Brightness contrast–contrast induction model predicts assimilation and inverted assimilation effects

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In classical assimilation effects, intermediate luminance patches appear lighter when their immediate surround is comprised of white patches and appear darker when their immediate surround is comprised of dark patches. With patches either darker or lighter than both inducing patches, the direction of the brightness effect is reversed and termed as “inverted assimilation effect.” Several explanations and models have been suggested, some are relevant to specific stimulus geometry, anchoring theory, and models that involve high level cortical processing (such as scission, etc.). None of these studies predicted the various types of assimilation effects and their inverted effects. We suggest here a compound brightness model, which is based on contrast–contrast induction (second-order adaptation mechanism). The suggested model predicts the various types of brightness assimilation effects and their inverted effects. The model is composed of three main stages: (1) composing post-retinal second-order opponent receptive fields, (2) calculations of local and remote contrast, and (3) adaptation of the second-order (contrast–contrast induction). We also utilize a variation of the Jacobi iteration process to enable elegant edge integration in order to evaluate the model is performance.

Keywords: computational model, assimilation, inverted assimilation, brightness, contrast–contrast induction, contrast gain control


Introduction

The effects of brightness (or chromatic) assimilation and contrast induction both depend on the contextual inducing of the light or surfaces (Anstis, 2005; Hong & Shevell, 2004). Brightness induction and assimilation effects have been regarded in the literature as two conflicting brightness effects. The brightness induction effect (first-order induction) exhibits an appearance of a shift in brightness away from the brightness of the surrounding surface, whereas the assimilation effect exhibits a perceived shift toward the brightness of adjacent surround surfaces (Hong & Shevell, 2004; Shapley & Reid, 1985). An additional induction effect that has been described is the contrast–contrast induction which is the effect of modulating the perceived contrast of a central area due to the contrast of its surrounding area (Ozak & Laurinen, 1999, 2005; Singer & D'Zmura, 1999; Xing & Heeger, 2001).

Helson (1963) suggested that the assimilation and contrast induction are actually continuum effects, which probably derive from the same mechanism. He suggested that spatial frequency of the lines determine the type of effect. Due to this approach a recent computational model has been suggested (Vanrell, Baldrich, Salvatella, Benavente, & Tous, 2004).

One of the prominent and most investigated assimilation effects is the White’s effect in lightness (White, 1979; Figure 1). In this effect the gray stripes superimposed on black stripes appear much brighter than when they are positioned between the black stripes. Different explanations have been suggested to explain this effect. Among the more recent explanations are

1. Anderson’s (1997) suggestion that the mechanism of visual “scission” treats the gray lines as separate transparent layers;
2. some explanations and models of the effect related to the stimulus geometry (Ross & Pessoa, 2000; Todorovic, 1997; Zaidi, Spehar, & Shy, 1997). Todorovic (1997) explained, for example, the White’s effect using the T-junction rule;

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Furthermore, Howe (2005) presented a circular variant of White’s effect, suggesting that the junctions are not an important consideration in all versions of White’s effect. The computational model of White’s effect both brightness contrast and assimilation mechanism plays a role (Blakeslee & McCourt, 2004). However, in the classical spatial arrangement presented in Figure 1, it is likely that assimilation mechanism plays a major role (Blakeslee & McCourt, 2004; White, 1981). It has been shown that the White’s effect occurs only when luminance of the test regions lies between the minimum and maximum luminance regions (Spehar, Gilchrist, & Arend, 1995; Spehar, Iglesias, & Clifford, 2005). An opposite effect occurs, “inverted White’s effect,” when the luminance of the test regions lies outside the luminance range of the surrounding strips (Figure 11A; Ripamonti & Gerbino, 2001; Spehar, Clifford, & Agostini, 2002). Many of the explanations (including 2 and 4, above) and models such as the oriented filter of Blakeslee and McCourt (1999), which have been given for the classical White’s effect, cannot account for the inverted White’s effect (Figure 11A). This opinion was previously expressed by Spehar and colleagues (2002), who claimed that models such as contrast-integration models which also account for the context (Ross & Pessoa, 2000) or T-junction (Todorovic, 1997; Zaidi et al., 1997) would be unsuccessful in including the inverted White’s effect. Furthermore, Howe (2005) presented a circular variant of White’s effect in which all the junctions have been removed without significantly affecting the strength of the illusion, suggesting that the junctions are not an important consideration in all versions of White’s effect.
model of Blakeslee and McCourt (2004) and the more elaborated version of this ODOG model (Robinson et al., 2007) cannot account for all of the geometrical variations of the effect, which appear also with random patterns (Anstis, 2005). As far as we know, there is no suggested computational model in the literature which succeeded in predicting the “conflicting” effects of assimilation, “inverted assimilation,” or even one of set of these effects such as the White’s effect and its inverted effect (Figures 7A and 11A). Our suggested model presents a single mechanism and computational model that shows predictions of these different brightness effects, which are perceived as opposite effects.

**Model**

The model aimed at suggesting a post-retinal mechanism for performing achromatic induction of the second order, i.e., adaptation of the second order. The model is composed of three main stages. The first stage describes the transformation of the visual stimulus into the response of post-retinal second-order opponent receptive fields (SORFs). These SORFs might refer to cortical levels even though the receptive fields are not necessarily oriented. These SORFs have various spatial resolutions, in compliance with the diversity found in the visual system. The second stage of the model describes the calculations of local and remote contrast, based on the multi-scale SORF responses. The third stage describes the adaptation of the second order (contrast–contrast induction) (Chubb, Sperling, & Solomon, 1989; Spitzer & Barkan, 2005; Xing & Heeger, 2001). This adaptation mechanism plays a role as a contrast gain control and is expressed by the adapted responses of the SORF cells. In order to evaluate the performance of the computational model, we performed a transformation of the adapted SORF cells’ responses to a perceived image in a standard intensity space.

