Numerical Experiments in Fluid Mechanics with a Tank and Draining Pipe

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ABSTRACT

This article emphasizes the use of interactive simulation packages as an integral part of the problem solution in both engineering education and practice. A homework problem in fluid mechanics is presented. Using numerical simulation of a tank and draining pipe arrangement, the students were required to find the effect of the viscosity of the fluid, draining pipe length, and initial fluid level in the tank on the initial draining velocity and the time required for complete draining of the tank. The results obtained were surprising and unexpected, encouraging the students to investigate further the model equations in order to find an explanation for the unexpected system behavior. © 1995 John Wiley & Sons, Inc.

INTRODUCTION

Computer-related exercises and assignments have been given to engineering students for about 25 years. The objective of these assignments is to teach the students to use numerical and optimization methods and computer programming in order to solve problems where either an analytical solution is not available, or it requires too many simplifying assumptions which may render very restricted or even misleading results.

Since all the engineering students have been exposed to computer exercises, we could have expected that nowadays most practicing engineers would use the computer to solve more complicated problems. A recent survey by the CACHE (Computer Aids for Chemical Engineering) Corporation among practicing chemical engineers [1] has indicated that <20% of them do any programming and <3% use numerical methods or mathematical libraries and packages. This is in comparison to about 97% who use spreadsheets programs on the computer. The interpretation of this survey was that among the engineers there is a small group of "professional" computer users who use the computer in very advanced and sophisticated way, while the great majority use only the simplest computer tools.

These results indicate clearly that computer-related education has not been successful so far in reaching the majority of the engineers, and a new approach is required in order to reach the ones who are still not comfortable with the use of computers. There are several interactive numerical simulation and analysis packages available today (for example, MAPLE [2], MATLAB [3], and POLYMATH [4]) which can serve as a basis for a new approach in computer-related education.

Here, we use a problem in fluid mechanics to demonstrate the new approach proposed by us.

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Fluid mechanics is a course being taught in most engineering disciplines (aerospace, agricultural, chemical, civil, environmental, mechanical, naval, etc.) so that the example presented here can be of interest to most readers.

The first widely known computer-related problems and assignments in fluid mechanics were included in the classical textbook by Carnahan et al. [5]. Detailed computer assignments including solutions and FORTRAN programs can be found in a CACHE publication from 1972 [6]. Typically, 3 weeks of elapsed time was allowed for the completion of an assignment. Most of this time was spent on programming and debugging the program. Since the computer assignment was so demanding, only the most complicated and difficult problems were worthy of being solved with the help of a computer.

The emergence of the user-friendly, interactive computer simulation packages has changed the situation considerably. With the help of these packages, there is no need to code and debug a program and recode well-known numerical algorithms. Consequently, the same "3-week" computer assignment can be completed in one afternoon. The "user friendliness" of these new programs makes the computer solution appealing to all students, and the change of time scale allows the use of computer differently from the way that it was used in the past.

The computer can be made an integral part of the solution of most problems, not just the very difficult ones. This approach has been already adopted in certain areas (such as chemical reaction engineering [7]) but not yet in fluid mechanics courses.

The new software tools offer the possibility to learn by simulation. In laboratory and research work, we first detect and observe a physical phenomenon, and then attempt to explain it. This is the natural order of learning. In the classroom the order is reversed. First, a mathematical model is presented. At this time, its physical significance is not clear to the student. Instead of observing the physical behavior of the system, the student must attempt to predict the behavior from mathematical analysis of the model equations.

It is possible to return to the natural order of learning by carrying out simulated "experiments." The student first carries out "experiments" in order to investigate a phenomenon and "discovers" the different aspects involved. Investigation of the mathematical model by analytical means can verify the results of the "experiments." The observations made in those experiments may then be analyzed in view of the mathematical model.

