Design of Low Update Rate Phase Locked Loops with Application to Carrier Tracking in OFDM Systems

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Abstract: **In this paper, we develop design procedures for carrier tracking loop for orthogonal frequency division multiplexing (OFDM) systems or other systems of blocked data. In such communication systems, phase error measurements are made infrequent enough to invalidate the traditional loop design methodology which is based on analog loop design. We analyze the degradation in the OFDM schemes caused by the tracking loop and show how the performance is dependent on the rms phase error, where we distinguished between the effect of the variance in the average phase over the symbol and the effect of the phase change over the symbol. We derive the optimal tracking loop including optional delay in the loop caused by processing time. Our solution is general and includes arbitrary phase noise and additive noise spectrums. In order to guarantee a well behaved solution, we have to check the design against margin constraints subject to uncertainties. In case the optimal loop does not meet the required margin constraints subjected to uncertainties, it is shown how to apply a method taken from control theory to find a controller. Alternatively, if we restrict the solution to first or second order loops, we give a simple loop design procedure which may be sufficient in many cases. Extensions of the method are shown for using both pilot symbols and data symbols in the OFDM receiver for phase tracking. We compare our results to other methods commonly used in OFDM receivers and we show that a large improvement can be gained.**

Index Terms: **Carrier tracking, loop design, low rate digital PLL, orthogonal frequency division multiplexing (OFDM), phase locked loop (PLL), synchronization.**

I. INTRODUCTION

The phase locked loop (PLL) principle has been successfully used for decades for tracking the carrier phase and the bit timing. Digital implementation of PLL in most cases is based on sampling frequency which is much higher than the loop bandwidth, and the PLL behavior can be approximated by its analog counterpart. There are situations which invalidate this assumption, and new design methods need to be developed. Such situations occur in cases where the loop bandwidth is desired to be as wide as possible. There are many possible variations on how the PLL is sampled. In this paper we are interested in the case where continuous time phase is tracked by mixed analog/digital PLL in which the phase detector output is sampled in low rate and fed to the loop filter. The sampling of the phase detector is undesired but is an unavoidable consequence of the communication system if the data is blocked and information about the phase can be extracted only at the end of a block. As an example to such a system is orthogonal frequency division multiplexing (OFDM) receivers. Another example is a system which uses block code, and the uncoded symbols are not reliable enough to be used in a PLL, but after decoding, the phase of the block can be estimated using the decoded symbols. In the examples above the continuous time section of the analog/digital PLL is replaced by a high sampling rate digital implementation. Since the sampling rate (of the digital front-end, rather than the matched filter output) is high relative to the loop bandwidth, the approximation by continuous waveforms is good for all practical purposes.

The loop filter is designed under the conditions of minimizing its *rms* phase error at the sampling point, with given phase noise spectrum and additive noise level. In some cases, the optimal design fails to satisfy restriction of gain and phase margins with required gain uncertainty.

A method from control theory is used to derive a nearly optimal loop filter minimizing the *rms* phase error like the loop filter above, but with the additional restriction to satisfy gain and phase margins with some gain uncertainty. After the high order solution (near optimal solution) is described, restricted order solutions are also given. First order solutions have closed form when the delay is large enough, and for second order solutions easy to use graphs are given for extracting the range in which the optimal solution can be searched. A similar method had been used to treat a related problem of loop design in presence of a delay [1].

Many papers have been devoted to the problem of phase noise effect in OFDM and synchronization loops. Several papers have investigated the effect of phase noise on OFDM [2]–[6] and show that the sensitivity of OFDM is orders of magnitude more than single carrier schemes with the same bit rate. However, none of these papers considered carrier tracking loop for relaxing of the need of very stable local oscillators. The common solution is to use differential detection or phase extracted from the pilot signal within the symbol and slow AFC loop [4], [7], [8]. Differential detection is known to lose at least 3 dB in performance for QPSK or QAM. Mignone [9], used phase estimation from the previous symbol to be used in the current symbol. Managing to obtain to get good phase estimate extracted from pilot signals requires that the signal to noise is good enough, which is not the case for relatively small FFT sizes (e.g., 64 points). The assumption that the carrier phase does not vary during the two symbol period, as in [9] or in differential detectors, requires a very low level of phase noise.

By using a properly designed loop, it is possible to considerably reduce the degradation from phase noise. We show that OFDM fits the model used in this paper; therefore, the design techniques can be applied. The phase detector estimates can be

Manuscript received November 3, 2004; approved for publication by Stefano Buzzi, Division I Editor, April 12, 2005.

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obtained either from the data of the previous symbol or from pilot symbols. Improvements for the solution for phase estimation in OFDM are also presented. In the improved version, a loop is used for generating preliminary phase estimation, and then the phase estimation is improved by adding non-causal information from pilot symbols, or, by passing the preliminary phase estimation through an additional, non-causal, filter.

The degradation of OFDM receiver from phase noise is due to two effects [3], [4]. The first is the average phase error over the symbol, which rotates the constellation of all subchannels. The second is the phase change over the symbol which causes loss in orthogonality. The latter is similar in its effects to additive Gaussian noise. Though the optimization in this paper is done with respect to the average phase error, we observe in many cases that using the loop significantly decreased the phase change over the symbol. Loop optimization with regard to the phase change is not treated here, but it is reasonable to assume that minimizing the average phase error lead, at least approximately, to minimization of the phase change.

II. STATEMENT OF THE PROBLEM

A. Tracking Loop for OFDM: Derivation of Phase Error Indication

Assuming perfect timing synchronization during a symbol period T, the complex envelope of the transmitted OFDM signal can be expressed as [3]

$$s(t) = e^{j\theta(t)} \sum_{m=0}^{N-1} a_m e^{j2\pi \frac{m}{T} t}, \;\; 0 \le t \le T \tag{1}$$

where a_m is the data symbol of the m frequency bin, and $\theta(t)$ is the carrier phase, which is common to all subcarriers. We assume that at the receiver front end there is a phase derotator which multiplies the signal by $e^{-j\hat{\theta}(t)}$, where $\hat{\theta}(t)$ is the estimated phase by the loop. Let us denote the phase error by

$$e(t) = \theta(t) - \hat{\theta}(t)$$

then the signal at the FFT output of the receiver can be written as

$$r_k = a_k I_0 h_k + \sum_{m=0, m\neq k}^{N-1} a_m I_{k-m} h_m + N_k \tag{2}$$

where

$$I_k = \frac{1}{T} \int_0^T e^{j2\pi \frac{k}{T} t} e^{je(t)} dt,$$

h_k is the channel attenuation of the k'th subcarrier, and N_k is the thermal noise contribution.