**Response of the second-order opponent receptive field**

The SORF cells receive their input from the retinal ganglion cells through several processing layers. The retinal ganglion cells perform an adaptation of the first-order and the SORF cells receive their responses after the adaptation of the first order. This model focuses on the adaptation of the second order (contrast–contrast induction) due to the current scope of the paper. In the current scope, when only brightness contrast–contrast induction is examined, the brightness first-order adaptation is not modeled for the sake of simplicity. Accordingly, the effect of brightness simultaneous contrast is not processed here since it is related to the adaptation of the first order (Spitzer & Barkan, 2005).

The SORF cells have an opponent type receptive field with a center-surround spatial structure. Since adaptation of the first order is not modeled here, the RF input was taken from signals of photoreceptors rather than from retinal opponent cells responses.

The spatial response profile of the two subregions of the SORF, “center” and “surround,” is expressed by the commonly used difference-of-Gaussians (DOG).

The “center” signals that feed the SORF are defined as an integral of the photoreceptors quantum catches over the center subregion, with a Gaussian decaying spatial weight function, \( f^c(x - x_0, y - y_0) \) (Shapley & Enroth-Cugell, 1984):

\[
L^k_{cen}(x_0, y_0) = \int \int L_{photo-r}(x, y) \cdot f^k_c(x - x_0, y - y_0) \cdot dx \cdot dy, \tag{1}
\]

while the \( L_{photo-r} \) at locations \( x_0, y_0 \) represents the response of the center area of the SORF, which is centered at location \( x_0, y_0 \). The following equations are similarly expressed, but for the sake of simplicity, \( x_0, y_0 \) at the following equations are substituted by zero: \( x_0 = y_0 = 0 \). \( k \) is an index that represents the specific spatial resolution, where \( k = 1 \) is the finest resolution. \( f^c_c \) is defined as:

\[
f^k_c(x, y) = \frac{\exp(-x^2 + y^2/\rho^2_{cen})}{\pi \cdot \rho^2_{cen}}; y \in center\_area, \tag{2}
\]

where \( \rho \) represents the radius of the center region of the receptive field. The “Surround” signals are similarly defined, with a spatial weight function three times larger in diameter than that of the “center.”

\[
L^k_{srnd}(x_0, y_0) = \int \int L_{photo-r}(x, y) \cdot f^k_s(x, y) \cdot dx \cdot dy, \tag{3}
\]

where \( f^k_s \) is defined as a decaying Gaussian over the surround region:

\[
f^k_s(x, y) = \frac{\exp(-x^2 + y^2/\rho^2_{sur})}{\pi \cdot \rho^2_{sur}}; x, y \in surround\_area. \tag{4}
\]

The total weight of \( f^c_c \) and \( f^k_s \) is 1.

The response of the SORF, \( L_{sort} \), is the subtraction of the “center” and “surround” signals:

\[
L^k_{sort} = L^k_{cen} - L^k_{srnd}. \tag{5}
\]

The calculations of the SORF response might be alternatively derived from an achromatic double-opponent...
Local and remote contrast

We propose that the adaptation mechanism acts as a remote contrast gain control on the SORF response. It is therefore necessary to calculate local and remote contrasts. We suggest that contrast calculation can be obtained from the multi-scale SORF.

Since the presented model processes contrast domain, it is reasonable to suggest a gain control mechanism that is dependent on zones which are spatially larger than edges which are commonly modeled as classical opponent receptive fields. A single classical opponent RF cannot account as a measure to contrast, as it only measures a local edge of a stimulus. We propose, therefore, that local and remote contrasts are measured within local and remote regions. In these regions the contrast is measured by considering a variability of opponent RF sizes which represent opponent RF with different scales.

Our model presumes that in the remote area there is higher sensitivity to higher contrasts than to low contrast in the same spatial location. Therefore, the contrast integration across the remote area of the opponent RF is not necessarily a linear spatial integration. Such a non-linear integration, as suggested here, in where the higher responses to higher contrast obtain larger weight-function in the spatial integration operation, has physiological plausibility (Discussion).

“Local” contrast

The suggested local contrast calculation is performed by calculating a local smoothing absolute response for each resolution and by summing these responses, Equation 6. Use of absolute signals enables the system to relate to the achromatic SORF responses as a contrast quantifier (Spitzer & Barkan, 2005; Spitzer & Hochstein, 1985).

$$C_{\text{local}}(x, y) = \sum_{k=\text{all scales}} \iint_{\text{local-area}} |L_{\text{sorf}}^k(x, y)| \cdot W^k(x, y) \cdot dx \cdot dy$$

(6)

where $W^k$ is a non-linear spatial weight function which is dependent also on the local opponent response, $L_{\text{sorf}}^k$. (Equation 7). Due to the nature of opponent signals (Equation 5) which perform edge detection at different resolutions, they yield a signal only at locations adjacent to edges. Thus, a common local averaging of such signals would lead to an erroneous low value of average contrast as it would always consider low values that derive from locations that are not adjacent to edges. In order to overcome that obstacle, we propose that in this local area, higher SORF response obtain higher weight function, when calculating their contribution to the local contrast.

$$W^k(x, y) = |L_{\text{sorf}}^k(x, y)| \cdot \exp\left(-\frac{x^2 + y^2}{\rho_{\text{local}}^2}\right).$$

(7)