To demonstrate the proposed new approach, we will use a "tank-draining" problem. This is a fairly simple problem but yields unexpected and intriguing results. This problem is based on the following question that was presented in a letter to the editor of the *Chemical Engineering* journal [8]: "Two 2-inch pipes extend from the bottom of an open tank. One is 4 ft. long and the other 10 ft. long. Both are open at the ends. Which, if either, will drain the tank faster? . . . This problem may seem simple, but out of about 50 engineers concerned, the opinion was almost equally divided."

Any of the general-purpose numerical software packages (such as MAPLE [2], MATLAB [3], etc.) can be used for solving the tank-draining problem. We have used the POLYMATH [4] package, which we found appropriate for educational use because of the ease of learning and user friendliness [9].

The POLYMATH software package was originally developed for the mainframe Plato educational computer system [10]. The current version of POLYMATH (3.0 PC) is distributed by the CACHE Corporation, a nonprofit organization that disseminates educational computer programs to chemical engineering department. This version runs on IBM PCs, PS/2s, and compatibles. The package includes programs for solving nonlinear algebraic and ordinary differential equations, curve fitting, and data correlation. POLYMATH has been widely used in chemical engineering education, mostly in courses of chemical reaction engineering [7,11] and process control [12].

THE "DRAINING TANK" ASSIGNMENT

Figure 1 shows the tank with draining pipe arrangement which is available for "experimentation." The tank is a cylindrical tank of diameter D with a flat bottom. The initial height of the liquid level above the draining pipe exit is H_0 and the final height is H_f. The tank is equipped with a heating coil so that the temperature, and thus the kinematic vis-

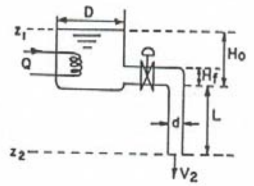

Figure 1 Tank with draining pipe.

cosity of the fluid inside, can be changed. The draining pipe diameter is d and its length L can be changed from 1 in to 12 ft by adding and removing sections of pipe. It is assumed that the valve on the draining pipe provides negligible resistance to the flow.

The following numerical data are recommended for use during the experimental runs: D = 3 ft, H_0 = 3 ft, and H_f = 1 in. The draining pipe is a nominal 0.5-in schedule 40 steel pipe with roughness ϵ = 0.00015. Two different fluids can be used for the experiments: water at 60°F (kinematic viscosity $\nu = 1.22 \times 10^{-5}$ ft²/s) and hydraulic fluid (MIL-M-5606). The kinematic viscosity of the hydraulic fluid is 41.1×10^{-5} ft²/s at 30°F and 20.9×10^{-5} ft²/s at 60°F [13, p. 775].

The "draining tank" apparatus can be used to demonstrate several different concepts in fluid mechanics. Some possible assignments follow.

Model Equations

Develop the model equations to express the exit velocity V_2 and the liquid height H as function of the dimensions of the tank, the draining pipe, and the elapsed time. Consider both laminar and turbulent flows.

For turbulent flow, use the following explicit equation [15]:

$$\frac{1}{\sqrt{f_D}} = -2 \log\left[\frac{\epsilon}{3.7d} - \frac{5.02}{Re} \times \log\left(\frac{\epsilon}{3.7d} + \frac{14.5}{Re}\right)\right], \tag{1}$$

where $Re = vd/\nu$ is the Reynolds number and f_D is the friction factor. Assuming a constant friction factor, show that the total draining time (t_f in seconds) is given by:

$$t_f = \frac{D^2}{d^2}\sqrt{\frac{2}{g}\left(1 + \frac{f_D L}{d}\right)} \times (\sqrt{H_0 + L} - \sqrt{H_f + L}). \tag{2}$$

Change of Initial Velocity and Friction Factor As a Function of L and H_0

To study the variation of the initial exit velocity and friction factor as a function of the draining pipe length and initial liquid level, carry out "experimental" runs in the following range of the variables: 1 in $\le H_0 \le$ 3 ft, 1 in $\le L \le$ 12 ft. The experiments are to be carried out with both water and hydraulic fluid. Measure and plot the initial velocity as function of L with H_0 as a parameter. Can you observe any change in the trend? Use the model equations to explain such a change. Is the use of Equation (2) justified for predicting the draining time in these experiments?

