4.2 Calculation of the Flow Rate in a Pipeline

4.2.1 Concepts Demonstrated

Application of the general mechanical energy balance for incompressible fluids, and calculation of flow rate in a pipeline for various pipe diameters and lengths.

4.2.2 Numerical Methods Utilized

Solution of a single nonlinear algebraic equation and alternative solution using the successive substitution method.

4.2.3 Excel Options and Functions Demonstrated

Absolute and relative addressing, use of the “goal seek” tool, programming of the successive substitution technique.

4.2.4 Problem Definition

Figure 4–9 shows a pipeline which delivers water at constant temperature $T = 60$ °F from point 1 where the pressure is $p_1 = 150$ psig and the elevation is $z_1 = 0$ ft to point 2 where the pressure is atmospheric and the elevation is $z_2 = 300$ ft.

The density and viscosity of the water can be calculated from the following equations.

$$\rho = 62.122 + 0.0122T - 1.54\times10^{-4}T^2 + 2.65\times10^{-7}T^3 - 2.24\times10^{-10}T^4 \quad \textbf{(4-18)}$$

$$\ln\mu = -11.0318 + \frac{1057.51}{T + 214.624} \quad \textbf{(4-19)}$$

where T is in °F, ρ is in lb_m/ft^3, and μ is in $lb_m/ft \cdot s$.

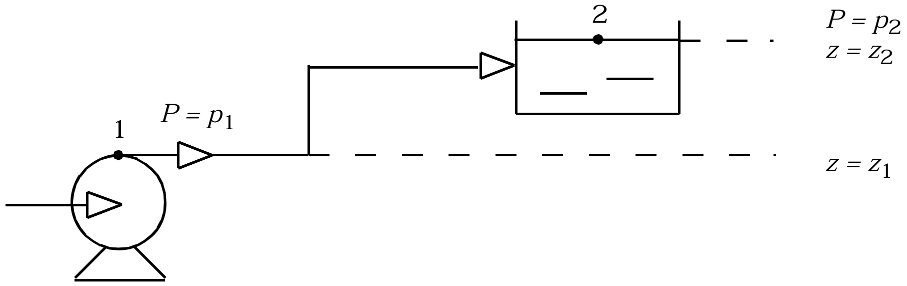

Figure 4–9 Pipeline at Steady State

(a) Calculate the flow rate q (in gal/min) for a pipeline with effective length of L = 1000 ft and made of nominal 8-inch diameter schedule 40 commercial steel pipe. (Solution: v = 11.61 ft/s, gpm = 1811 gal/min)
(b) Calculate the flow velocities in ft/s and flow rates in gal/min for pipelines at 60 °F with effective lengths of L = 500, 1000, ... 10000 ft and made of nominal 4, 5, 6 and 8-inch schedule 40 commercial steel pipe. Use the successive substitution method for solving the equations for the various cases and present the results in tabular form. Prepare plots of flow velocity v versus D and L and flow rate q versus D and L.
(c) Repeat part (a) at temperatures T = 40, 60 and 100 °F and display the results in a table showing temperature, density, viscosity, and flow rate.

4.2.5 Equations and Numerical Data

The general mechanical energy balance on an incompressible liquid applied to this case yields

$$-\frac{1}{2}v^2 + g\Delta z + \frac{g_c\Delta P}{\rho} + 2\frac{f_F L v^2}{D} = 0 \tag{4-20}$$

where v is the flow velocity in ft/s, g is the acceleration of gravity given by g = 32.174 ft/s^2, $\Delta z = z_2 - z_1$ is the difference in elevation (ft), g_c is a conversion factor (in English units g_c = 32.174 ft·lb$_m$/lb$_f$·s^2), $\Delta P = P_2 - P_1$ is the difference in pressure lb$_m$/ft^2), f_F is the Fanning friction factor, L is the length of the pipe (ft) and D is the inside diameter of the pipe (ft). The use of the successive substitution method requires Equation (4-20) to be solved for v as

$$v = \sqrt{\left(g\Delta z + \frac{g_c\Delta P}{\rho}\right) \Big/ \left(0.5 - 2\frac{f_F L}{D}\right)} \tag{4-21}$$