“Remote” contrast

The remote contrast represents the peripheral area that extends beyond the borders of the opponent classical receptive field (Creutzfeldt, Crook, Kastner, Li, & Pei, 1991; Creutzfeldt, Kastner, Pei, & Valberg, 1991; Solomon, Peirce, & Lennie, 2004). The “remote” area is composed of an annulus-like shape around the entire RF region (Spitzer & Barkan, 2005). The second-order remote signal ($C_{\text{remote}}$) represents the average contrast in this remote area. The remote signal is defined as an inner product of the local contrast ($C_{\text{local}}$, Equation 6) response at each location of the remote area with a non-linear decaying Gaussian weight function ($W_c$) (or as a convolution for all spatial locations). This weight function obtains higher values for higher local contrast values (as was also explained in local contrast calculations, Equation 6).

$$C_{\text{remote}}(x, y) = \frac{\iint_{\text{remote-area}} |C_{\text{local}}(x, y)| \cdot W_c(x, y) \cdot dx \cdot dy}{\iint_{\text{remote-area}} W_c(x, y) \cdot dx \cdot dy},$$

(8)

$$W_c(x, y) = C_{\text{local}}(x, y) \cdot \exp\left(-\frac{x^2 + y^2}{\rho_{\text{remote}}^2}\right).$$

(9)

Adaptation of the second order

We propose that the result of the second-order adaptation can be expressed as a simple gain control mechanism. The local gain factor is determined by the ratio of local contrast and remote contrast. The gain factor (GF) is therefore defined as follows:

$$GF(x, y) = \frac{C_{\text{local}}(x, y) + \beta}{C_{\text{remote}}(x, y) + \beta},$$

(10)

where $\beta$ is a constant that determines the degree of gain. The response of each SORF cell after adaptation is therefor:

$$L_{\text{sorf-adapted}}^k(x, y) = GF(x, y) \cdot L_{\text{sorf}}^k.$$
These SORF responses are the result of the adaptation and are used by higher visual stages as inputs for further processing. In order to evaluate how these signals are interpreted, a transformation from the SORF responses to an achromatic space is required.

**Methods**

This section describes the different parameters used in the model simulation and also presents intermediate results for enhancing the clarity of the model. In this section, we demonstrate the model performance on a specific example—the well known White’s effect image (Figure 1, at a resolution of 300 × 400 pixels). Intermediate model calculations and parameters will be introduced throughout this example. The same parameters were used for all simulated images, which are presented in the Results section.

The model parameters were chosen to be roughly in the range of physiological findings, at least for parameters which have been studied.

**Response of the second-order opponent receptive field**

The “center” signal represents the response of the central area of each SORF. At the finest resolution ($k = 1$), this central area is simulated by a single pixel. Accordingly, the “center” size for the finest grid was chosen as 1 pixel, i.e., $L_{cen}$ (Equation 1) is equal to the original image luminance (at each spatial location). For different $k$ values, the center size was chosen as a diameter of $k$ pixels. The “surround” diameter (Equations 3 and 4) was chosen as 3 times larger than the center diameter (Shapley & Enroth-Cugell, 1984). This ratio was applied for all $k$ resolutions. The “center” and “surround” calculations were applied, and $L_{sorf}$ calculation (Equation 5) was applied by a convolution of the original luminance values with a filter that represent the subtraction of center and surround contributions, for example, when $k = 1$ the convolution kernel is:

\[
\begin{pmatrix}
-0.1095 & -0.1405 & -0.1095 \\
-0.1405 & 1 & -0.1405 \\
-0.1095 & -0.1405 & -0.1095
\end{pmatrix}. \tag{12}
\]

The SORF response (before adaptation; Equation 5) was tested on the White’s effect image, and the result for $k = 1$ is presented in Figure 2. As expected, the responses have been obtained only adjacently to the edges, where the background stripes (black and white in the original image; Figure 1) yield higher “gradient” response (blue and red striped in Figure 2) than the responses to the stripes along the location of the “test” stripes (cyan and yellow in Figure 2).

**Local and remote contrast**

The next step in the model is the calculation of the local and remote contrast, in order to process the texture stimuli (contrast).

**Calculation of local contrast**

The calculation of $C_{local}$ (Equation 6) was performed with a $\rho_{local}$ diameter of 10 pixels for all resolutions. In order to enable the influence of more distant edges, the cutoff of this calculation was taken in a diameter of $5^\circ \rho_{local}$. This large spatial range of cutoff is required due to the significant weight given to high contrast values even at distant locations (Equation 7). The resolutions where calculated with $k = 1, 3, 5, 7$ (model, Equations 1–5).

The intermediate result of the local contrast ($C_{local}$) of the White’s effect is demonstrated in Figure 3. The figure clearly demonstrates the different local contrast regions. The white background represents the high background contrast, while the two gray squares represent the lower contrast squares, derived as a result of regions with the gray stripes. Note that the squares’ contrast differ from each other due to the value of the gray stripe which is not the median value between the values of the white and the black stripes.
Calculation of remote contrast

The calculation of $C_{\text{remote}}$ (Equation 8) was performed with a $\rho_{\text{remote}}$ diameter of 10 pixels for all resolutions. The cutoff of this calculation was taken as a diameter $15^\circ \rho_{\text{remote}}$. This chosen “remote” size is also within the range of the reported electrophysiological findings (Creutzfeldt, Crook, et al., 1991; Creutzfeldt, Kastner, et al., 1991). The intermediate result of this remote contrast is demonstrated in Figure 4. It can be shown that the boarders of the squares are not existent any more due to the larger spatial weight function. Note that remote contrast is calculated over the local contrast response (Equation 6).