The first term in (2) is the useful signal, $a_k h_k$, which was rotated by $e_a(n)$, the average value of $e(t)$ during the symbol. The phase of I_0 is well approximated by $e_a(n)$ as will be shown in the following. The second term is called inter bin interference (IBI) and results from the phase error change over the symbol. It is difficult to extract information from the IBI regarding the phase error. However, the first term can be used to extract information about the average value, $e_a(n)$. The phase detector used in this paper which is based on the approximation of the phase error for small errors is

$$\hat{e}_a = \frac{1}{NE_N} \sum_{m=0}^{N-1} \mathrm{Im}\{r_m a_m^* h_m^*\}. \tag{3}$$

Here, we assumed that a_m has been decoded successfully, and h_m is well estimated. High error rate in a_m can be tolerated since N phase estimates are averaged. Alternatively, a_m after re-encoding can be used [9], or pilot symbols with known a_m can be used. The scaling factor E_N is the total energy in the symbol, $E_N = \frac{1}{N}\sum_{m=0}^{N-1} |a_m|^2 |h_m|^2$, which for large N can be approximated by the average symbol energy E_s multiplied by the average channel gain $E_h = \frac{1}{N}\sum_{m=0}^{N-1} |h_m|^2$.

B. Degradation of Performance Due to Phase Errors

The degradation in the OFDM performance is due to both $e_a(n)$ which cause rotation in the constellation and $e_c(t) = e(t) - e_a(n)$ which causes the IBI. We shall analyze the effect of $e_c(t)$ on the performance. Let us analyze the second term in (2)

$$\begin{aligned}
\nu_k &\stackrel{\text{def}}{=} \frac{1}{T} \sum_{m=0, m\neq k}^{N-1} a_m h_m \int_0^T e^{j2\pi \frac{k-m}{T} t} e^{je(t)} dt \\
&= \frac{1}{T} \sum_{m=0, m\neq k}^{N-1} a_m h_m \int_0^T e^{j2\pi \frac{k-m}{T} t} e^{je_c(t)} dt \\
&\approx \frac{j}{T} \sum_{m=0, m\neq k}^{N-1} a_m h_m \int_0^T e^{j2\pi \frac{k-m}{T} t} e_c(t) dt \\
&= j \sum_{m=0}^{N-1} a_m h_m \frac{1}{T} \int_0^T e^{j2\pi \frac{k-m}{T} t} e_c(t) dt.
\end{aligned}$$

The approximation assumes $|e_c(t)| << 1$, and the last equality results from $\int_0^T e_c(t)dt = 0$. Since a_m are uncorrelated,

$$\begin{aligned}
\sigma_\nu^2 &\stackrel{\text{def}}{=} \mathrm{VAR}[\nu_k] \\
&= E_s E_h \cdot E\left[\sum_{n=0}^{N-1} \left| \frac{1}{T} \int_0^T e^{j2\pi \frac{n}{T} t} e_c(t) dt \right|^2 \right]
\end{aligned} \tag{4}$$

where n stands for $k-m$. Assuming that the energy contribution of the frequencies of $e_c(t)$ above $(N-1)/T$ can be neglected, then

$$\begin{aligned}
\sigma_\nu^2 &= \mathrm{VAR}[\nu_k] \\
&\approx E_s E_h \cdot E\left[\sum_{n=0}^{\infty} \left| \frac{1}{T} \int_0^T e^{j2\pi \frac{n}{T} t} e_c(t) dt \right|^2 \right].
\end{aligned} \tag{5}$$

We can then use the Parseval relation to get

$$\sigma_\nu^2 = E_s E_h \cdot E\left[\frac{1}{2T} \int_0^T e_c^2(t) dt \right].$$

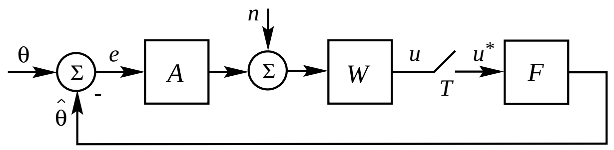

Fig. 1. The sampled PLL linear approximated feedback system.

Thus, the signal to IBI ratio is

$$\frac{E_s E_h}{\sigma_\nu^2} = \frac{2}{\sigma_c^2}$$

where

$$\sigma_c^2 \stackrel{\text{def}}{=} E\left[\frac{1}{T}\int_0^T e_c^2(t)dt\right].$$

The influence of $e_c(t)$ on I_0 is negligible since it contributes only to the second order term in the following series expansion

$$I_0 = \frac{1}{T}\int_0^T e^{je(t)}dt = 1 + \frac{1}{T}\int_0^T [je(t) + O(e(t)^2)]dt.$$

III. LOOP DESIGN

A. Mathematical Model

Since the sampling rate of the front-end receiver is much higher than the symbol rate, the OFDM receiver operation can be well approximated by a matched filter operating on a continuous signal. The received samples are phase-corrected by multiplying by a numerically implemented oscillator fed by a loop filter, where this loop filter is fed by low rate phase detector outputs. Both this oscillator and loop filter are implemented in the high sampling rate. In the linear model of the PLL, the numerically controlled oscillator is equivalent to integration, and the cascade of the actual loop filter and integrator is represented by a loop filter F. For the analytic convenience, the loop filter which operates in the front-end sampling rate is approximated by a continuous filter $F(s)$. The actual implementation is purely digital, where $F(s)$ will be implemented digitally by s to z transform (where z in this case is the inverse delay element of a front end high sampling rate, not the symbol rate). The linear model of the PLL is presented in Fig. 1 where,

- T denotes a periodic sampling with sampling period T (in the OFDM case it is the length of OFDM symbol);
- W denotes a decimating filter. In this paper we choose an integrator along a period T, and its transfer function is $W(s) = \frac{1-e^{-sT}}{Ts}$;
- A denotes the phase detector gain;
- $F(s)$ denotes a linear time invariant filter to be designed; and
- $\theta(t)$ and $n(t)$ are mutually uncorrelated zero mean stochastic signals with known spectral densities, modeled by stable minimum phase linear time invariant systems driven by white noise.

Using the notation $[x]^*$ for the sampled representation of the continuous signal $x(t)$ with sampling period T [12, p. 266] (that is $x(t)$ multiplied by the train of pulses $m(t) = \sum_{k=-\infty}^{\infty}\delta(t - kT)$), the error signal e of the system in Fig. 1 is the solution of the following equation

$$\begin{aligned} e(s) &= \theta(s) - F(s)\left[W(Ae+n)\right]^*(s) \\ &= \theta(s) - F(s)\left[WAe\right]^*(s) - F(s)\left[Wn\right]^*(s). \end{aligned}$$

Our problem is to design a loop filter, $F(s)$, which minimizes the phase error signal, $e(t)$, subject to the following performance index, data, and constraints.

