The object of this column is to enhance our readers' collections of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested, as well as those that are more traditional in nature and which elucidate difficult concepts. Please submit them to Professor James O. Wilkes (e-mail: wilkes@engin.umich.edu) or Mark A. Burns (e-mail: maburns@engin.umich.edu), Chemical Engineering Department, University of Michigan, Ann Arbor, MI 48109-2136.

THE WIND-CHILL PARADOX

Four Problems in Heat Transfer

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Problems involving wind chill came into our consideration when we were looking for interesting exercises in heat transfer. The wind-chill index (or wind-chill temperature) is regularly reported by meteorologists during the winter months, and its purpose is to represent by a single number the combined effects of low temperature and high wind velocity on human comfort and tolerance to the cold. When we proceeded to calculate the wind-chill temperature using a model based on heat-transfer principles, we obtained results that differ significantly from those announced by the weatherman.

Trying to find the reason for this discrepancy, we stumbled upon what we call the "wind-chill paradox." The model used by weathermen to calculate the wind-chill temperature is empirical and contains many apparent inaccuracies and inconsistencies. On the other hand, the wind-chill index has been in use for over forty years, and as the ASHRAE handbook[1] states, "This index has provided a reliable way of expressing combined effects of wind and temperature on subjective discomfort and has proven useful for ordering the relative severity of environment." But since the wind-chill index and temperature are not measurable quantities, their validity cannot be verified by experimental results.

The popularity of the wind-chill index and its importance in everyday life on one hand, and its imprecise definition, empirical and non-measurable nature on the other hand, can make it a very interesting subject to explore. The concept is simple enough so that it can be investigated using tools available to undergraduate engineering students.

In this paper we present four problems related to wind chill. Most of the numerical calculations involved in solving the four assignments can be carried out using either a spreadsheet or a numerical computation package such as Matlab or Polymath.

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PROBLEM STATEMENT

The wind-chill index (WCI), originated by Siple and Passel,[2] can be defined as the instantaneous rate of heat loss from bare skin at the moment of exposure. Siple and Passel developed a correlation for the WCI based on measurements of the freezing rate of water in a sealed cellulose acetate cylinder (length 5.875 in., diameter 2.259 in.) suspended on a pole above the roof of a building. These measurements were carried out in Antarctica in 1941 in the range of ambient temperatures -56°C < T_a < -9°C and wind velocities 1 m/s < V < 15 m/s.

Siple and Passel measured the time required for complete freezing of the water inside the cylinder in addition to measurements of ambient temperature and wind velocity. Based on these measurements, they calculated what they called the "Wind-Chill Factor (WCF)":

$$WCF = \frac{\Delta H / t}{(T_f - T_a)} \quad (1)$$

where

ΔH latent heat of melting for 1 liter of water (79.71 kcal/kg)

t total freezing time (hr)

T_f freezing temperature of the water (0°C)

Considering the method for calculating WCF, it is obvious that it represents an overall heat-transfer coefficient (U), which includes heat-transfer resistance of the cylinder wall and the air. Siple and Passel correlated WCF versus the wind velocity, and the best correlation they obtained was WCF=$\left(10\sqrt{V}+10.45-V\right)$.

Based on this correlation, the following expression for the WCI was suggested:

$$WCI = \left(10\sqrt{V}+10.45-V\right)(T_s - T_a) \quad (2)$$

where WCI is the wind-chill index (kcal/h-m^2) and T_s = 33°C is the exposed skin temperature. It can be seen that WCI represents the rate of heat loss from the human body, based on a skin temperature of 33°C, which is considered a neutral, most comfortable skin temperature.