The $C_{\text{remote}}$ signals have a smaller response range than the range of local signals (Figure 4) due to the large integration area of the $C_{\text{remote}}$ signal. Since the $C_{\text{remote}}$ signal calculation in the “test” region includes also the background contrast (the white area in Figure 4), the result of the $C_{\text{remote}}$ signal is an average of the contrasts of both “test” and “background” regions. Consequently, the $C_{\text{remote}}$ signal in the “test” area obtains higher values than the local contrast at the same “test” area. In other cases (such as in the inverted White’s effect; Figure 11), the $C_{\text{remote}}$ signal of the “test” area can obtain lower values than the local contrast signal at the same “test” area.

Adapted SORF response

The adapted SORF signals, $L_{\text{ sorf-adapted}}$, are then calculated (Equations 10–11) with $\beta = 0.4$. A smaller value of beta causes a stronger contrast–contrast induction.

Figure 3. Local contrast. Illustration of an intermediate stage of the model, local contrast (Equation 6) which has been performed on the White’s effect image at each $x, y$ location. The contrast degree is presented by the luminance value (color bar on the right) at each location. The dark squares present the texture of the regions with gray stripes. Note that the darker square has been obtained due to smaller contrast of the gray stripes while they are adjacent to the white stripes.

Figure 4. Remote contrast. Illustration of an additional intermediate stage of the model, remote contrast. The “remote” contrast (Equation 8) is higher in the background than in the “test” regions. Note that the border of the test regions is blurred due to the larger area the remote signals extends. The remote contrast is presented in a similar format as presented in Figure 3.

Both the adapted SORF absolute responses and the original SORF absolute responses are plotted, while $k = 1$, along a vertical line located at $x = 300$ (yellow vertical line in Figure 7A) in Figure 5. The adapted SORF, $L_{\text{ sorf-adapted}}$, presents the perceived local DOG, Equation 11. Therefore, suppression or facilitation of these responses, in

Figure 5. Adapted SORF response. Illustration of the final stage of the model (Equation 11) performed on the White’s effect image. The figure presents the suppression of the perceived contrast in the test area due to the adaptation of the second order. Absolute values of SORF responses ($k = 1$), $L_{\text{ sorf}}$ (blue line, Equation 5), and calculated absolute perceived opponent response, $L_{\text{ sorf-adapted}}$ (red dotted line, Equation 11) are plotted along location $x = 300$ (vertical yellow line in Figure 7A). The perceived contrast, $L_{\text{ sorf-adapted}}$ (red dotted line), in the region of the test area ($y = 110–240$), is lower than the original “contrast,” $L_{\text{ sorf}}$ (blue line).
comparison to the original DOG ($L_{o}p$), presents contrast suppression (assimilation effects) or contrast facilitation (inverted assimilation effects) respectively. Figure 5 demonstrates the contrast suppression in the “test” area, which predicts the assimilation effect. In order to transform these perceived values, $L_{Sorf\text{-adapted}}$, into perceived luminance values, $L_{\text{per}}$, a further inverse function has to be performed.

**Transformation of the adapted color cells’ response to a perceived image (inverse function)**

In order to evaluate the performance of our model, we chose to perform a transformation from the SORF values to intensity values ($L_{\text{per}}$) to reflect the perceived luminance values. These values are presented as perceived images (Figures 7–17, results). Since the gain factor (Equation 10) is uniform over all resolutions, it is plausible to demonstrate the inverse function only through the finest resolution ($k = 1$). Since this stage is presented mainly for evaluation of the model, different numerical approaches could be applied at this stage.

For the sake of simplicity and clarity of the calculations the finest resolution was taken as a discrete Laplace operator, which is commonly used as an approximation to DOG function. The finest resolution was, therefore, calculated as a convolution of the original image with a kernel of:

$$\begin{pmatrix} 0 & -1/4 & 0 \\ -1/4 & 1 & -1/4 \\ 0 & -1/4 & 0 \end{pmatrix}. \quad (13)$$

The adapted response ($L_{Sorf\text{-adapted}}$) is approximately a convolution of the above kernel (Equation 13) with $L_{\text{per}}$, therefore,

$$L_{(Sorf\text{-adapted})}^{k=1}(n,m) = L_{\text{per}}(n,m) - 1/4 \cdot (L_{\text{per}}(n-1,m) + L_{\text{per}}(n,m-1) + L_{\text{per}}(n+1,m) + L_{\text{per}}(n,m+1)). \quad (14)$$

where $n, m$ represent the location index. An extraction of $L_{\text{per}}$ would yield an iterative function:

$$L_{\text{per}}^{i+1}(n,m) = L_{(Sorf\text{-adapted})}^{k=1}(n,m) + 1/4 \cdot (L_{\text{per}}^{i}(n-1,m) + L_{\text{per}}^{i}(n,m-1) + L_{\text{per}}^{i}(n+1,m) + L_{\text{per}}^{i}(n,m+1)), \quad (15)$$

where $i$ represents the $i$th iteration. This is in fact, a variation of the Jacobi method (Quarteroni & Valli, 1994).

The first guessed perceived luminance, $L_{\text{per}}^{(0)}$, at the inverse function according to the variation of the Jacobi method was chosen as the original image at all locations, for this example. After applying Equation 15, the estimated perceived luminance iterations: $L_{\text{per}}^{(1)}, L_{\text{per}}^{(100)},$ and $L_{\text{per}}^{(1000)}$ are obtained and shown in Figure 6. After 100 iterations (Figure 6B), the process starts to converge.

Figure 6. Calculation of perceived luminance. The figure presents iterations 1, 100, and 1000 (A, B, and C, correspondingly). A is the original image. The convergence of the iterations can be seen at C.
Iteration 1000 (Figure 6C) shows the predicted luminance image. At this stage, we emphasize the feasibility of the model and not the efficiency of this inverse procedure.