- The power spectral density of the noise, n, and phase noise, θ, are $\phi_n(\omega)$ and $\phi_\theta(\omega)$, respectively, and it is assumed that θ and n are uncorrelated.
- The phase detector gain, A, is fixed but only known to belong to an interval $A \in [A_1, A_2]$, where A_1 and A_2 are known. Variations in A are the result of change in symbol energy E_N or fading.
- The performance index is to minimize the *rms* value of $e_a(n) = \frac{1}{T}\int_{nT}^{nT+T} e(t)dt$, the average phase error during a symbol.
- The open loop response should have some gain and phase margins in order to guarantee a well damped closed loop response and stability in case of some gain uncertainty. These margins are defined by a constant γ such that

$$\left|\frac{L^*(j\omega)}{1+L^*(j\omega)}\right| \le \gamma,\ \forall\omega \ge 0,\ A \in [A_1, A_2]; \qquad L^*(s) = [AWF]^*(s).$$

B. Derivation of Optimal Loop—No Margins Specified

From (6),

$$e + F\left[WAe\right]^* = \theta - F\left[Wn\right]^*,$$

$$\begin{aligned} \left[We\right]^* &= \frac{\left[W\theta\right]^*}{1+\left[AWF\right]^*} - \frac{\left[WF\right]^*\left[Wn\right]^*}{1+\left[AWF\right]^*} \\ &= \frac{\left[W\theta\right]^*}{1+L^*} - \frac{L^*\left[Wn\right]^*}{1+L^*}\frac{1}{A}. \end{aligned}$$

We want to minimize the *rms* value of $e_a(n) = [We]^*$, which is

$$\begin{aligned} \sigma_e^2 &= E\left[\frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left|[We]^*(j\omega)\right|^2 d\omega\right] \\ &= E\left[\frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left|\frac{L^*(j\omega)\left[Wn\right]^*(jw)}{A(1+L^*(j\omega))} - \frac{\left[W\theta\right]^*(jw)}{1+L^*(j\omega)}\right|^2 d\omega\right] \\ &= E\left[\frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}}\left(\left|\frac{L^*(j\omega)\left[Wn\right]^*(j\omega)}{A(1+L^*(j\omega))}\right|^2 + \left|\frac{\left[W\theta\right]^*(j\omega)}{1+L^*(j\omega)}\right|^2\right)d\omega\right]. \end{aligned}$$

Let us denote by $[Wn](z)$, $[W\theta](z)$, and $L(z)$ the z-transform representation of the sampled signals, $[Wn]^*(s)$, $[W\theta]^*(s)$, and the impulse response of $AWF(s)$, respectively. Using the equality $L(z = e^{sT}) = L^*(s)$ [12, p. 280] we get

$$\sigma_e^2 = E\frac{T^{-1}}{2\pi j}\oint_{|z|=1}\left|\frac{L(z)\left[Wn\right](z)}{A(1+L(z))}\right|^2\frac{dz}{z}$$

$$+\frac{T^{-1}}{2\pi j}\oint_{|z|=1}\left|\frac{[W\theta](z)}{1+L(z)}\right|^2\frac{dz}{z}$$
$$=\frac{T^{-1}}{2\pi j}\oint_{|z|=1}\left|\frac{\phi_\theta(z)}{1+L(z)}\right|^2\frac{dz}{z}$$
$$+\frac{T^{-1}}{2\pi j}\oint_{|z|=1}\left|\frac{L(z)\phi_n(z)}{A(1+L(z))}\right|^2\frac{dz}{z}$$

where ϕ_n and ϕ_θ are the following spectral factorization

$$E\left[|[Wn](z)|^2\right]=\phi_n(z)\phi_n(z^{-1}),$$
$$E\left[|[W\theta](z)|^2\right]=\phi_\theta(z)\phi_\theta(z^{-1}).$$

Since $L\propto A$, the argument of each integral in (6) in low frequencies is approximately proportional to $1/A^2$, and since in general the spectral density of ϕ_θ is concentrated in low frequencies and that of ϕ_n is white, the following assumption is valid

Assumption III.1: For a given $F(s)$, the maximum of $\sigma_e^2(A)$ over $A\in[A_1,A_2]$ is $\sigma_e^2(A_1)$.
This assumption means that a solution that minimizes $\sigma_e(A_1)$ subjected to all other constraints is the solution to our problem.

Using the notation

$$Q(z)=\frac{L(z)}{1+L(z)},\ 1-Q(z)=\frac{1}{1+L(z)} \tag{6}$$

where $Q(z)$ is a stable rational function with poles inside the unit circle, gives ($A=1$ is used for simplicity)

$$\begin{aligned}2\pi jT\sigma_e^2 &= \oint_{|z|=1}|\phi_n(z)Q(z)|^2\frac{dz}{z}\\ &+\oint_{|z|=1}|\phi_\theta(z)-\phi_\theta(z)Q(z)|^2\frac{dz}{z}.\end{aligned} \tag{7}$$

By simple completion to square it can be shown that

$$\begin{aligned}2\pi jT\sigma_e^2 &= \oint_{|z|=1}\left(\left|\alpha(z)+\beta(z)Q(z)\right|^2\right.\\ &\left.+\phi_\theta(z)\phi_\theta(z^{-1})-\alpha(z)\alpha(z^{-1})\right)\frac{dz}{z}\end{aligned} \tag{8}$$

where $\beta(z)$ and $\alpha(z)$ satisfy the following equations

$$\beta(z)\beta(z^{-1}) = \phi_n(z)\phi_n(z^{-1})+\phi_\theta(z)\phi_\theta(z^{-1}) \tag{9}$$
$$\alpha(z^{-1})\beta(z) = -\phi_\theta(z)\phi_\theta(z^{-1}). \tag{10}$$

$Q(z)$ has the same zeros outside the unit circle as $L(z)$ and these can only be pure delays due to the process W and the delay in the feedback loop. We, therefore, denote

$$Q(z)=z^{-k}Q_0(z) \tag{11}$$

where k represents the delay and $Q_0(z)$ is minimum phase. Hence,

$$\begin{aligned}2\pi jT\sigma_e^2 &= \oint_{|z|=1}\left(\left|z^k\alpha(z)+\beta(z)Q_0(z)\right|^2\right.\\ &\left.+\phi_\theta(z)\phi_\theta(z^{-1})-\alpha(z)\alpha(z^{-1})\right)\frac{dz}{z}.\end{aligned} \tag{12}$$