This correlation is being used by weathermen in the range of

$$-90°C < T_a < 30°C \quad \text{and} \quad 0.05\ m/s < V < 25\ m/s$$

TABLE 1
Characteristic Thermal and Transfer Properties of the Human Body

Variable	*Value*
Body's heat-transfer resistance r_b	0.08 (°C - m^2 - hr)/kcal
Body's characteristic diameter (face) D	15 cm
Body's characteristic diameter (finger) D	2 cm
Body's temperature	36.7°C
Neutral skin temperature[7] T_s	33°C
Metabolic heat production, resting[7]	50 kcal/(hr - m^2)
Matabolic heat production, walking[7]	100 kcal/(hr - m^2)
Metabolic heat production, running[7]	400 kcal/(hr - m^2)
Thermal diffusivity of the skin tissues,[10] α	10^{-7}m^2/s
Thermal conductivity of the skin tissues,[10] k	1 W/(m - °C)
Heat input from solar radiation to bare skin[7]	$q_{sr} = 144\left(1-C_1^{1.33}\right),\ 0 < C_1 < 1$
C_1 represents a correction factor for cloud cover	

Problem 1
Calculation of Momentary Heat Loss

Derive an equation for calculating the WCI based on heat-transfer principles. Calculate and plot the WCI in the range of

$$1.34\ m/s \le V \le 35\ m/s \quad \text{and} \quad -60°C \le T_a \le 10°C,$$

using Siple and Passel's correlation (Eq. 1) and your model. For thermal and transport properties of the human body, use the pertinent data from Table 1. For the characteristic diameter of the body use D = 15 cm (face). Compare results from your model with those of Siple and Passel. Comment on the validity of the extrapolation when Siple and Passel's correlation is used beyond the range of the experimental data.

Solution The momentary heat loss per unit area (q_s) from exposed surface of the skin is

$$q_s = h_c(T_s - T_a) \quad (3)$$

where T_s is the skin temperature (°C) and h_c is the forced convection heat-transfer coefficient (kcal/°C-m^2-h). For the range of wind velocities indicated in the question, the equation proposed by Eckert and Drake[4] for a body of a cylindrical shape can be used:

$$Nu = 0.25\,Re^{0.6}\,Pr^{0.38} \qquad Re > 1000 \quad (4)$$

where

$Nu = h_c D / k_f$

$Re = VD/\upsilon_f$

$Pr = \upsilon_f / \alpha_f$

D = diameter of the cylinder (m)

k_f = air thermal conductivity (kcal/h-m-s)

α_f = thermal diffusivity (m^2/s)

υ_f = kinematic viscosity (m^2/s)

The values of WCI (Eq. 2) and q_s (Eqs. 3-4) are plotted in Figure 1 versus the wind velocity for different ambient temperatures. Consistent with Siple and Passel, T_s = 33°C is used in Eq. (3). Therefore, q_s (and WCI) represents the instantaneous heat loss rate from bare skin at the moment of exposure when its temperature is at the neutral, comfortable level. Equations were fitted to data of air thermal conductivity and air viscosity given in Welty, *et al.*,[5] in order to calculate these properties at the average temperature between T_s and T_a. The set of equations for calculating pertinent physical and transfer properties of air in the region of interest is shown in Table 2.

Figure 1 shows that the two plots are very different. The rate of rise of q_s with the wind velocity is higher than that of WCI, which passes a maximum at about V = 25 m/s and diminishes for higher wind velocities. There is no maximum for q_s. Explaining the reasons for the difference between the results can be a basis for a stimulating class discussion after the wind-chill homework assignment is completed.

As indicated earlier, WCF represents an overall heat-transfer coefficient, which includes the heat-transfer resistance of the cylinder wall and the air. In calculating WCI (Eq. 2), the human skin temperature (33°C) is used (and not the body temperature, 37°C). Thus, in this equation the WCF is used as if it includes only the heat-transfer resistance of the air, ignoring the heat-transfer resistance of the human body. The body's resistance at low wind velocities may turn out to be negligible, but it becomes the controlling resistance at high wind velocities.