Discuss the difficulties that can be encountered if the flow is in the transition region ($2100 \le Re < 4000$).

Note: For the "experimental" assignments you can use a general-purpose interactive software package such as MAPLE [2], MATLAB [3], or POLYMATH [4] to solve the model equations.

Draining Time for Laminar Flow

Based on result of the friction factor variation in the laminar region, derive an expression (similar to Equation [1]) for total draining time in laminar flow.

Draining Time Studies

Measure and plot draining times for the following range of the variables: 1 in $\le L \le$ 12 ft, 1 ft $\le H_0 \le$ 6 ft using both water and hydraulic fluid. Explain your results.

SOLUTIONS

Model Equations

The equations representing the tank during draining are fairly simple and have been discussed in detail elsewhere (see, for example, Sommerfeld [14]).

Apply the Bernoulli equation to points 1 and 2 in Figure 1, assuming that $V_1 = 0$ and that $P_1 = P_2$ yields:

$$Z_1 = \frac{V_2^2}{2g} + Z_2 + h_f. \tag{3}$$

Using the Darcy friction factor (f_D) for the draining pipe gives:

$$h_f = \frac{f_D L}{d}\frac{V_2^2}{2g}, \tag{4}$$

Substituting Equation (4) into Equation (3) and defining $H + L = Z_1 - Z_2$, Equation (3) can be rearranged to obtain an expression for V_2:

Table 1 Summary of Results of Initial Velocity Change as Function of Pipe Length for Water at 60°F*

Variable	Initial Value	Max. Value	Min. Value	Final Value
t	0.0	143.00	0.0	143.00
v	13.784	13.784	11.324	11.324
l	0.0833	12.000	0.0833	12.000
eps	0.150×10^{-3}	0.150×10^{-3}	0.150×10^{-3}	0.150×10^{-3}
d	0.0518	0.0518	0.0518	0.0518
et	0.782×10^{-3}	0.782×10^{-3}	0.782×10^{-3}	0.782×10^{-3}
re	0.586×10^{5}	0.586×10^{5}	0.481×10^{5}	0.481×10^{5}
f	0.0281	0.0285	0.0281	0.0285
H	3.0000	3.0000	3.0000	3.0000

* Notation used as in Appendix A.

$$V_2 = \sqrt{\frac{2g(H+L)}{1+f_D L/d}}. \qquad (5)$$

Equation (5) looks like an explicit equation for V_2. This is not true, however, as the friction factor is a function of the draining fluid's Reynolds number, and thus a function of V_2. In the turbulent region, Equation (1) can be used for calculating the friction factor. For laminar flow, the pressure drop and flow rate are related by the Hagen-Poiseuille equation [13, p. 204], which yields the following expression for the friction factor:

$$f_D = \frac{64}{Re}. \qquad (6)$$

The change of the liquid level in the tank can be obtained from dynamic material balance on the tank's content:

$$\frac{dH}{dt} = -\frac{d^2}{D^2} V_2. \qquad (7)$$

Assuming a constant friction factor, the expression for V_2 from Equation (5) can be introduced into Equation (7). Integration from H_0 to H_f yields Equation (2).

Change of Initial Velocity and Friction Factor

Calculation of the initial velocity and friction factor involves simultaneous solution of Equations (1) and (5) for turbulent flow or Equations (6) and (5) for laminar flow. In terms of a numerical solution, this constitutes a solution of a single nonlinear algebraic equation. The equations as they should be entered into the POLYMATH algebraic equation solver are shown in Appendix A.

In order to obtain the plot of V_2 versus L, or f_D versus L, for a constant value of H_0, it is more convenient to change L continuously and calculate V_2 and f_D along the way. A differential equation for continuously changing L from $L = 1$ in up to $L = 12$ ft (such as $dL/dt = 1/12$) should be added to the system of equations. Addition of a differential equation to a single nonlinear algebraic equation converts the problem, in numerical terms, into the solution of a differential-algebraic system. There are several ways to solve such a system; the simplest and most easily understood by fluid mechanics students involves the conversion of the implicit algebraic equation into a differential equation by differentiation.

