Linear Equalizers for Turbo Equalization A New Optimization Criterion for Determining the Equalizer Taps

D. Raphaeli* and A. Saguy *

*Electrical Engineering-Systems Dept., Tel-Aviv University, 69978 Tel-Aviv ISRAEL.
Phone : (+972) 3 640-7051
E-mail: danr@eng.tau.ac.il
E-mail: asaguy@helioss.co.il

Abstract: *This paper investigates the subject of turbo equalizations in which a receiver combines the equalization and decoding process in an iterative fashion. In order to save in complexity, a linear equalizer with two inputs, one for the received signal and one for the extrinsic information, replaces the aposteriori probability (APP) processor used for the equalization. The equalizer tap gains are determined as a mean square approximation to the APP. We show that such optimization criterion for determining the equalizer tap gains is better than minimum mean square error (MMSE) optimization of such taps. Simulation results are presented for parallel-concatenated turbo code with BPSK modulation over channels that introduce severe amplitude distortion.*

Keywords: Turbo Codes, Equalization.

1. INTRODUCTION

Turbo Codes have aroused much exciting in the past years due to their ability to achieve performance very close to the Shannon limit at moderate BERs for large enough information sequence block lengths [1]. Extension of the turbo decoding principle to the equalization process referred to as Turbo equalization has also been investigated [2]. It was shown in [2], that Turbo equalization is superior to independent decoding and equalization. In [4], in order to save in complexity, a linear equalizer with two inputs, one for the received signal and one for the extrinsic information, replaces the APP used for the equalization. The equations for determining the equalizer taps were based on the minimum mean square error (MMSE) criterion. In this paper, we show that it is possible to view this equalizer as an approximation to the APP, and using this view to optimize the coefficients. The optimization criterion presented in this paper, called minimum mean log likelihood ratio (LLR) square error, is used for determining the tap gains of the equalizer.

The equalizer tap gains are a function of the channel impulse response, extrinsic information and SNR. The extrinsic information is modeled as a Gaussian sequence with variable variance throughout the training process. For each channel, the equalizer tap gains are determined for different SNRs and extrinsic information variance. The tap gains are stored in a look-up table.

The least mean square (LMS) algorithm is used to train the equalizer tap gains. The training scheme consists of a linear equalizer and an APP processor connected in parallel. The received signal and extrinsic information are supplied to both a linear equalizer and APP processor. The output of the APP processor is used as the target sequence to adapt the linear equalizer tap gains. A linear analytical approximation to the APP, which is currently being pursued, will absolve the need for the APP processor.

During the equalization process, the SNR and the variance of the extrinsic information are calculated and serve as an index to the look-up table from which the equalizer tap gains are read. The received signal and extrinsic information are linearly filtered and the result is provided to the turbo decoders.

We will show through simulation, that optimization of the tap gains using the minimum mean LLR square error criterion is better in performance than MMSE and saves in complexity compared to the APP processor. The disadvantage of such a criterion compared to the MMSE criterion, is that the channel must be known and stationary and its performance is poorer than the APP processor.

2. SYSTEM MODEL

Consider that the information bits $\{u_k\}$, selected

encoded by a turbo encoder. The turbo encoder consists of two identical recursive systematic (RSC) encoders separated by a random interleaver. The coded bits are BPSK modulated, block interleaved and sent over a linear baseband channel. The linear baseband channel is the equivalent of the cascade of a transmitter filter, transmission path and receiver filter [3]. The symbols at the output of the transmission path filter are corrupted by an additive zero-mean white Gaussian noise and a flat two-sided power spectral density of No/2. The receiver filter consists of a matched filter and a noise-whitening filter. The received waveform is sampled once per data symbol to give the received samples r_n:

$$r_n = \sum_{j=-L_1}^{L_2} s_{n-j} y_j + w_n \quad (1)$$

where s_n are the transmitted symbols, y_j are the components of the sampled impulse response of the baseband channel with L_1 precursor ISI terms and L_2 postcursor terms and w_n, the noise components, are statistically independent Gaussian random variable with zero mean and variance ^{2} = No/2.

3. TURBO EQUALIZATION

The turbo equalization process is performed in an iterative fashion as illustrated in figure 1. Each iteration, the equalizer attempts to improve its mitigation of the deleterious affects of the channel induced distortion and noise by using side-information or extrinsic symbol information obtained from the turbo-code decoders. The equalizer output, in turn, feeds the turbo-decoders, which yield a better estimate of the symbols due to the improved input from the equalizer. The process is iterated until no further improvement is gained from another iteration.