**Results**

Our model predictions are presented for a variety of brightness assimilation effects and contrast–contrast induction effects. The results are presented first, for the well known White’s effect (Figure 7; White, 1979), the recent Hong and Shevell’s achromatic ring patterns (Figure 8; Hong & Shevell, 2004), the random Stuart’s rings (Figure 9; Anstis, 2005), and a new “liquefied” image which demonstrates the assimilation effect (Figure 10). We added the “liquefied” image and its inverted assimilation image (not presented) in order to demonstrate the generalization of the effects that are not connected to specific periodicity, oriented lines or expected shapes. The prediction of the “inverted White’s effect,” that was first described by Rippamonti and Gebino (2001), is also shown (Figure 11). In addition, we present the model

![Figure 7](image)
prediction of new inverted effects, that we have found, the “inverted achromatic ring patterns” (Figure 12) and “inverted Stuart rings” (Figure 13).

To further challenge the model, we present predictions to a challenging stimuli presented by Bindman and Chubb (2004), the bull’s-eye illusion and checkerboard contrast illusion (DeValois & DeValois, 1988; Gilchrist et al., 1999; Figures 14 and 15, respectively). In addition, we present the model predictions to controlled stimuli suggested by Spehar and colleagues (Spehar, Debonet, & Zaidi, 1996; Spehar & Zaidi, 1997; Figures 16 and 17). All of the predictions were simulated with the same set of model parameters (Methods).

The results show that the model succeeded in predicting all of these effects correctly, with the same set of parameters (Figures 7–17). The images and the curve results of all the tested assimilation effects show the correct direction of the brightness shift. Figures 7–10 show the prediction of higher luminance of the gray surfaces, while their immediate surround is white and vice versa. This trend of result has also been obtained for the new “liquefied” image (Figure 10), checkerboard contrast illusion (Figure 14), and bull’s-eye illusion (Figure 15). The same trend has also been obtained for the controlled stimuli of Spehar and colleagues (Figures 16 and 17).

In all of the tested inverted assimilation effects (Figures 11–13), the model images results and curves show the correct prediction of the opposite effect, i.e., the predicted test surfaces (highest luminance) obtained higher values, while the immediate surround region is black rather than when their immediate surround is gray.

In all of the model predictions of the assimilation effects (Figures 7–10) and inverted assimilation effects (Figures 11–13), we can observe that the predicted
perceived brightness shift is larger in the “test” regions (gray regions) than in the background (black and white) regions. This trend of results is demonstrated in the red curves in comparison to the blue curves in Figures 7C–13C.

Discussion

We developed a brightness adaptation model of the second order for the different brightness assimilation effects and their inverted effects. The model is based on physiological building blocks, second-order receptive fields (SORF), and their adaptation mechanisms of gain control.

The model succeeds in predicting different assimilation effects and inverted assimilation effect with a single gain control mechanism of contrast–contrast induction. However, the presented model cannot predict simultaneous contrast effects.

The model predictions support the idea that a single second-order induction mechanism (contrast–contrast induction) rules opposite perceived effects, i.e., assimilation and “inverted” assimilation effects. Although simultaneous contrast effects express similarity to the “inverted” assimilation effect regarding the direction of the brightness shift, we suggest that these two effects are distinguished and derive from two different mechanisms. The simultaneous contrast effect is derived from the first-order adaptation and can be seen as induction of the first order, while the “inverted” assimilation effect is derived from the adaptation of the second order (induction of the second order).

In order to evaluate the model performance in image presentation (luminance domain) rather than only SORF response, we propose a numeric inverse function. The
inverse function includes a variation of Jacobi iteration process (Quarteroni & Valli, 1994), which enables an inverse of the model outputs after adaptation to the perceived image, including the transformation of "gradient" values into luminance values. This suggested numeric method is one of many possible methods. We choose this method due to its relatively simple implementation.

Our model contains several physiologically based elements and building blocks, and some plausible physiological mechanisms. The post-retinal receptive fields modeled here have been suggested as having balanced center-surround regions.

The post-retinal receptive fields building blocks can also be built from basic components of non-balanced center and surround regions, which are more physiologically plausible, as found for the retinal receptive fields (Croner & Kaplan, 1995). We have also tested implementing the unbalanced opponent response as the input to balanced second-order receptive fields and found a similar trend of results. This type of second-order receptive field would resemble the chromatic double-opponent structure that was suggested also previously for the chromatic second-order induction (Spitzer & Barkan, 2005). We chose the first method because it leads to relatively simple calculations, and more crucial, it enables faster and smoother iterative inverse function.

In addition, the suggested model consideration of having higher sensitivity to higher remote contrasts than to low contrast in the same spatial location has to be systematically tested psychophysically. Such an effect can be derived from electrophysiological mechanisms, such as a mechanism of change in a conductivity of excitatory ion channels, which depend on the electrical potential (Hodgkin-Huxley model).

Figure 10. Modified White’s effect and its model predication. See Figure 7.
We tested the model predictions for a variety of brightness assimilation effects, among them: White’s effect (White, 1979; Figure 7), achromatic ring patterns (Hong & Shevell, 2004; Figure 8), random “Stuart rings” (Anstis, 2005; Figure 9), and a new display of “liquefied” patterns (Figure 10).

Anstis (2005) presented a challenging stimulus and wrote: “Some geometric theories of White’s effect invoke the role of T-junctions in the stimulus, or of elongated receptive fields in the stimulus. However, a new isotropic brightness illusion called “Stuart’s Rings,” which can be stronger than White’s effect, seems to rule out these theories.” Figure 9 demonstrates the performance of our model on this type of random stimuli. It demonstrates the correct prediction of perceived luminance shift to this type of assimilation stimulus and its inverted effect (Figure 13). The model also predicted, correctly, additional random assimilation patterns that are designed from curved patterns Figure 10.