Differentiating Equation (5) with respect to L, assuming a constant value of f_D (turbulent flow), yields:

$$\frac{dV_2}{dL} = \frac{gd(d - f_D H)}{V_2(d + f_D L)^2}. \qquad (8)$$

For laminar flow, $f_D = 64/Re$ and the expression obtained is:

$$\frac{dV_2}{dL} = \frac{2gd(d - f_D H)}{V_2(d + f_D L)(2d + f_D L)}. \qquad (9)$$

Since $dV_2/dt = (dV_2/dL)(dL/dt)$, this equation can be integrated simultaneously with the differential equation describing the change of L with time, to yield the change of V_2 associated with the continuous variation of the pipe length.

The equations' initial and final values, as they should be entered into the POLYMATH differential equation solver, are shown in Appendix B. The numerical values of the various constants

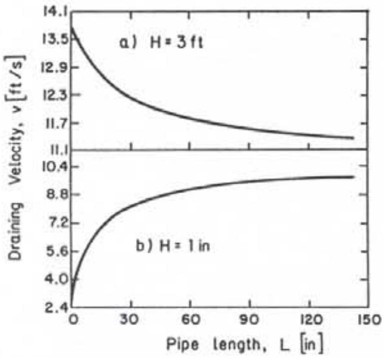

Figure 2 Exit velocity as a function of pipe length.

correspond to water at 60°F and $H = H_0 = 3$ ft. Table 1 summarizes the initial, maximum, minimum, and final values of all the variables from $L = 1$ in up to $L = 12$ ft.

It can be seen from the table that the friction factor changes very little (from a value of $f_D = 0.0281$ for $L = 1$ in, it increases to $f_D = 0.0285$ for $L = 12$ ft) as a result of the decrease in the initial draining velocity with an increase in the length of the draining pipe.

Repeating the calculations for $H_0 = 1/12$ ft shows that the friction factor does not change very much in this case either. Its value for the shortest pipe length is $f_D = 0.0332$ and for the longest, $f_D = 0.0288$. Thus, f_D in the range of draining velocities obtained in these experiments can be expressed as $f_D = 0.03065 \pm 0.00255$.

The results for the draining velocities in the two cases corresponding to $H = 3$ ft and $H = 1$ in deserve further attention. These velocities are shown in Figure 2. It can be seen that for $H = 3$ ft, the velocity decreases with increasing pipe length, while for $H = 1$ in the velocity increases.

This change of trend can be explained with reference to Equation (8). It can be seen from this equation that the sign of dV_2/dL is determined by the sign of the term: $d - f_D H$. When $f_D H > d$, the velocity will decrease with increasing the pipe length, while for $f_D H < d$ the velocity will increase with increasing length. It is obvious that there must be a critical value of H for which the velocity will not change with increasing the draining pipe length. This critical height is: $H_C = d/f_D$. Using the data in Table 1 yields $H_C = 1.81$ ft.

When the "experiments" for observing the change in the initial draining velocity are carried out with the hydraulic fluid at 30°F (kinematic viscosity $\nu = 41.1 \times 10^{-5}$ ft^2/s), the flow in the pipe is laminar for the whole range of H and L investigated. Nevertheless, the variation of the velocity is very similar to what is shown in Figure 2. Table 2 summarizes some of the results for this case. It can be seen that the friction factor changes very significantly between the start and the end of the draining. For example, for a draining pipe of 1 in, the friction factor changes from $f_D = 0.0371$ at the start to $f_D = 0.1754$. This is over a fourfold increase, and it is obvious that neglecting the change of the friction factor will cause a very significant error in calculating the draining period.