3.1. Equalizer Structure

The equalizer structure is similar to the structure presented in [4]. For the first iteration, the equalizer consists of one feedforward transversal linear filter. For the following iterations, the equalizer consists of two filters. The first filter, receives the samples at the output of the channel and has M_1+M_2+1 taps with gains given by

$$D = \lfloor d_{-M_1} \quad d_{-M_1+1} \quad \dots \quad d_{M_2} \rfloor \quad (2)$$

The second filter, receives the soft-symbol estimation $\bar{s}_n$ provided by the turbo decoders and has N_1+N_2+1 taps with gains given by

$$F = \lfloor f_{-N_1} \quad f_{-N_1+1} \quad \dots \quad f_{N_2} \rfloor \quad (3)$$

The output of the equalizer is given by x_n:

$$x_n = \sum_{j=-M_1}^{M_2} r_{n-j} d_j - \sum_{j=-N_1}^{N_2} \bar{s}_{n-j} f_j \quad (4)$$

The soft-symbol estimate is taken to be the mean symbol estimate, which for BPSK modulation reduces to

$$\bar{s}_n = \frac{e^{L_e(s_n)} - 1}{e^{L_e(s_n)} + 1} \quad (5)$$

where $L_e(s_n)$ is the extrinsic symbol information supplied by the turbo decoders from the previous iteration.

3.2. Equalizer Tap Gain Training

The criterion for optimizing the tap gains of the linear filters is the mean Log Likelihood Ratio (LLR) square error:

$$E\left\{ \left(x_n - LLR(s_n)\right)^2 \right\} \quad (6)$$

which differs from the MMSE, applied in [4]:

$$E\left\{ \left(x_n - s_n\right)^2 \right\} \quad (7)$$

In order to adapt the tap gains of the equalizer filters for the different channel impulse response SNR and extrinsic information, the symbol LLR during the training process, must be supplied by an external source. The adaptation structure is illustrated in figure 2. A symbol-by-symbol APP processor that yields the symbol LLR is used to obtain the target training sequence. The APP processor is based on the fact that the sampled impulse response of the channel can be modeled as a Markov chain and its behavior can be represented by a trellis diagram.

The extrinsic information is modeled as a zero-mean Gaussian process and variable variance. The motivation for this model is based on inspection the histogram of the extrinsic information [1]. It can be seen that the histogram has a bell-shaped curve for low SNRs and the first few iterations. For high SNRs or as the iteration increases, the hypothesis of zero-mean is not satisfied. However, at this point the extrinsic information of the turbo decoder is more reliable and has mo
the turbo decoder than the received channel symbols. Therefore, the assumption of Gaussian distribution is valid in the region where the equalizer output is critical for correct turbo decoding and is not valid in the region where the turbo decoder has reached a clear decision and gains minimal information from the equalizer output.

Through simulation, it has been determined that the equalizer tap gains attain a more stable steady state value if the input to filter F is replaced the mean symb ol value evaluated from the Gaussian distributed extrinsic information similar to [4].

The equalizer is trained for different SNRs and extrinsic information variance. For each SNR and variance, a sequence of received data symbols and zero-mean Gaussian sequence are generated. The two sequences enter the APP processor and the appropriate filter. The output of the APP processor is used to adapt the filter tap gains. The equalizer tap gains are stored in a look-up table where the SNR and the Gaussian variance serve as indexes. Because LLR(s_n) is a non-linear function of the baseband channel sampled impulse response components, the tap gains are adapted using the least mean square (LMS) algorithm:

$$D_n = D_{n-1} - mR_n\left(x_n - LLR(s_n)\right) \quad (8)$$

$$F_n = F_{n-1} + m\overline{S}_n\left(x_n - LLR(s_n)\right) \quad (9)$$

where m is a scale factor, F_n and D_n are the vectors of filter tap gains, R_n is the vector of received channel data symbols

$$R = \left[r_{n-M_1} \quad r_{n-M_1+1} \quad \dots \quad r_{n+M_2} \right] \quad (10)$$

and S_n is the vector of mean symbol extrinsic information at time n

$$\overline{S}_n = \left[\overline{s}_{n-N_1} \quad \overline{s}_{n-N_1+1} \quad \dots \quad \overline{s}_{n+N_2} \right] \quad (11)$$

3.3. Equalizer Operation

Once the equalizer tap gains have been evaluated for the specific channel, the equalization process is performed according to figure 3. Each iteration, the variance of the extrinsic information from the decoders is evaluated and along with the SNR serve as an index to a look-up table from which the tap gains are read. The received signal and extrinsic information after processing by the mean symbol estimator are independently filtered, combined and presented to the turbo decoders.

4. PERFORMANCE AND RESULTS

The performance of the turbo equalizer and decoder has been evaluated by simulation. A turbo code of rate 1/3 with two identical recursive systematic encoders with generating polynomials $(37,21)_8$ separated by a random interleaver of length 10000 was simulated. The block interleaver rows and columns are 300 and 100 respectively. The lengths of the filter D and F were fixed at 31.

The components of the baseband channel are

$$y_0 = 0.4097 \quad y_1 = 0.815 \quad y_2 = 0.4097$$

The channel has two zeros on the unit circle at z = -0.9945 0.1044i and no poles and thus introduces severe amplitude distortion. A comparison between an equalizer whose tap gains were optimized according to LLR square error criterion (referred as equalizer A) and an equalizer whose tap gains were optimized according to the MMSE criterion (referred as Equalizer B) is shown in figure 4. Also shown is the sixth iteration of an APP processor (referred as Equalizer C).