In addition, we tested the model predictions for the above assimilation inverted effects, such as the inverted White’s effect (Ripamonti & Gerbino, 2001; Figure 11) and new
inverted assimilation effects, among them inverted achromatic ring patterns and inverted Stuart rings (Figures 11–13). The results show that the model succeeded in predicting all these effects correctly, with the same mechanism, model, and with the same set of parameters (Figures 7–17). The predicted images and curves of all the tested assimilation effects show the correct direction shift of brightness. It shows in all cases higher luminance of the gray surfaces while the immediate surround are the white zones and lower luminance while the immediate surround are the black zones. The curves show in all the tested inverted assimilation curves the corrected prediction of opposite effect, i.e., the predicted white zone obtained higher values while the immediate surround region were the black zones and have higher values than in the case where the white zones immediate surround region are the gray zones.

Research of the assimilation effects in recent years pose challenges to existing and future explanations of the effect. These studies present specific “difficult” stimuli that show the assimilation effect or its inverted effect. These stimuli could not be predicted by previous computational models and explanation for the assimilation effects. Two of these stimuli are presented in the study of Bindman and Chubb (2004). They wrote in reference to the failure of different brightness models in predicting the bull’s-eye assimilation effects (Figure 14) that pose an important challenge for the brightness model. We tested these types of bull’s-eye assimilation effect stimuli and the checkerboard contrast illusion (DeValois & DeValois, 1988; Gilchrist et al., 1999; Figures 14 and 15, correspondingly) and showed the correct predictions of our model for these stimuli.

Figure 12. “Inverted” Shevell’s achromatic ring patterns rings and its model predication. See description in Figure 11.
Figure 18 in the discussion shows the results regarding the classical contrast–contrast effect (induction of the second order, similarly as has been shown with similar chromatic stimuli (Spitzer & Barkan, 2005). These results of contrast–contrast induction predicted by the same model support the basic assumptions of this model.

In the Discussion section we added three additional stimuli, which are referred to in the literature as unique stimuli. Figures 19–21 show stimuli of Spehar and colleagues (Spehar et al., 1996; Spehar & Zaidi, 1997) who refer to them as “phenomena of perceived contrast and brightness.” These stimuli present an effect that is yielded from different background contrast levels of texture but have identical luminance values.

Figures 19–21 also present the successful predictions of all tested target stimuli. The trend of the model results show that the mechanism of induction of the second order can solely predict the correct trend of the perceived results. Thus, we conclude that the adaptation of second order plays the major role in these effects (Figures 19–21). However, additional mechanism may contribute to these effects.

This prediction is in conflict with the previous argument that suggested that the effect in these configurations (Figures 19–21) cannot be explained in terms of simple lateral contrast–gain control (Spehar & Zaidi, 1997) as actually suggested in this model.

We suggest that the above effects cannot be derived from induction of the first order since the average luminance is identical in all stimulus images. However, the results of the model show that contrast gain control suffices in order to predict this trend of effects.

The rational of these predictions is that the perceived luminance is a result of both the contrast–contrast induction and the edge integration (part of the inverse function, Equations 11–15). See, for example, Figure 20.
presuming all four target rectangles are suppressed by similar contrast induction. This induction is derived from the remote contrast that is similar for the four stimuli targets due to the non-linearity integration of the remote area (Equation 8). Since the remote contrast is almost identical and larger than the local contrast of target’s edge, the contrast gain control will cause contrast suppression for all target stimuli. However, the direction of the perceived luminance shift is determined by the luminance of the contact area. Accordingly, the right target stimuli obtain dark shift of their appearance since the longer horizontal contacting stripes are darker in each location than the luminance of the target stimuli. In the left target stimuli there is no significant luminance shift, since the longer horizontal contacting stripes contain contradicting higher and lower luminance values than the gray target stimuli.

The model for the different assimilation effects and their inverted effects is based on the same suggested mechanism that activates the contrast–contrast phenomenon. Two previous studies suggested the idea that a texture appears to be perceived with lower contrast when surrounded by high-contrast texture, and same mechanism is accounted for assimilation effects (Bindman & Chubb, 2004; Guclu & Farell, 2005). This contrast–contrast mechanism activates the dual effects (suppression and enhancement) of the contrast induction effects Figure 18. We suggest here that the assimilation effects and their inverted effects can be comprehended as a specific case of the dual contrast–contrast effects.

We would like to emphasize that we believe that the induction of the first order (simultaneous contrast) is a mechanism that enhances the relevant brightness or color and not the texture, i.e., contrast–contrast domain. The role of the adaptation of the second order is to enhance the texture or contrast differences between object or surface and its context. It has to be note that these two processes
appears as contradicting each other at the achromatic domain, while in chromatic domain they can both coexisted (Anstis, 2005).

The contrast–contrast effects have been investigated psychophysically thoroughly and have been summarized by Bidman and Cubb (2004), which related only to the suppression aspect of the effect: “medium contrast texture patch surrounded by high contrast texture appears to have a lower contrast than an identical patch surrounded by low contrast texture (Cannon & Fullenkamp, 1996; Chubb et al., 1989; Olzak & Laurinen, 1999; Singer & D’Zmura, 1994; Solomon, Sperling, & Chubb, 1993; Spehar et al., 1995).” The enhancement aspect of the effect has been reported less intensively; it is the opposite effect which shows that a test patch contrast can be enhanced by context lower contrast (Olzak & Laurinen, 2005; Spitzer & Barkan, 2005; Xing & Heeger, 2001).