After removing terms not depending on $Q_0(z)$, it is clear that $Q_0(z)$ which minimizes σ_e^2 is the same one which minimizes

$$\sigma_{e1}^2\overset{\text{def}}{=}\oint_{|z|=1}\left(\left|z^k\alpha(z)+\beta(z)Q_0(z)\right|^2\right)\frac{dz}{z}. \tag{13}$$

Since $\beta(z)^*=\beta(z^{-1})$ it can be chosen as minimum phase (no zeros and poles outside the unit circle). Thus, (13) can be split into its causal and anti-causal parts

$$\sigma_{e1}^2=\oint_{|z|=1}\left(|\alpha_-(z)|^2+|\alpha_+(z)+\beta(z)Q_0(z)|^2\right)\frac{dz}{z} \tag{14}$$

where $z^k\alpha(z)=\alpha_+(z)+\alpha_-(z)$ is the series expansion of $z^k\alpha(z)$ with terms inside and outside the unit circle, respectively. Therefore, the optimal Q_0 is

$$Q_0=-\frac{\alpha_+(z)}{\beta(z)}\overset{\text{def}}{=}Q_{opt}(z) \tag{15}$$

and the optimal open loop, $L_{opt}(z)$, is by (6) and (11)

$$L_{opt}(z)=-\frac{z^{-k}\alpha_+(z)}{\beta(z)+z^{-k}\alpha_+(z)}. \tag{16}$$

The same results, without explicit consideration of the expected delays, can be found in many papers, for example in [10].
An example: $\phi_\theta=\frac{1}{1-z^{-1}}$, $\phi_n=\sqrt{N_0}$ and one delay ($k=1$). Then,

$$\begin{aligned}\beta(z)\beta(z^{-1}) &= N_0+\frac{1}{z-1}\frac{1}{z^{-1}-1}\\ &= \frac{2N_0+1-N_0(z+z^{-1})}{(z-1)(z^{-1}-1)}.\end{aligned}$$

Denoting $\beta(z)=\frac{Az^{-1}-B}{1-z^{-1}}$ where $A>0$ and $|A/B|<1$, implies

$$\begin{aligned}A &= \frac{-1+\sqrt{1+4N_0}}{2},\ B=\frac{1+\sqrt{1+4N_0}}{2}\\ z\alpha(z) &= \frac{-z\frac{1}{1-z^{-1}}\frac{1}{1-z}}{\beta(z^{-1})}\\ &= \frac{-z}{1-z^{-1}}\frac{1}{Az-B}\\ &= \frac{1}{1-z^{-1}}-\frac{B}{A-Bz^{-1}}\\ &\Rightarrow\quad \alpha_+=\frac{1}{1-z^{-1}}\\ &\Rightarrow\quad Q_{opt}=-\frac{\alpha_+}{\beta(z)}=\frac{1}{Az^{-1}-B}\\ &\Rightarrow\quad L_{opt}(z)=\frac{-z^{-1}\frac{1}{1-z^{-1}}}{\beta(z)+z^{-1}\frac{1}{1-z^{-1}}}=\frac{z^{-1}}{B(1-z^{-1})}\\ &\Rightarrow\quad \tilde{F}_{opt}(s)=\frac{1/B}{s}.\end{aligned}$$

The filter, $\tilde{F}_{opt}(s)$, such that $[WF_{opt}(s)]^*(s)=L_{opt}(z)$ is the solution we seek, only if the closed loop satisfies the gain and phase margin constraints, γ, over all $A\in[A_1,A_2]$. However if the open loop gain interval is large and/or the desired margins are large compared to T, $\tilde{F}_{opt}(s)$ will not be a satisfactory solution. It might even be an unstable solution for possible higher open loop gains. In the next paragraph it is shown how to synthesize $F(s)$ by modifying Q_{opt}.

C. Second Synthesis Method—with Margin Specs

The gain and phase margin specifications can be expressed as

$$\left|\frac{L(z)}{1+L(z)}\right| = \left|\frac{\frac{A}{A_1}L_1(z)}{1+\frac{A}{A_1}L_1(z)}\right| \le \gamma; \quad \forall\, A \in [A_1, A_2],\ |z|=1 \tag{17}$$

where $L_1(z)$ is the open loop when $A = A_1$. Let Q_1 be the new closed loop response that will satisfy the margins and define λ by

$$Q_1 = Q_{opt} - \frac{\lambda(z)}{\beta(z)},\ \text{where } A = A_1. \tag{18}$$

Then (14) for $Q_0 = Q_1$ gives

$$\sigma_{e2}^2 = \oint_{|z|=1} |\lambda(z)|^2 \frac{dz}{z}, \tag{19}$$

which by assumption III.1 reduces our problem to design a stable $\lambda(z)$ which minimizes (19) subjected to the constraint (17), which is

$$\left|\frac{\tilde{A}L_1}{1+\tilde{A}L_1}\right| = \left|\frac{\tilde{A}Q_1(z)}{1+(\tilde{A}-1)Q_1(z)}\right| \le \gamma; \quad \forall \tilde{A} \in [1, A_2/A_1],\ |z|=1. \tag{20}$$

Substituting (18) into (20) gives

$$\left|\frac{\tilde{A}Q_{opt}(z) - \lambda(z)\frac{\tilde{A}}{\beta(z)}}{1+(\tilde{A}-1)Q_{opt}(z) - \lambda(z)\frac{\tilde{A}-1}{\beta(z)}}\right| \le \gamma; \quad \forall \tilde{A} \in [1, A_2/A_1],\ |z|=1. \tag{21}$$

Optimization problem (19) subjected to (21) reduces our optimization problem into a simpler problem, in the sense that it guarantees stability, it has less optimization parameters and it can be solved to find suboptimal solutions within the framework of the quantitative feedback theory (QFT) [11]. A more detailed description of using the technique is given in the example.

Having λ, Q_{opt}, and β, Q_1 can be calculated using (18). $L(z)$ is then calculated by (6) where by assumption III.1 $Q = Q_1$. Since $L(z)$ is the sampled transfer function of $F(s)$ using a zero order hold (ZOH), it can be calculated from $L(z)$, how to do it and conditions under which it is unique are given in [12].

C.1 Practical Design Example

This is a practical design example. A very high speed, 155 Mbps, microwave link at around 30 GHz is designed. Let us assume an OFDM system is used, with 48 carriers with 16-QAM and code rate 3/4 and a symbol duration $T = 144/155 \times 10^{-6}$. We assume that the channel is flat (the shape of the channel has no effect on this example). The required minimum E_b/N_0 of the information bits is 10 dB. The normalized noise two-sided spectral density is $\phi_n = -92$ dB and the phase spectral density (units are radians and seconds) is well approximated by $\phi_\theta = (2.75 \times 10^7/\omega^2)^2$ at the region of interest. The gain, A, is any value in the interval $A \in [1, 2]$. The margins constraint is of the form $\left|\frac{L}{1+L}\right| < 3$ dB, which guarantees $45°$ phase margin and 5 dB gain margin for $A = 2$ and 11 dB for $A = 1$. These margins are even less that the lowest one can choose for a proper PLL operation [13].

