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E. Fridman, F. Gouaisbaut, M. Dambrine & J.-P. Richard

Department of Electrical Engineering Systems, Tel-Aviv University, Tel-Aviv 69978, Israel
LAIL UMR 8021 Ecole Centrale de Lille, F-59651 Villeneuve d'ascq cedex, France

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Sliding mode control of systems with time-varying delays via descriptor approach

E. Fridman†, F. Gouaisbaut‡, M. Dambrine‡ and J.-P. Richard‡

A descriptor approach to stability and control of linear systems with time-varying delays, which is based on the Lyapunov–Krasovskii techniques, are combined with a recent result on the sliding mode control of such systems. The systems under consideration have norm-bounded uncertainties and uncertain bounded delays. The solution is given in terms of linear matrix inequalities and improves the previous results based on other Lyapunov techniques. A numerical example illustrates the advantages of the new method.

1. Introduction

During the last decade a rich literature has been dedicated to robust control of time-delay systems (e.g. Boyd et al. 1994, Dugard and Verriest 1998, Fridman 2001, 2002, Fridman and Shaked, 2002a, b, 2003, Fu et al. 1997, Gouaisbaut et al. 2002, Ivanescu 2000, Kolmanovskii and Richard 1999, Kolmanovskii et al. 1999, Kolmanovskii and Myshkis 1999, Li and de Souza 1997, Mahmoud 2000, Moon et al. 2001, Niculescu 2001 and references therein). Many existing results concern systems with unknown but constant delays. However, in some applications, such as networked control or teleoperated systems, the assumption of a constant delay is too restrictive; this can lead to bad performances or, even worse, to unstable behaviours.

This paper combines two previous results to obtain a more efficient sliding mode controller for uncertain systems with time-varying delays and norm-bounded uncertainties. Other results (Gouaisbaut et al. 2002) concern varying delays but may lead to strong conditions which reduce the dynamic performances.

The first of these results is the sliding mode design (Gouaisbaut et al. 2002), which copes with stabilization of systems with time-varying delays. The approach relies on the construction of a Lyapunov-Razumikhin function that allows fast variations of the delay but leads to some conservatism on the upper bound of the time-delay.

The second result given in Fridman (2001) concerns the construction of a new class of Lyapunov–Krasovskii functionals using a descriptor model transformation. Unlike previous transformations, the descriptor model leads to a system that is equivalent to the original one (from the point of view of stability) and requires bounding of fewer cross-terms. Furthermore, following this approach, stability criteria have been given in Fridman and Shaked (2003) for systems with time-varying delays without any assumption on their derivatives (which was the case with the usual Lyapunov–Krasovskii functionals).

The paper is organized as follows. Section 2 develops a Lyapunov–Krasovskii approach on a descriptor representation for an uncertain, linear, time-delay system. This provides a stability condition expressed in term of feasibility of a linear matrix inequality (LMI) (Boyd et al. 1994). Then the design of a stabilizing memoryless state feedback is derived. Section 3 deals with the design of a sliding mode controller. This is achieved through the resolution of a generalized eigenvalue problem that can be solved efficiently using semidefinite programming tools. Section 4 solves an illustrative example using the present approach and compares it with previous results.

Throughout, the superscript T stands for matrix transposition, \(\mathbb{R}^n\) denotes the \(n\) dimensional Euclidean space, and \(\mathbb{R}^{n\times n}\) is the set of all \(n \times n\) real matrices. The notation \(P > 0\) for \(P \in \mathbb{R}^{n\times n}\) means that \(P\) is symmetric and positive definite. \(I_n\) represents the \(n \times n\) identity matrix.
2. Stabilization of linear systems with norm-bounded uncertainties by delayed feedback

This section considers the following uncertain linear system with a time-varying delay:

\[
\dot{x}(t) = (A_0 + H \Delta(t) E_0)x(t) + (A_1 + H \Delta(t) E_1)x(t - \tau(t)) + (B_0 + H \Delta(t) E_2)u(t) + B_1u(t - \tau(t)),
\]

\[
x(t) = \phi(t), \quad t \in [-h, 0],
\]

where \( x(t) \in \mathbb{R}^n \) is the state, \( u(t) \in \mathbb{R}^m \) is the control input, \( h \) is an upper-bound on the time-delay function \( 0 \leq \tau(t) \leq h, \forall t \geq 0 \). The matrix \( \Delta(t) \in \mathbb{R}^{p \times q} \) is a matrix of time-varying, uncertain parameters satisfying

\[
\Delta^T(t) \Delta(t) \leq I_q \quad \forall \ t.
\]  

(2)

For simplicity, we consider only one delay, but the results here may be easily generalized to the case of multiple delays.

We seek a control law

\[
u(t) = Kx(t)
\]

(3)

that will asymptotically stabilize the system.