The chosen specific procedure to perform the inverse function has been determined due to the iteration of Jacobi (Quarteroni & Valli, 1994), which enabled us to apply a

Figure 15. Checkerboard contrast illusion and its model prediction. See description in Figure 7.

Figure 16. The classical White’s effect and its inverted White’s effect in a controlled configuration (Spehar et al., 2002) and the model predictions (right column). The specific configuration is such that the effect is manifested on test stimuli that have the same contrast range among the inducing stripes in both classical and inverted variants. The results are presented along the colored axis in the same format as Figure 7. The model’s predictions show the correct trend for all the test stripes. The capital letters (A1/2–D1/2) indicate the specific locations in the original stimulus image and in the result curves, correspondingly. Note that in the original stimulus (left column), identical capital letters (A1/2–D1/2) refer to equal intensity values.
variation of a numerical algebraic mathematical tool to perform a “filling-in” process that is required in order to transform the “contrast” (or gradient) responses, $L_{\text{surf}}$-adapted (Equation 11) to a luminance image. This method can be compared to previous filling-in used methods suggested for mechanisms of assimilation effects (Grossberg, 2003; Spehar et al., 2002). As far as we know, the previous used filling-in methods, which relate to modeling psychophysical effects (such as assimilation and Kanizsa effects), do not enable application with real images. The modified Jacobi method (Quarteroni & Valli, 1994) used here (Equation 15) enables this application and with less arbitrary constrains. It has to be mentioned that previous models which applied filling-in components used these components as part as the model. In the presented model, however, the filling-in component is used only as part of the inverse function in order to transform the “gradient” image to luminance image.

The usage of the specific method for inverse function leads to good image’s results, which are not strongly influenced by the spatial resolution and are mildly impaired with the common simulated results such as with “halo” artifacts (Reinhard, Ward, Pattanaik, & Debevec, 2005). Results images in this quality enable an evaluation of the predicted psychophysical effects (Figures 7–21). The quality level of output images can be compared to few published predicted output images, mainly from the predicted White’s effect done by ODOG and LDOG model in the computational studies (Blakeslee & McCourt, 2004; Robinson et al., 2007). Our results show images with minimal distortions in the shape of the predicted stimulus and predicted modulation in the test regions not less than in the surrounding region as observed in some predicted results in previous studies.

Although the main reason of implementing the inverse function was to evaluate the model performance, it does not exclude the possibility that such neuro-physiological process might exists. Such an application can be performed by a simple neurological network consisting of several layers. This can be done by utilizing a parallel network, containing lateral connections which has excitatory connection to its surrounding neighboring neurons (the “surround” area in Equation 15) and also a feed-forward longitude excitatory connection ($L_{\text{surf}}$-adapted in Equation 15).

The model predicts also that higher spatial frequency or smaller retinal size of the pattern contribute to yield stronger assimilation effect (Figure 17). This crucial property is in agreement with many experimental studies in assimilation effect (McCourt, 2005). This property is predicted by model by yielding larger influence of the contrast values in the remote area (Equation 6) to stimuli with high spatial frequencies. In such stimuli, more edges are present in the remote area than in the remote area of low frequency stimuli. In addition, in a similar rational, the model predicts larger assimilation effect while the context area contains larger number of “number of surround-bands” in the different assimilation stimuli for the same spatial frequency stimulus components (Bindman & Chubb, 2004) because more edges will be summed at the remote area.

Psychophysical results showed (Spehar et al., 2002) that the shift of the brightness is smaller at inverted White’s effect than in the White’s effect. We believe that this occurs commonly in psychophysical stimuli since in inverted stimuli usually the contrast ratio (between test and surrounding) is smaller comparing to the classical effect.

Spehar and colleagues (2002) address experimentally the question regarding the differing views on whether the classical and inverted White’s effects are mediated by common or separate underlying mechanisms. They varied the aspect ratio of the test and inducing regions in the classical and inverted White’s effects and compared two specific sets of stimuli. They found that in the “portrait” orientation of the inverted White’s effect configuration (Figure 2 in Spehar et al., 2002) there is an opposite relative brightness shift than the one they found in the “landscape” inverted White’s effect. Due to these experimental results (see Figure 3 in Spehar et al., 2002), they suggested that induction in the classical and inverted White’s configurations originated from different underlying mechanisms.

We speculated that the contrast relation between their type of stimuli (the Classical landscape and Classical portrait) is different and expected to lead to opposite predictions according to our model in relation to the test region. Thus, this contrast difference can cause the different trend of results. To test this assumption, we ran our computational model and found significant smaller contrast induction in the “portrait” stimuli (due to the smaller spatial frequency). We speculated that the effect in the “portrait” configurations is mainly mediated by the induction of the first order. To test this assumption we created a “revised” version of the “classical portrait” with decreased area of the black zones adjacent to the target stimuli (Figure 2C in Spehar et al., 2002). In this “revised” configuration, the direction of the perceived brightness shift is inverted. This brightness shift in the “revised” portrait configuration has also been predicted by the current presented model. Consequently, it supports the idea that the different trend of results (Figure 3 in Spehar et al., 2002) is derive from the induction of the first order (simultaneous contrast), which deals with brightness induction rather than contrast–contrast induction. The computational analysis of this first-order induction is not in the scope of this current paper but was processed previously for the chromatic domain (Spitzer & Barkan, 2005).