The change of the initial flow rate with increasing pipe length is very similar to what is shown in Figure 2. Obviously, there is a critical value for H also in laminar flow. Equation (9) reveals that the expression for H_C is the same as that obtained for turbulent flow. For the case studied we find that $H_C = 0.68$ ft, where the draining velocity is $V_2 = 6.57$ ft/s and the friction factor is $f_D = 0.0077$.

For observing the system behavior at the transition region, hydraulic fluid at 60°F with a kinematic viscosity of 20.5×10^{-5} ft^2/s is used. It turns out that for this fluid the initial flow is in the transition region ($2100 \leq Re < 4000$) for $H = 3$ ft and in the laminar region for $H = 1$ in. In the transitional region, the friction factor can get significantly reduced (by half or more) when the flow regime transition from turbulent to laminar takes place. If the system is modeled based on the assumption that the flow changes from turbulent to laminar at some specific

Table 2 Summary of Results for the Change of Initial Velocity as a Function of Pipe Length for Hydraulic Fluid at 30°F

	$H = 3$ ft		$H = 1$ in	
	$L = 1$ in	$L = 12$ ft	$L = 1$ in	$L = 12$ ft
V_2 (ft/s)	13.689	7.7118	2.8935	6.2886
Re	1726.4	972.58	364.91	793.08
f_D	0.0371	0.0658	0.1754	0.0807

value of the Reynolds number (for example at Re = 2100), the numerical solution oscillates between laminar and turbulent flow once the transitional point is reached. When the Reynolds number becomes a little less than 2100 and the flow becomes laminar, f_D is reduced to, say, half of its previous value, resulting in increased flow rate and Reynolds number, corresponding to turbulent regime. This in turn increases f_D, thus decreasing the velocity and the Reynolds number back to the laminar regime. These oscillations may happen in practice. Moody [16], for example, indicates that in the critical zone the flow is pulsating rather than steady.

Draining Time for Laminar Flow

The draining time for laminar flow can be calculated by introducing the expression $f_D = 64/Re$ into Equation (5). Substituting V_2 into Equation (7) and integrating the differential equation between $H = H_0$ and $H = H_f$ yields the draining time for laminar flow, $t_{f(L)}$:

$$t_{f(L)} = \left(\frac{D}{d}\right)^2 \frac{\alpha L}{2g}\left(Y_f - Y_0 + \ln\frac{Y_0}{Y_f}\right),$$

where

$$\alpha = \frac{64\nu}{d^2},$$
$$Y_0 = 1 - \sqrt{1 + \frac{8g(H_0 + L)}{\alpha^2 L^2}}, \qquad (10)$$

and

$$Y_f = 1 - \sqrt{1 + \frac{8g(H_f + L)}{\alpha^2 L^2}}.$$

Another approach for calculating t_f in laminar flow is to use Equation (2) with an average value of the friction factor, $f_{D,avg.}$. It can be verified "experimentally" that when the geometric average friction factor $f_{D,avg.}$ is used, the error in calculating t_f using Equation (2) never exceeds 0.5%. The geometric-average friction factor is defined as:

$$f_{D,avg.} = \sqrt{f_{D,0} \times f_{D,f}}, \qquad (11)$$

where $f_{D,0}$ is the friction factor calculated using the draining velocity when $H = H_0$ and $f_{D,f}$ is the one calculated based on the velocity at $H = H_f$.

Draining Time Studies

Studies of the friction factor variation revealed that for water at 60°C the change of the friction factor during draining is negligible, and Equation (2) can be used to calculate the draining time. To do the actual calculation, Equation (2) can be added to the set of equations given in Appendix B.

Figure 3 shows the draining time as a function of the draining pipe's length, for an initial water level of H_0 = 3 and 6 ft. It can be seen that for H_0 = 3 ft the draining time decreases continuously. It is 1150 s for L = 1 in and 913 s for L = 1/12 ft.