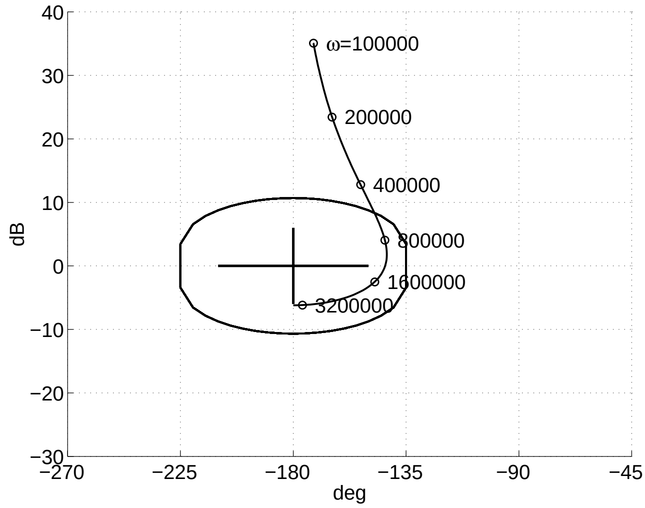

Fig. 2. The Nichols plot of the optimal open loop for $A = A_1$ without considering margin specs.

The optimal filter, $L(z)$, and other parameters involved, calculated by the algorithm described above for $A = 1$ ignoring the margin specifications are

$$\begin{aligned}
L(z) &= \frac{1.2200z - 0.7399}{z^2 - 2z + 1} \\
\beta &= \frac{0.05129z^2 - 0.04z + 0.01334}{z^2 - 2z + 1} \\
\alpha_+ &= \frac{-0.06257z^2 + 0.03795z}{z^2 - 2z + 1} \\
\phi_n(z) &= 0.02606 \\
\phi_\theta(z) &= \frac{z^2 + 0.4737z + 0.0186}{60.6(z-1)^2} \\
\sigma_e &= 2.53°
\end{aligned}$$

where $\phi_n(z)$ and $\phi_\theta(z)$ were derived using the appendix.

Using the Bode or the Nichols plot, it can be shown that the gain and phase margins are approximately $37°$ and 6 dB, respectively (see Fig. 2), which does not satisfy the phase margin $45°$ and gain margin 5 dB for all $A \in [1, 2]$, as required by the specified phase margin parameter $\gamma = 3$ dB. For example, if $A = 2$, the open loop gain margin is almost zero, therefore the PLL will be unstable.

The solution of (21) for a given $z = z_0$ and $\tilde{A}$ is a circle in the complex plane. If $\lambda(z_0)$ is inside that circle, the inequality is satisfied for that $\tilde{A}$ and z_0. The intersection of all of these circles for the allowed $\tilde{A}$'s is the valid region for $\lambda(z_0)$. That is, only if $\lambda(z_0)$ belongs to this region then (21) is satisfied for that $z = z_0$.

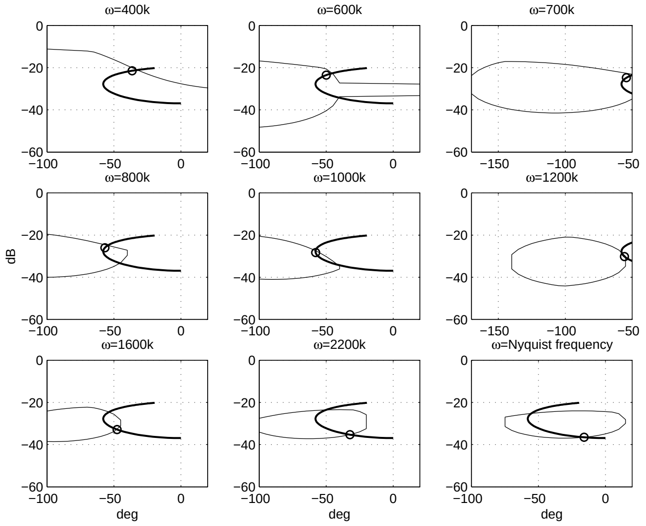

Fig. 3. Rigid heavy line is the amplitude versus phase plot of a shaped $\lambda(z) = \lambda(e^{j\omega})$. The allowed regions for $\lambda(e^{j\omega})$ at the frequency ω marked above each figure is inside the closed curves or below it if it is open. The value of $\lambda(e^{j\omega})$ is marked by a circle, note that it is inside its allowed region.

These regions, for several z_0 values, are shown in Fig. 3, which also include the plot of a chosen transfer function $\lambda(z)$ such that $\lambda(z_0)$ is inside its allowed region for each z_0. Choosing a transfer function satisfying restrictions is a well known loop shaping problem in control theory, see, for example, the QFT technique [11], although other technique are available.

The controller is calculated by (18) and (6) where $Q = Q(A = 1)$, which gives for L at $A = 1$,

$$\begin{aligned} L(z) &= \frac{0.6734z^2 - 0.9452z + 0.3255}{z^3 - 2.659z^2 + 2.318z - 0.6591} \\ &= \frac{0.6734(z - 0.7976)(z - 0.6060)}{(z-1)^2(z - 0.6599)} \end{aligned}$$

whose Nichols plot, including the margin specs, is shown in Fig. 4. The phase error result for this controller is $\sigma_e = 3.64°$. Note that decreasing the sampling time to 50% or less of its original value guarantees satisfaction of the margins for all uncertainty by the optimal solution (16). Also its bandwidth is about 0.8 Mrad/sec which is about $1/T$ the loop frequency updating. The filter $F(s)$ can be calculated from $L(z)$, which gives

$$F(s) = \frac{2.17(s + 535000)(s + 243000)}{s^2(s + 449000)}.$$

IV. RESTRICTED ORDER LOOP FILTERS

A *restricted order* loop filter is a loop filter which has less poles and zeros than an optimal loop filter. The reasons for using a restricted order loop filter are (i) reduction of computation effort in real time; (ii) the design of a restricted order loop filter may be simpler and faster; and (iii) the restricted order loop filter may be close enough to an optimal one. The drawback of using a restricted order loop filter is when (iii) is not satisfied, therefore produce too much error compared to a non-restricted order design. Two restricted order filters will be described in the sequel, $F(s) = a/s$ and $F(s) = a/s^2 + b/s$.

