = High Capacity
- **Yellow** = Medium Capacity
- **Red** = Low Capacity
HOMOGENEOUS COSTS

Constructively design a strategy profile (flow), whose corresponding cost vector is

(i) socially optimal
(ii) satisfies the five axioms of the NBS.
Homogeneous Costs

*How to get there?:*

Given the NEP, we can find the optimal NBS solution in $O(L \cdot N)$ time.
For heterogeneous players (i.e. different optimization objectives),
the Price of Anarchy (PoA) is unbounded
and the Price of Selfishness (PoS) is unbounded.
The problem here is not due to selfish behavior but rather due to the poor performance of the network, as seen from the perspective of user 2.

\[ J_{\text{system}} = \sum_{i=\{\text{players}\}} \sum_{l=\{\text{links}\}} J_i \]
How much might the performance of a user deteriorate, due to the selfish behavior of the other users, with respect to the case where all the traffic would be optimally controlled according to its own cost function?

What if player 2 controlled all traffic?
The Price of Heterogeneity of a user $i$:

$$PoH^i = \frac{\text{Cost experienced by that user at the NEP}}{\text{Social Cost it would experience according to its own perception}}$$

$$PoH^i \leq \frac{R}{r^i}$$

The $PoH^i$ could be interpreted as the $PoA$ “as seen by that user”.
Conclusions

• **Homogeneous Players:**
  
  • Through bargaining, the system optimum can be reached, while a lack of bargaining can be disastrous.

PoS = 1
PoA is unbounded!
Conclusions

• **Homogeneous Players:**
  - Through bargaining, the system optimum can be reached, while a lack of bargaining can be disastrous.

• **Heterogeneous Players:**
  - Bargaining might not help.

• **In a Heterogeneous Scenario,** You’d better be Big!

PoS and PoA are unbounded!
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