This can be understood by recalling that the critical height in this case is H_C = 1.81 ft. Below this critical height, the draining velocity increases with increasing the pipe length, but above this value it decreases. When H_0 = 3 ft, the larger portion of the water mass is below H_C; thus, the increasing trend of the draining velocity with lengthening the pipe is dominant, resulting in a net decrease in the draining time with pipe lengthening. If this argument is correct, increasing the water level above H_C should lead to a change in this trend. Indeed, for H_0 = 6 ft, the shortest draining pipe gives t_f = 1755.9 s. The draining time goes down to t_f = 1658.8 s for L = 15 in and increases again to t_f = 1758.2 s for L = 12 ft.

The existence of the minimum in Figure 3b can be explained with reference to Figure 2 and to the

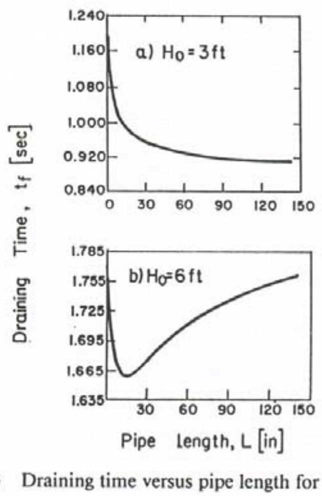

Figure 3 Draining time versus pipe length for water.

expression of dV_2/dL in Equation (8): dV_2/dL is much larger for small values of V_2. Therefore, for short pipes the slope of the increasing velocity below H_C is much larger than the slope of the decreasing velocity above H_C. This will make the velocity increase dominant for a certain range of pipe lengths. With a further increase in the pipe length, the draining time may continue to decrease or start to increase, depending on the initial difference between H_0 and H_C. The decreasing trend in the draining velocity can be made dominant, thus completely eliminating the minimum by further increasing H_0.

For draining time studies involving hydraulic fluid, Equation (10) should be used instead of Equation (1). Figure 4 shows the variation of the draining time with the pipe length for the hydraulic fluid at 30°F for two different values of H_0. As expected, the shapes of the curves are very similar to those obtained for turbulent flow in the pipe. With $H_0 = 1$ ft, there is a monotonic decrease of the draining time for increasing pipe length, while for $H_0 = 2$ ft, there is a marked minimum at about $L = 1$ ft.

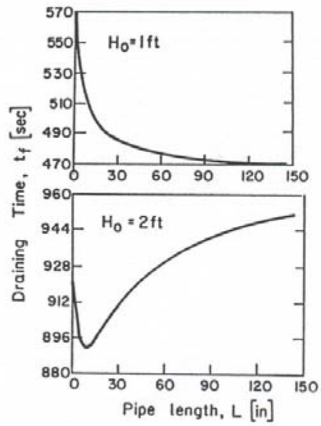

Figure 4 Draining time versus pipe length for hydraulic fluid.

Which One of the Pipes Drains the Tank Faster?

Getting back to the question in the letter to the editor of the *Chemical Engineering* journal—which motivated this exercise—the reason for the disagreement among the engineers answering the question is quite obvious now. Based on the results of this study, the answer should be, "it depends." Whether the shorter or longer pipe will drain faster depends on the initial level and physical properties of the liquid as well as the pipe length.

CLASSROOM IMPLEMENTATION

The tank-draining problem was given as a homework assignment to a class of about 150 chemical engineering students during a fluid mechanics course at the University of Michigan. Most of these students learned to use the POLYMATH software in an introductory chemical engineering course, which was given in the semester preceding the fluid mechanics course. The introduction included two 1-hour lectures. After this introduction, the students were given several homework assignments that required the use of different programs in the POLYMATH package: solution of ordinary differential and nonlinear algebraic equations and polynomial and multiple linear regression. From this point onward, the students were expected to use the software on their own, without additional help. The students can get a copy of the software free of charge and install it on their own computer, but at the University of Michigan most students do not have their own computer and they have to use the public computer laboratories.

The "tank-draining" assignment was given to the students about a month before the end of the semester as part of a computer assignment, which included calculation of flow rates in a pipeline network and the trajectory of a baseball in addition to the "tank-draining" assignment.