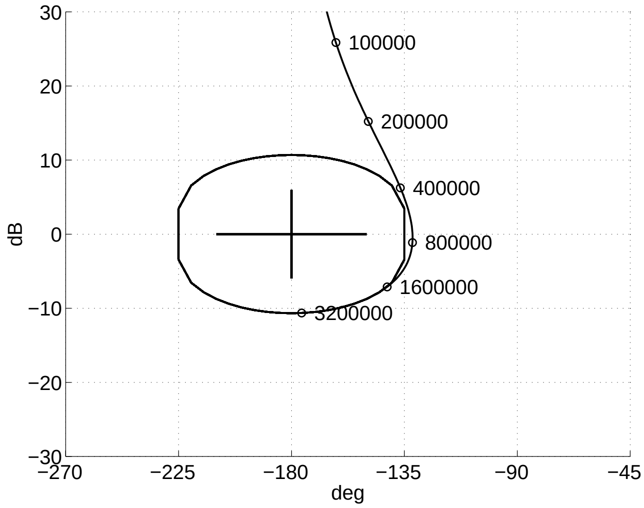

Fig. 4. The Nichols plot of $L_1(j\omega)$ which satisfies the margin specs.

A. $F(s)$ of the Form a/s

The pulse transfer function of the open loop, $L(s)$, for $A = A_1$ is

$$L_1(z) = \frac{A_1 a}{z - 1}. \tag{22}$$

We look for the range of a values for which the PLL is stable and the margin specification (17) is satisfied. Substitution of (22) into (17) reduces the specification to the form

$$\left| \frac{Aa}{z - 1 + Aa} \right| \le \gamma; \ \ \forall\, Aa \in [A_1, A_2],\ |z| = 1. \tag{23}$$

Stability requires $0 < Aa < 2$. If $Aa > 1$, (23) is maximum at $z = -1$, hence $Aa \le \frac{2\gamma}{\gamma+1}$; and if $Aa \le 1$ (23) is always satisfied. Therefore, Aa must lie in

$$0 < Aa \le \frac{2\gamma}{1+\gamma},\ Aa \in [A_1, A_2]$$

therefore

$$0 < a \le \frac{2\gamma}{(1+\gamma)A_2}$$

and the filter, $F(s)$, can be any one of the following

$$F(s) = \frac{2\gamma}{v(\gamma+1)}\frac{1}{s},\ v \ge A_2.$$

Optimization to minimize σ_e^2 subjected to this structure can now be executed on the single parameter v. Clearly for large enough T and/or low noise with reasonable phase noise, the optimal $v = A_2$.

B. $F(s)$ of the Form $\tilde{a}/s^2 + \tilde{b}/s$

The pulse transfer function of the open loop, $L(s)$, for $A = A_1$ is

$$\begin{aligned} L_1(z) &= T\left[\left(\frac{\tilde{a}(1+\tilde{b}s)}{s^2}\right)\right]^*_{\mathrm{ZOH}} \\ &= \frac{\tilde{a}T^3(z+1)}{2(z-1)^2} + \frac{\tilde{a}\tilde{b}T^2}{z-1} \\ &= \frac{a(z+1)}{2(z-1)^2} + \frac{ab}{z-1}, \end{aligned} \quad (24)$$

where the subscript ZOH denotes that a ZOH is included and where

$$a = \tilde{a}T^3,\ b = \tilde{b}/T. \quad (25)$$

We now look for the range of (a, b) values for which the PLL is stable and the margin specification (17) is satisfied. Substitution of (24) in (17) reduces the specification to

$$\left|\frac{a(z+1) + 2ab(z-1)}{2(z-1)^2 + a(z+1) + 2ab(z-1)}\right| \le \gamma,\ \ \forall\, |z| = 1 \quad (26)$$

which can be replaced, using the bilinear transformation $z = \frac{1+jw}{1-jw}$, by

$$\left|\frac{a + 2ab\omega^2 + j(2ab-a)\omega}{a + (2ab-4)\omega^2 + j(2ab-a)\omega}\right| \le \gamma,\ \ \forall\, \omega \ge 0 \quad (27)$$

which is satisfied if and only if for all $\omega > 0$

$$\begin{aligned} &(-4a^2b^2 + \gamma^2(2ab-4)^2)w^4 + (\gamma^2(4a^2b^2 - 8a + a^2) \\ &\quad -a^2 - 4a^2b^2)w^2 + a^2(\gamma^2 - 1) \ge 0. \end{aligned} \quad (28)$$

The solution of (28) is either (i) the coefficients of ω^4 and ω^2 are positive or (ii) the minimum of the left side of (28) is positive (the coefficient of $\omega^4 > 0$ and the discriminant of (28) is negative). For a given b, the range of a due to (i) is

$$\frac{8\gamma^2}{(\gamma^2-1)(1+4b^2)} < a < \frac{2\gamma}{(1+\gamma)b} \quad (29)$$

and the range of a due to (ii) is

$$\begin{aligned} a < \frac{2\gamma}{(1+\gamma)b}\ \text{ and }\ &(4b^2-1)^2a^2 + 16\frac{\gamma^2(2b-1)^2}{(1-\gamma^2)}a \\ &+64\frac{\gamma^2}{(1-\gamma^2)^2} < 0. \end{aligned} \quad (30)$$

The solution is depicted in Fig. 5 by curves, each curve is the boundary of the allowed (a, b) values for given γ.

Note that for $b > 1$, which is guaranteed if the required phase margin is greater than 28°, conditions (30) includes conditions (29). Thus in practice, conditions (30) is the dominant condition.

Fig. 5 can be used to find the region for which $|\frac{AL}{1+AL}| < \gamma$ for phase detector uncertainty $A \in [A_1, A_2]$. Simply shift the upper bound of the (a, b) region down by $20\log\frac{A_2}{A_1}$ dB. For example, if 40° phase margin is required with 18 dB gain margin, then we first shift down the upper bound of the curve marked 40° in Fig. 5 by $(18-4.5)$ dB (4.5 dB is reduced because the parameter γ for 40° guarantees 4.5 dB gain margin). The area below that curve and above the lower curve marked 40° gives the allowed (a, b) values. The optimal (a, b) pair(s) can then be calculated by minimizing σ_e^2 over (a, b) values in the appropriate region.