In order to show that this mix-up between U and h is the main reason for the difference between Siple and Passel's correlation (Figure 1) and the results obtained based on heat-transfer principles, we have correlated Siple and Passel's experimental data using

$$U = \frac{1}{r_d + 1/h_c}; \qquad \frac{h_c D}{k_f} = C R_e^m \tag{5}$$

where

r_d thermal resistance of the cylinder (°C-m^2-hr-kcal^{-1})
D diameter of the cylinder
C and m constants

The values for the constants (r_d, C, and m) are found by correlating the data of Siple and Passel. The optimal numerical values of the constants, obtained by nonlinear regression (numbers rounded to three decimal digits) are r_d = 0.0202, C = 0.823, and m = 0.535.

Figure 2 shows the momentary heat loss, q_s, calculated when h_c in Eq. (2) was replaced by U as obtained from Eq. (5) and values of WCI as calculated from Eq. (2). It can be seen that in this case the values of q_s and WCI are very similar, except that q_s continues to rise monotonously with increasing wind velocity. The unreasonable behavior of Siple and Passel's WCI correlation above wind speeds of about 25 m/s reflects the consequences of extrapolating an empirical correlation beyond the range of wind velocities where measurements were carried out. No measurement was taken above V = 15 m/s.

Additional obvious inaccuracies in using Siple and Passel's WCI result from ignoring the difference between the thermal resistance of the plastic cylinder and that of the human body and the effect of the body geometry on the heat-transfer coefficient. The use of WCF in Eq. (2) implies that the difference between the geometries of a human body and the experimental cylinder and the temperature dependence of the heat-transfer coefficient are both neglected.

At this point, questions will probably arise: How, in spite

TABLE 2
Viscosity, Thermal Conductivity, and Density of Air in the Pertinent Temperature Range

Density (from ideal gas law)

$\rho = 353.603/T$ where T = average temperature (°K)
ρ = density of the air (kg/m^3)

Thermal conductivity (from polynomial fit to data from Welty *et al.*[5])

$k = (0.0035 + 8.809\times10^{-5}T - 3.5\times10^{-8}T^2)/4186.8$
where k = thermal conductivity (kcal/s-m-°K)

Viscosity (from curve fit to data from Welty, *et al*[5])

$\mu = 1.5359\times10^{-6}\sqrt{T} - 8.1619\times10^{-6}$
where μ = viscosity (kg/m-s)

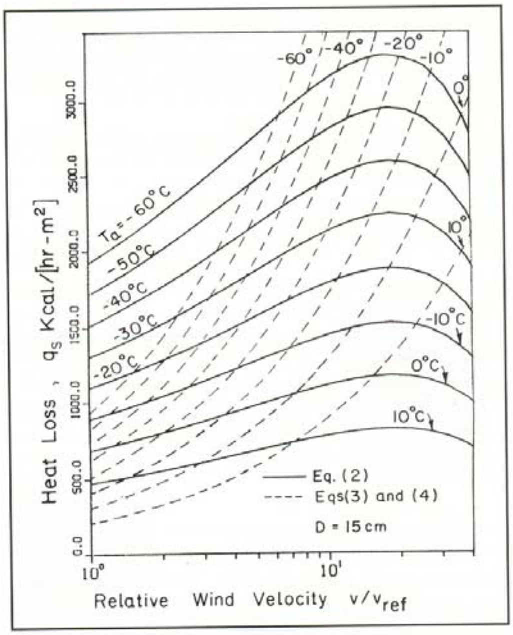

Figure 1. *Comparison of Siple and Passel's wind-chill index and heat loss from exposed skin at the moment of exposure.*