The complete computer assignment turned out to be too demanding, and the students expressed doubts in being able to complete it. The main reason for the difficulty was that toward the end of the semester the computer laboratories were very crowded and there was a long wait for the use of a computer. Since the tank draining assignments consists of several parts, we could make it easier by removing parts of it, in light of the technical difficulties.

No formal evaluation of the students' attitude toward the type of question presented and the software used was carried out. However, we can quote from such an evaluation that was carried out by Howard [17] after exposing junior engineering students to the use of both the Lotus 1-2-3 spreadsheets program and POLYMATH. The response of the overwhelming majority of the students was in favor of the use of POLYMATH. A typical response from one of the students in this group was:

> After repeated disasters with Fortran 77 programming, I was anxious to try a different type of com-

puter work. The two types of software, Lotus 1-2-3 and POLYMATH we learned, actually gave me hope that I wasn't as computer illiterate as I once thought when doing Fortran. . . .

Very few students were resentful to the restrictions being put on them by the "user-friendly" programs, which are more limited in the options they offer compared to using a programming language. A typical response from this group of students was:

. . . It is difficult to master the use of these two new programs (Lotus 1-2-3 and POLYMATH) because they are limited to the programs already written in them. Fortran has more computing capability. . . .

CONCLUSIONS

A new approach for the use of computers in fluid mechanics courses is presented. According to this approach, the use of an interactive numerical software and simulation package becomes an integral part of the solution of a problem. This new approach is demonstrated by presenting a problem for which the intuitive answer can be controversial. The problem is investigated by carrying out simulated "experimental" runs. The model equations are used to critically analyze and explain the experimental results.

We expect that the new approach to computers will be beneficial in two respects: a) Learning by simulation is a more interesting and effective way for understanding the subject matter; b) the sophisticated use of computers in education will affect a more comprehensive and sophisticated use of computers by all practicing engineers.

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APPENDIX A

Calculation of the Initial Exit Flow Velocity for Water at 60°F

```
(1) f(v) = ((2*32.2*(h+L))/(1+f*L/
    d))**0.5-v
(2) L = 1/12
(3) eps = 0.00015
(3) d=0.622/12
(4) et=eps/(3.7*d)
(5) re=v*d/(1.22*10**(-5))
(6) f=1/(-2*log(et-5.02*log(et
    +14.5/re)/re))**2
(7) h=3
    v(min)= 2, v(max) = 15
```

—To change pipe length change the value of L in Equation (2)

—To solve for a different fluid or temperature change the kinematic viscosity in Equation (5)

—For laminar flow replace Equation (6) by f=64/re

—For flow in the transient region (2100<=Re<4000) add the equation:

```
(8) fc=(re-2100)*f/1900+((4000-re)/
    1900)*(64/2100)
```

and change f in the right hand side of Equation (1) to fc.

APPENDIX B

Calculation of Initial Velocity as Function of Pipe Length for Water at 60°F

```
(1) d(v)/d(t)=32.2*d*(d-f*H)/
    ((d+f*L)**2*v)*(1/12)
(2) d(L)/d(t)=1/12
(3) eps=0.00015
(4) d=0.622/12
(5) et=eps/(3.7*d)
(6) re=v*d/(1.22*10**(-5))
(7) f=1/(-2*log(et-5.02*log(et
    +14.5/re)/re))**2
(8) H=3
t(0)=0, v(0) = 13.784, L(0) = 0.08333
t(f)= 143
```

—Note: any change of data or equation requires changing the initial velocity v(0)

—To solve for a different fluid or temperature change the kinematic viscosity in Equation (6)

—For laminar flow replace Equation (7) by the equation f=64/re

—To calculate the draining time as function of the pipe length add the following equation:

```
(9) tf=3349.8*(2*(f*L/d+1)/
     32.2)**.5*
     ((H+L)**.5-(1/12+L)**.5)
```

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