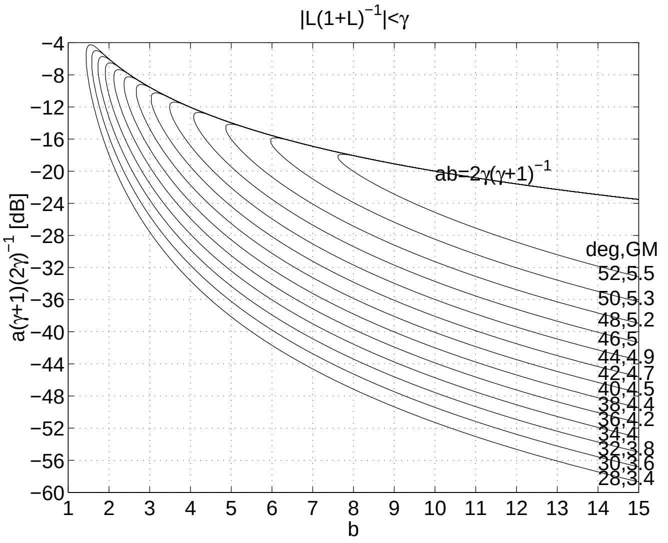

Fig. 5. Boundary curves of the a, b values that satisfy $|\frac{L}{1+L}| < \gamma$. Marked on the right is the phase and gain margins, deg, GM, for the appropriate γ.

B.1 The Practical Design Example

The design example in Section III-B requires 6 dB uncertainty and margin $\gamma = |L/(1+L)| \le 3$ dB which gives the 41.5° curve of Fig. 5 calculated by $\gamma = 1/(2\sin(41.5/2))$. For these parameters the largest $a(\gamma+1)/2\gamma) = -17$ dB and $b = 3.5$. The value of σ_e for this solution is $\sigma_e = 3.67^\circ$ which is about the same as for the solution in Section III-C.1. We repeat the calculation for a larger uncertainty assuming $A \in [1, 4]$, that is, 12 dB uncertainty. In this case, the PI design gives σ_e which is larger by 26% than the one designed by the proposed algorithm in Section III-B.

C. Comparison between First/Second Order Loops for $\phi_\theta \propto \frac{1}{\omega^2}$

From the appendix (31), $\phi_\theta \propto \frac{az+1}{z-1}$, $a = 2+\sqrt{3}$. For the discrete second order open loop

$$L_2(z) = \frac{a(z+1)}{2(z-1)^2} + \frac{ab}{z-1}$$

where b is chosen so that $L(z)$ touch the margins conditions twice. The *rms* value of the phase noise is given in Fig. 6 as a function of a. The normalization σ_0 is the *rms* phase noise of the first order open loop $L_1 = \frac{c}{z-1}$, where c was chosen such that $L_1(z)$ touch the margins conditions. Clearly for reasonable phase and gain margins, using a second order loop ($a \neq 0$), for increasing lock-in range, increase the *rms* noise by less than 20%. Note that addition of noise, $\phi_n \neq 0$, reduces this figure for the relative total *rms* noise.

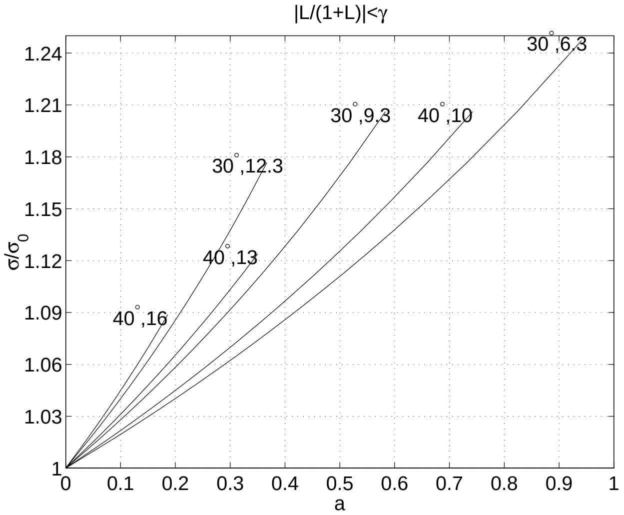

Fig. 6. Normalized phase noise for several gain and phase margin values (marked on top of each curve), the range of a values is all that satisfy the margin $|\frac{L(j\omega)}{1+L(j\omega)}| \le \gamma$ where γ fits the appropriate margins.

V. IMPROVEMENT OF THE METHOD

A. Pilot Assisted Phase Estimation

Until this point we assumed a causal operation of the loop, which means that the phase error estimation uses the previously detected symbols a_m. In many cases pilot signals are added to the OFDM symbol. These are unmodulated sub-carriers which can be used for phase estimation. An estimate of the average carrier phase can be extracted from the pilot signals. This causal information can be combined with the loop estimation of the average phase to improve the average phase estimation. The average value of $\theta(t)$ during the symbol will be denoted by θ_a. Let the pilot estimate of θ be denoted by $\tilde{\theta}_a$. $\tilde{\theta}_a = \theta_a + n_1$ where n_1 is Gaussian noise with spectrum $(1/\mu)\phi_n^2$, where μ is the ratio of pilots energy relative to total symbol energy. The estimate phase which combines the sources of information in an optimal way is

$$\hat{\hat{\theta}}_a(t) = \alpha\hat{\theta}_a(t) + (1-\alpha)\tilde{\theta}_a$$

where the noise contribution should be summed as independent random variables. Since we assume statistical independence between the estimation errors due to the previous symbols to the error induced by the current symbol noise. Thus the error mean squared is $(\alpha n_1)^2 + ((1-\alpha)e_a)^2$, which is minimized when

$$\frac{\alpha}{1-\alpha} = \frac{E[n_1^2]}{E[e_a^2]}.$$

A more convenient implementation is to measure the pilot phase after the correction of the phase by $\hat{\theta}$, so the pilot is measuring $\tilde{e}_a = \tilde{\theta}_a - \hat{\theta}_a$. Therefore,

$$\hat{\hat{\theta}}_a(t) = \hat{\theta}_a(t) + (1-\alpha)\tilde{e}_a.$$

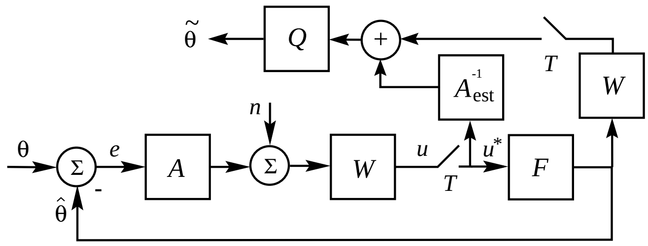

Fig. 7. The improved feedback system.

A.1 The Practical Example

Let us assume that there is a pilot with an energy which is 10 dB less than the symbol energy, i.e., -82 dB. The rms phase error in the pilot phase estimation is $3.34°$. Recall that the loop estimation was $\sigma_e = 3.64°$, combining these estimates gives an rms error of $2.46°$.