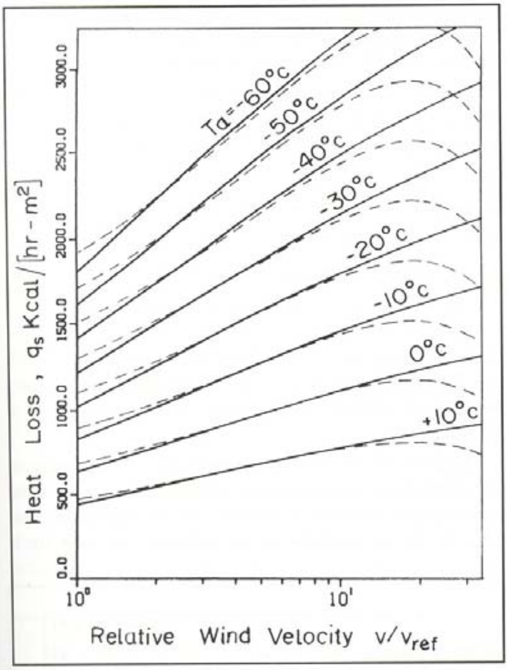

Figure 2. *Wind-chill index calculated based on improved correlation for Siple and Passel's WCF.*

—— $q_s = (T_s - T_a)/(r_d + 1/hc)$

----- $WCI = \left(10\sqrt{V} + 10.45 - V\right)(T_s - T_a)$

TABLE 3
Siple and Passel's Human Comfort Scale

WCI in kcal/m²-hr	
600	Conditions considered as comfortable when dressed in wool underwear, socks, mitts, ski boots, ski headband, and thin cotton windbreaker suits, and while skiing over level snow at about three miles per hour. (Metabolic output about 200 kcal/m²-hr)
1000	Pleasant conditions for travel cease on foggy and overcast days
1200	Pleasant conditions for travel cease on clear sunlit days
1400	Freezing of human flesh begins, depending upon degree of activity, amount of solar radiation, character of skin and circulation. Travel and life in temporary shelter becomes disagreeable.
2000	Conditions for travel and living in temporary shelter becomes dangerous. Exposed areas of face will freeze within less than one minute for the average individual.
2300	Exposed areas of face will freeze within less than a half minute for the average individual.

of the inaccuracies and inconsistencies in Siple and Passel's correlation, it is still the basis for a useful human confort indicator of the wind-chill effect? This paradox is probably the most interesting aspect of the wind-chill concept.

A very important reason for the success of their correlation is that Siple and Passel have also carried out a calibration of the WCI scale that relates it to human comfort, as shown in Table 3. Because of the calibration, it becomes less important whether WCI really represents what it is supposed to represent, as long as similar wind velocities and ambient temperatures give the same WCI as used in the calibration. The peculiar behavior of the correlation at high wind velocities is mostly irrelevant since the practical range of wind velocities is inside the range of Siple and Passel's measurements.

WCI clearly gives some representation of heat loss from the human body, but in no way can it be interpreted as a steady heat loss. The human metabolism rate under normal activities is in the range of 50-200 kcal/hr-m², while the WCI is already over 400 kcal/hr-m² for "cool" conditions and reaches values up to 2400 kcal/hr-m² under severe weather conditions. Obviously, humans cannot survive with such rates of heat losses. (Note that the metabolism rate is usually expressed in the literature[6,7] per unit area rather than per unit volume, so that it can be directly related to heat loss.)

In order to calculate the steady heat loss from the whole body, a complete heat balance must be carried out, taking into consideration additional heat effects such as metabolism rate, resistance of clothing, heat loss through the lungs, effective wind speed at ground level,[6] radiative heat input and losses, and conductive heat loss (to the ground), in addition to convective heat loss. Heat loss due to evaporation from the bare skin is negligible at low temperatures.[6]

Problem 2
Wind-Chill Equivalent Temperature

Wind-chill temperature (or wind-chill equivalent temperature) was introduced by Falconer,[3] who realized that different combinations of wind velocity and ambient temperature yield the same WCI. He used this observation as the basis for deriving the wind-chill equivalent temperature (T_{wc}), which is the ambient temperature that would yield the same WCI at a reference wind velocity (V_r = 3 mph ~ 1.34 m/s, average walking velocity), as the actual temperature yields at the actual wind velocity. Falconer used Siple and Passel's correlation to prepare a nomogram, which is the one still being used by meteorologists for announcing this temperature as part of their daily weather reports.