The equation for calculation of the Fanning friction factor depends on the Reynold's number, Re = $v\rho D/\mu$, where μ is the viscosity in lb$_m$/ft·s. For laminar flow (Re < 2100), the Fanning friction factor can be calculated from the equation

$$f_F = \frac{16}{\mathrm{Re}} \tag{4-22}$$

For turbulent flow (Re > 2100) the Shacham equation[5] can be used

$$f_F = 1/16\left\{\log\left[\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re}\log\left(\frac{\varepsilon/D}{3.7} + \frac{14.5}{Re}\right)\right]\right\}^2 \tag{4-23}$$

where e/D is the surface roughness of the pipe (ε = 0.00015 ft for commercial steel pipes).

The flow velocity in the pipeline can be converted to flow rate by multiplying it by the cross section are of the pipe, the density of water (7.481 gal/ft^3), and factor (60 s/min). Thus q has units of (gal/min). The inside diameters (D) of nominal 4, 5, 6 and 8-inch schedule 40 commercial steel pipes are provided in Appendix Table D-5.

4.2.6 Solution

(a) The problem is set up first for solving for one length and one diameter value with POLYMATH. The POLYMATH Nonlinear Algebraic Equation Solver is used for this purpose. It should be emphasized that Equation (4-21) (or Equation (4-20)) cannot be solved explicitly for the velocity in the turbulent region as in that region the friction factor is a complex function of the Reynolds number (and the velocity, see Equation (4-23)). Thus Equation (4-21) should be inputted as an "implicit" (nonlinear) equation. The implicit equations are entered in the form: f(x) = an expression, where x is the variable name, and f(x) is an expression that should have the value of zero at the solution. Bounds for the unknown x should be provided. Minimal and maximal values between which the function is continuous and one or more roots are probably located should be provided. For the velocity calculation, the following equation and bounds are used:

```
f(v) = v - sqrt((32.174 * deltaz + deltaP * 144 * 32.174 /
   rho) / (0.5 - 2 * fF * L / D)) # Flow velocity (ft/s)
v(min) = 1
v(max) = 20
```

Note that the program looks for a solution where f(v) = 0; thus, there is no need to write this out explicitly. The complete set of equations is shown in Table 4–3.

Table 4–3 Equation Input to the POLYMATH Nonlinear Equation Solver Program - File **P4-02A.POL**

Line	Equation
1	f(v) = v - sqrt((32.174 * deltaz + deltaP * 144 * 32.174 / rho) / (0.5 - 2 * fF * L / D)) # Flow velocity (ft/s)
2	fF = If (Re < 2100) Then (16 / Re) Else (1 / (16 * (log(eoD / 3.7 - 5.02 * log(eoD / 3.7 + 14.5 / Re) / Re)) ^ 2)) # Fanning friction factor (dimensionless)
3	eoD = epsilon / D # Pipe roughness to diameter ratio (dimensionless)
4	Re = D * v * rho / vis # Reynolds number (dimesionless)
5	deltaz = 300 # Elevation difference (ft)
6	deltaP = -150 # Pressure difference (psi)
7	T = 60 # Temperature (deg F)
8	L = 1000 # Effective length of pipe (ft)
9	D = 7.981 / 12 # Inside diameter of pipe (ft)
10	pi = 3.1416 # The constant pi
11	epsilon = 0.00015 # Surface rougness of the pipe (ft)
12	rho = 62.122 + T * (0.0122 + T * (-1.54e-4 + T * (2.65e-7 - T * 2.24e-10))) # Fluid density (lb/cu. ft.)
13	vis = exp(-11.0318 + 1057.51 / (T + 214.624)) # Fluid viscosity (lbm/ft-s)
14	q = v * pi * D ^ 2 / 4 * 7.481 * 60 # Flow rate (gal/min)
15	v(min) = 1
16	v(max) = 20

The solution obtained by POLYMATH is the same as specified in the problem statement (v = 11.61 ft/s, q = 1811 gal/min).