5. CONCLUSIONS

In this paper a new optimization criterion, the mean Log Likelihood Ratio square error, for determining the tap gains of a linear equalizer has been proposed. The performance of the equalizer was compared to [4] where the optimization criterion was the MMSE. For the example shown, the asymptotic gain at the sixth iteration compared to MMSE is about 0.2dB and is poorer than an APP processor by about 0.1dB.

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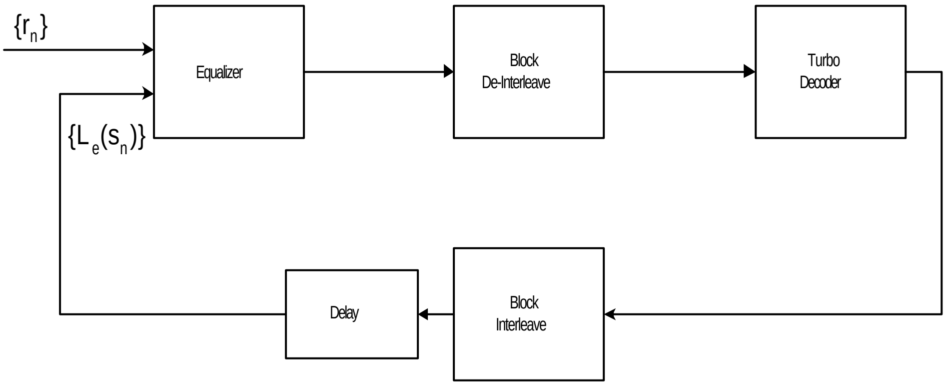

Figure 1: Turbo Equalizer

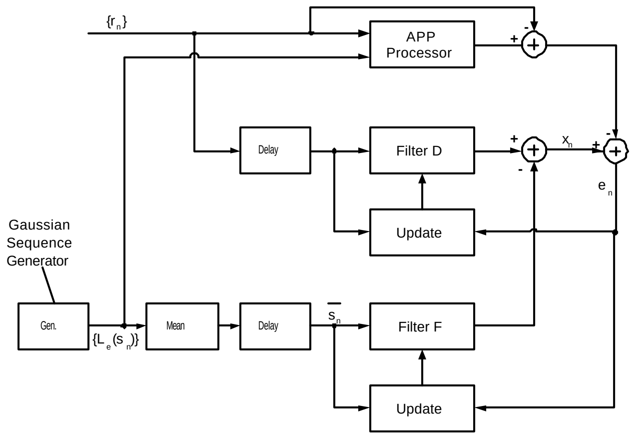

Figure 2: Training Scheme

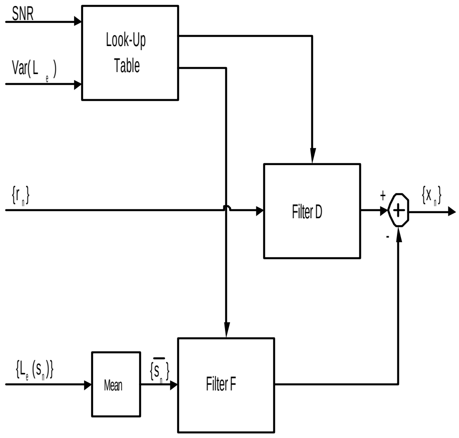

Figure 3: Equalizer Operation

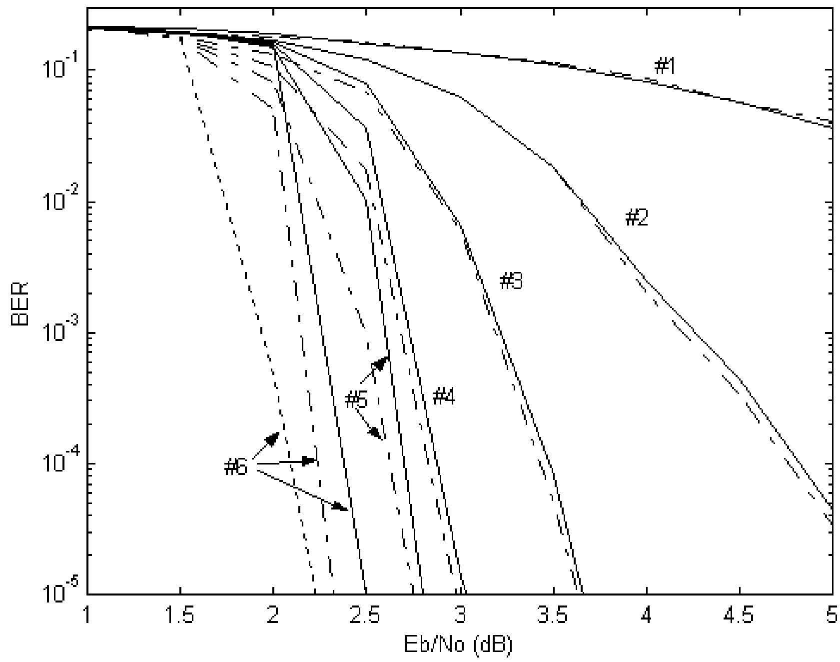

Figure 4: Equalizer A (dashed), B (solid) and C (dotted)