Calculate and plot T_{wc} using Siple and Passel's correlation and the model that was developed in Problem 1. Compare the results in the range of

$$-60°C < T_a < 10°C,\ 1.34\ m/s < V < 35\ m/s.$$

Solution Using Siple and Passel's correlation at the reference velocity yields

$$WCI = \left(10\times\sqrt{1.34}+10.45-1.34\right)\left(33-T_{wc}\right) \tag{6}$$

which should be equal to the WCI at the actual temperature and actual wind velocity. Introducing Eq. (6) into Eq. (2) and solving for T_{wc} yields

$$T_{wc} = 33-\left(10\sqrt{V}+10.45-V\right)\left(33-T_a\right)/20.686 \tag{7}$$

Using the same considerations and taking into account that the skin temperature is the same under the actual and reference conditions, Eq. (3) yields

$$T_{wc} = T_s - \frac{h_c}{h_{c_r}}\left(T_s - T_a\right) \tag{8}$$

where h_{c_r} is the heat-transfer coefficient at the reference velocity. Neglecting the effect of temperature on the physical properties of air due to $T_{wc} \neq T_a$,

$$\frac{h_c}{h_{c_r}} = \left(\frac{Re}{Re_r}\right)^{0.6} = \left(\frac{V}{V_r}\right)^{0.6}$$

and thus

$$T_{wc} = T_s - \left(\frac{V}{V_r}\right)^{0.6}\left(T_s - T_a\right) \tag{9}$$

The wind-chill equivalent temperatures calculated using Eqs. (7) and (9) (based on $T_s = 33°C$) are plotted in Figure 3 versus the wind velocity for different ambient temperatures. Note that there are considerable differences between the T_{wc} calculated using the two models, which result from the same reasons discussed in the previous section. Taking into account the influence of the temperature on the physical properties of air has a minor effect relative to the discrepancy between the results.

Problem 3
Exposed Skin Temperature

The temperature we actually sense is the skin temperature. From the moment of exposure, the skin temperature will drop until it reaches a steady-state level. Thus, the most adequate indicator for the combined effect of temperature and wind velocity is the bare skin temperature at steady state. Derive the equations required to calculate the steady temperature of bare skin as function of the wind velocity and ambient temperature. Assume that only a small portion of the skin is exposed, thus the body temperature is constant. Consider two cases: no heat input from solar radiation (cloudy day) and with heat input from solar radiation (sunny day). Calculate and plot of the exposed skin temperature in the range of V and T_a described in Problem 2.

Solution At steady state the heat transferred from the body to the skin surface is equal to the heat transferred to the surroundings. Thus, neglecting radiation effects,

$$q_s = \frac{T_b - T_s}{r_b} = \frac{T_b - T_a}{r_b + 1/h_c} \tag{10}$$

where

T_b = body temperature (°C) and

r_b = the body thermal resistance (°C-m^2-hr/kcal)

Using the values of T_b and r_b from Table 1 and Eq. (3) for h_c, Eq. (10) can be solved for T_s. (Note that the viscosity and thermal conductivity of the air should be calculated at $T_f = (T_a + T_s)/2$.)

Radiation effects that can be taken into account include heat input to the skin from solar radiation and heat loss from the body due to radiation. The equation for calculating heat input is shown in Table 1; the radiative heat loss, q_r is

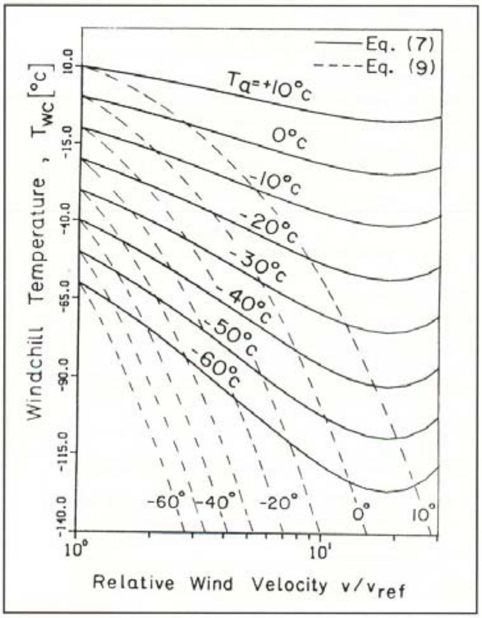

Figure 3. *Wind-chill equivalent temperature versus wind velocity and ambient temperature.*