Figure 17. The same classical White’s effect and its inverted White’s effect are the same as in Figure 16, but with two times higher the spatial frequency (left column). The model’s results (right column and relevant curves) show that the perceived effect obtained is stronger for the configurations with higher spatial frequency both for White’s and its inverted effect.
Since the predicted results of both assimilation effects and their inverted effects (Figures 7–13) are yielded from the very same model, we derive an opposite conclusion to theirs, i.e., that induction in the classical and inverted White’s configurations is due to the same underlying mechanism. To further support these predictions we tested (Figure 16) the algorithm performance also on the original “landscape” (Figure 2 in Spehar et al., 2002), which contain a careful configuration which use the same contrast range among the inducing stripes in both classical assimilation and inverted assimilation effect. Figure 16 shows the correct trend of results that is in agreement with the perceived results.

A further support to our model has been achieved by the results to our scattered tests on the role of the spatial frequency on the contrast–contrast effects. Figures 16 and 17 presents the model’s results to two sizes of the “landscape” orientation of the classical White and the inverted White effects (from Figure 2 in Spehar et al., 2002). The results show in this effect and additional assimilation stimuli (not presented) that the higher the spatial frequency the larger the assimilation and the inverted assimilations obtained effects, as perceived psychophysically.

Several studies tried to predict through computational models the two “contradicting” effects, simultaneous contrast and several assimilation effects (Bressan, 2006; Howe, 2005; Robinson et al., 2007). We believe that these two effects are ruled by two different mechanisms, the adaptation of the first order and the adaptation of the second order, correspondingly (Spitzer & Barkan, 2005). The adaptation of the first order deals with perception of the levels of brightness (or color) and the second order deals with brightness (or color) contrast–contrast stimuli (textures). The adaptation of the first order is processed in the retinal level and the second order is probably processed in the cortical level (Spitzer & Barkan, 2005). Evidences of existence of these two effects can be seen in a clearer way in chromatic stimuli that can manifest two different chromatic directions, while in achromatic stimuli the perceived result is obtained in one domain that is the gray level. The psychophysical results of colored White’s effect of Antis (see Figure 1.2 of Anstis, 2005) show the two different chromatic directions toward complementary color (upward arrow) and toward embedded color (rightward arrow). The same trend of results is obtained from our previous computational chromatic model of induction of the first and second orders. That model has been applied on real images which show both color constancy (first-order induction) and chromatic contrast enhancement (second-order induction) (Figure 12 in Spitzer & Barkan, 2005). Due to this view on the two induction orders, we do expect to get simultaneous contrast effects only from adaptation of first order (which is not dealt within this current brightness model that emphasizes the contrast–contrast predictions). This is a contrast to some other approaches presented in the literature, which expected to get both effects in the very same model, as presented in ODOG model (Blakeslee & McCourt, 1999, 2004; Robinson et al., 2007).

Our proposed model does not contradict the suggested idea that White’s effect is produced by a combination of both contrast and assimilation processes as suggested by Blakeslee and McCourt (2004). Although only one mechanism of adaptation of second order, contrast–contrast mechanism can predict the “classical” White’s effect and its inverted effect, as presented here, still in different spatial arrangements additional mechanisms can play even the major role.

In a following study we will present the predictions of the two orders of brightness inductions as a compound model, as we have shown for the two orders of chromatic inductions (Spitzer & Barkan, 2005). In any case an ideal computational model is supposed to predict all configurations of all effects that relates to the relevant mechanism. The presented model, however, presents only the effects that are refer only to the adaptation of the second order. Consequently, the presented model cannot predict phenomena that are govern mainly by adaptation of the first order or other brightness mechanisms such as Mach bands, Chevreul illusion, and Herman grid.

In addition, the current model cannot account for all spatial configurations of White’s effect. Depending also on the spatial scale White’s effect is produced by a combination of both contrast and assimilation processes (Blakeslee & McCourt, 2004). We believe that further research on the integration of first and second order of adaptation might succeed in predicting the effect in all spatial configurations.

Many of the studies mentioned the form of anchoring to explain the assimilation effect or as an additional factor which contributes to the assimilation effect (Gilchrist et al., 1999). This anchoring model causes the lightest surface in a scene or an image to appear white. Such an explanation might be useful for the inverted assimilation effect, but it cannot account for all configurations that exhibit the appearance of assimilation effect, such as the White’s effect (Economou, Zdravkovic, & Gilchrist, 2007). In addition, several studies reported that the anchoring model failed to predict additional assimilation effects (the inverted assimilation effect has not been referred to), such as in the study of Bindman and Cubb (2004), which are also presented here, Figures 14 and 15.

In this study we suggest that assimilation and contrast–contrast induction which includes both facilitation and suppression derive from the very same mechanism. The same induction mechanism has to be studied for the chromatic corresponding effects. On the other hand, our current model (and other such as the ODOG model; Blakeslee & McCourt, 2004) is not suspected to predict the grating contrast–contrast effects, as for example has been demonstrated in D’Zmura and Singer (1998). The issue of accounting the selectivity for orientation properties is beyond the scope of the current study and is planned to be study by us in future.
Figure 18. Illustration of the classical contrast–contrast effect (left column). The central gray squares in the left and right large squares images have identical contrast and luminance values. The appearance of the central squares is influenced by the surrounding contrast and this correct prediction’s effect is presented in the right column and the curve.
Figure 19. An additional contrast-contrast effect (Spehar et al., 1996). The horizontal three uniform patches are identical in right and left sides of the original image (left column). The model’s results image (right column) show the correct trend that is demonstrated in relevant curves.
Figure 20. First and second-order White’s configurations. Figures 20 and 21 (left columns) termed in the literature as combination of first and second-order White’s configurations (Spehar & Zaidi, 1997). Note that the average luminance of the inducing textures is identical; across, the two images and the luminance of the test stripes at each image share the same value. The result’s image (right column and the relevant curves) shows the correct trend of prediction, for all the test stimuli.
Figure 21. First and second-order White’s configurations on dark gray test stimuli. See description in Figure 20.
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