B. Improvement by Non-Causal Filters

The gain and phase margin constraints, the delay and the uncertainty dictate a suboptimal filter $F(s)$. It is possible to estimate the phase θ, by the sum of $\hat{\theta}$ and $u^*(kT)$ during the time interval $[kT, kT+T]$, see Fig. 7. Then, an optimal filter can be applied on this first estimate of θ, which can even be non-causal, in order to improve the estimation of θ in the sampling time and/or between them. The optimal filter is applied in open-loop and its output is given after delay. The loop itself continues to be causal. Using this method requires storing the frame and performing the FFT twice, once for the loop and once for the detection, the latter after correction by the new phase estimate. For our practical example, when the estimated A value is $A_{\text{est}} = 1$ and for $k = 0$ (no sample delay), the optimal filter (calculated as described in Section III-B) is $Q = \frac{0.06257z^2 - 0.03795z}{0.05129z^2 - 0.04z + 0.01334}$. Further improvement can be obtained if a longer delay is assumed. Moreover, the change during the symbol can be improved by replacing $Q(z)$ with $Q(s)$ which can be interpreted as interpolating filter.

C. Improvement in Phase-Change Error

As derived in Section II-B the OFDM receiver performance is dependent on both σ_e and σ_c. Although the loop was optimized with respect to σ_e, we have checked the contribution of the loop to the reduction of σ_c. If the phase noise process is random walk, we do not expect an improvement in phase change since the phase change in a symbol is independent on previous symbols. Only interpolation, as noted in the previous section, can improve the phase change (dramatically). However, for phase noise spectrum of higher order, like $\phi_\theta \propto \frac{1}{\omega^4}$, the phase change can be reduced.

C.1 The Practical Example

The results, for example, 1 are $\sigma_c = 0.6°$ with the loop. For comparison, let us assume a pilot based scheme with AFC to correct the frequency drift. We assumed a first order AFC operating on the frequency estimate extracted from the pilots. The bandwidth of the AFC was optimized for minimum σ_c. The result was $\sigma_c = 1.29°$, about 6 dB difference.

VI. CONCLUSIONS

We have presented a loop design methodology for cases where the phase error update rate is much lower than the receiver sampling rate. For OFDM, most often the symbols are long enough to invalidate the traditional “analog” loop design assumptions, leading to an unacceptable solution. Using the proposed close-to-optimal technique one can design a phase recovery loop with the best performance possible under requirement of margins for good dynamic response, and under conditions of uncertainty in the phase detector gain. We derive the optimal loop without the margin constraints, and in case the optimal design does not meet the margins over the entire parameter uncertainties, we have presented a method of finding a suboptimal solution. The solution is given for arbitrary phase noise and additive noise spectrum, margins and uncertainty, and loop delay. In addition, we give a simple low order loop design procedure which may be sufficient in many cases. The solution for OFDM covers both data aided without pilots and data aided with pilots. In the latter case, we combine the estimation from data and from pilots for the best performance. We analyzed a practical test case and showed better performance than that obtained by the conventional technique, where phase is estimated using pilots only and frequency is estimated using AFC.

APPENDIX

Let $y(kT)$ be the sampled output at time kT of the LTI system $H(s)$ whose input is the white noise $u(t)$ with mean value of zero and spectral density N_0. The spectral density of $y(kT)$ is

$$\phi_y(\omega) = \frac{N_0}{T}\sum_{n=-\infty}^{\infty}|H(jw_n)|^2,\ w_n = \omega - \frac{2\pi n}{T}.$$

Example 1: $H(s) = \frac{1-e^{-sT}}{sT}$.

$$\begin{aligned}\phi_y(\omega) &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{1-e^{-j\omega_n T}}{\omega_n T}\frac{1-e^{j\omega_n T}}{-\omega_n T}\\ &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{2-e^{-j\omega_n T}-e^{j\omega_n T}}{-\omega_n^2 T^2}\\ &= -\frac{N_0}{T^2}\left[\frac{2Tz^{-1}-Tz^{-2}-T}{(1-z^{-1})^2}\right]\\ &= \frac{N_0}{T}\frac{1-2z^{-1}+z^{-2}}{(1-z^{-1})^2}\\ &= \frac{N_0}{T}.\end{aligned}$$

Example 2: $H(s) = \frac{1}{s}\frac{1-e^{-sT}}{sT}$.

$$\begin{aligned}\phi_y(\omega) &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{1-e^{-j\omega_n T}}{\omega_n^2 T}\frac{1-e^{j\omega_n T}}{\omega_n^2 T}\\ &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{2-e^{-j\omega_n T}-e^{j\omega_n T}}{\omega_n^4 T^2}\\ &= \frac{N_0}{T^2}\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{6(1-z^{-1})^4}(2-z^{-1}-z)\\ &= -N_0T\frac{1+4z^{-1}+z^{-2}}{6(1-z^{-1})^2}\\ &= N_0T\frac{z+4+z^{-1}}{6(1-z^{-1})(1-z)}\\ &= N_0T\frac{(z^{-1}+a)(z+a)/a}{6(1-z^{-1})(1-z)},\ a=2+\sqrt{3}\\ &= \left|\sqrt{N_0}\sqrt{T}\frac{(z^{-1}+a)}{(1+a)(1-z^{-1})}\right|^2,\ a=2+\sqrt{3}.\end{aligned}$$

Example 3: $H(s) = \frac{1}{s^2}\frac{1-e^{-sT}}{sT}$.

$$\begin{aligned}\phi_y(\omega) &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{1-e^{-j\omega_n T}}{\omega_n^3 T}\frac{1-e^{j\omega_n T}}{\omega_n^3 T}\\ &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{2-e^{-j\omega_n T}-e^{j\omega_n T}}{\omega_n^6 T^2}\\ &= \frac{N_0}{T}\sum_{n=-\infty}^{\infty}\frac{1-e^{-j\omega_n T}}{\omega_n T}\frac{1}{\omega_n^5}-\frac{e^{j\omega_n}(1-e^{j\omega_n T})}{\omega_n T}\frac{1}{\omega_n^5}\\ &= N_0\frac{T^3}{120}\frac{z^4+26z^3+66z^2+26z+1}{(z-1)^5}(1-z)\\ &= N_0\frac{T^3}{120}\frac{z^2+26z^1+66+26z^{-1}+z^{-2}}{(1-z)^2(1-z^{-1})^2}\\ &= \left|\sqrt{N_0}\frac{T\sqrt{T}}{\sqrt{120}}\frac{(z^{-1}+a)(z^{-1}+b)}{\sqrt{ab}(1-z^{-1})^2}\right|^2\\ &\qquad a=23.2,\ b=2.32,\ ab=53.9.\\ &= \left|\sqrt{N_0}T\sqrt{T}\frac{(z^{-1}+a)(z^{-1}+b)}{(1+a)(1+b)(1-z^{-1})^2}\right|^2.\end{aligned}$$

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