calculated from

$$q_r = \varepsilon\sigma\left(T_s^4 - T_a^4\right) \qquad (11)$$

where

σ = 4.88 x 10^{-8} (kcal)/(m^2-hr-°K^2)

ε = 1 for bare skin

When solar radiation and radiative heat losses are included in the energy balance, Eq. (10) is replaced by

$$q_s = h_c\left(T_s - T_a\right) - q_{sr} + \sigma\left(T_s^4 - T_a^4\right) = \frac{T_b - T_s}{r_b} \qquad (12)$$

which can be solved for the skin temperature (EST) similarly to the way Eq. (10) is solved.

Figure 4 shows the EST when radiative heat losses and heat input due to solar radiation on a clear sunny day ($C_1 = 0$) are both included, compared to EST on a cloudy day (with no heat input from solar radiation, $C_1 = 1$). Note that at low wind velocities, solar radiation compensates for about 5 to 10°C ambient temperature difference. For example, at low wind velocities, an ambient temperature of -10°C on a sunny day will cause the same feeling of cold as 0°C on a cloudy day. The effect of radiation diminishes at higher wind velocities.

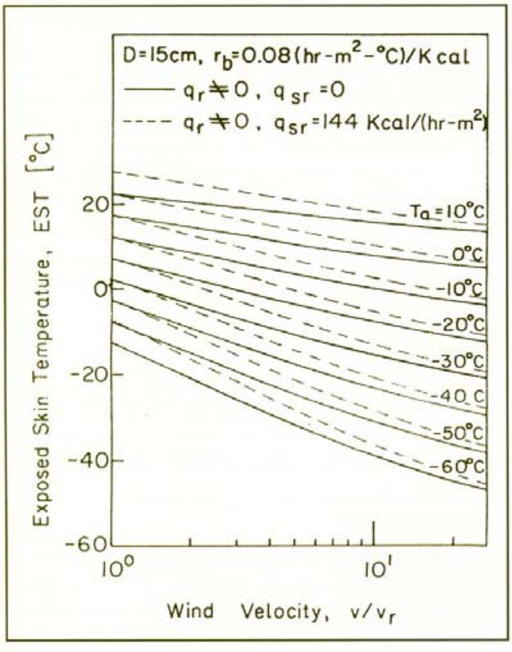

Figure 4. *Exposed skin temperature with and without solar radiation.*

Problem 4
Maximum Exposure Time (MET)

Another interesting question that may come up on a cold windy day is whether it will be possible to walk to the cafeteria without the danger of getting frostbite on the (exposed) face. Suggest a model for estimating the time it takes for the exposed skin to reach the freezing point from the moment you leave the inside of a building. Calculate and plot this time for different values of T_a and V. Use data from Table 1 as needed.

No solution is provided for this rather challenging problem. Instructors can probably best use it as a basis for a variety of open-ended problems. Two possible solutions for the problem are provided in reference 9.

CONCLUDING REMARKS

The "wind-chill paradox" can serve as a basis for several interesting and motivating exercises in heat transfer. The exercises presented in this paper also demonstrate some general principles of significance that are not limited to the heat transfer area. The danger of extrapolation beyond the interval where measurements were taken is demonstrated. It is shown that calibration, using experimental data, is the key to the consistency and reliability of a measuring device. However, modeling of a physical phenomenon by a purely empirical model does not contribute to the understanding of the phenomenon, and it can even be misleading.

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