2.1. Stability issue

This subsection considers the following equation:

\[
\dot{x}(t) = (\dot{A}_0 + H \Delta(t) \dot{E}_0)x(t) + (\dot{A}_1 + H \Delta(t) \dot{E}_1)x(t - \tau(t)).
\]

(4)

Representing (1) in an equivalent descriptor form (Fridman 2001):

\[
\dot{x}(t) = y(t),
\]

\[
0 = -y(t) + (\dot{A}_1 + H \Delta \dot{E}_1)x(t)
- (\dot{A}_1 + H \Delta \dot{E}_1) \int_{t-\tau(t)}^t y(s)ds
\]

or

\[
E \dot{x}(t) = \begin{bmatrix} 0 & I_n \\ \dot{A}_1 + H \Delta \dot{E}_1 & -I_n \end{bmatrix} x(t)
- \begin{bmatrix} 0 \\ \dot{A}_1 + H \Delta \dot{E}_1 \end{bmatrix} \int_{t-\tau(t)}^t y(s)ds,
\]

(5)

with

\[
\dot{x}(t) = \begin{bmatrix} x(t), \ y(t) \end{bmatrix}, \quad E = \begin{bmatrix} I_n, \ 0 \end{bmatrix},
\]

\[
\dot{A}_T = A_0 + A_1, \quad \dot{E}_T = \dot{E}_0 + \dot{E}_1,
\]

the following Lyapunov–Krasovskii functional is applied:

\[
V(t) = \dot{x}^T(t)EP\dot{x}(t) + V_2(t),
\]

(6)

where

\[
P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1 > 0, \quad EP = P^T E \geq 0,
\]

\[
V_2(t) = \int_{-h}^t \int_{t+\theta}^t y^T(s)(R + \delta_2 \dot{E}_1^T \dot{E}_1)y(s)ds d\theta.
\]

The following result is obtained:

Lemma 1: The system (4) is asymptotically stable if there exist \( n \times n \) matrices \( 0 < P_1, P_2, P_3, R > 0 \) and positive numbers \( \delta_1, \delta_2 \) that satisfy the following LMI:

\[
\begin{bmatrix} \Psi & hP^T \\ hP & \dot{A}_1 \\ \dot{A}_T & -I_n \\ \dot{A}_1^T & \dot{E}_1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & h \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \dot{A}_T \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} \leq 0
\]

(8)

where

\[
\Psi = \Psi_0 + \begin{bmatrix} \delta_1 \dot{E}_1^T \dot{E}_T \\ \delta_2 \dot{E}_1^T \dot{E}_T \end{bmatrix},
\]

\[
\Psi_0 = P_1 \begin{bmatrix} I_n \\ \dot{A}_T & -I_n \end{bmatrix} + P_3 \begin{bmatrix} 0 \\ \dot{A}_1 \end{bmatrix} P_3^T,
\]

and * denotes symmetrical entries.

Proof: Note that

\[
\dot{x}^T(t)EP\dot{x}(t) = x^T(t)P_1x(t)
\]

and, hence, differentiating the first term of (6) with respect to \( t \) gives:

\[
\frac{d}{dt}(\dot{x}^T(t)EP\dot{x}(t)) = 2x^T(t)P_1\dot{x}(t) = 2\dot{x}^T(t)P_1\dot{x}(t).
\]

(9)
Replacing \([\mathbf{x}(t)\ 0]_0\) by the right side of (5) we obtain:

\[
\frac{dV(t)}{dt} = \dot{x}^T(t)\Psi\dot{x}(t) + \eta_0 + \eta_1 + \eta_2
\]

\[\quad + h\dot{y}^T(t)[R + \delta_1 \bar{E}_1^T \bar{E}_1]y(t)
\]

\[\quad - \int_{t-h}^{t} y^T(s)[R + \delta_2 \bar{E}_1^T \bar{E}_1]y(s)ds,
\]

where

\[
\eta_0(t) \triangleq -2\int_{t-h}^{t} \dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{A}_1 \end{array} \right]y(s)ds,
\]

\[
\eta_1(t) \triangleq 2\dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right] \Delta(\bar{E}_0 + \bar{E}_1)x(t),
\]

\[
\eta_2(t) \triangleq -2\int_{t-h}^{t} \dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right] \Delta \bar{E}_1y(s)ds.
\]

Applying the standard bounding

\[a^Tb \leq a^TRa + b^TR^{-1}b, \quad \forall a, b \in \mathbb{R}^n, \forall R \in \mathbb{R}^{n \times n} : R > 0,
\]

and using the fact that \(r(t) \leq h\), we have

\[
\eta_0(t) \leq r\dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{A}_1 \end{array} \right] R^{-1}[0 \begin{array}{c} 0 \\ \bar{A}_1^T \end{array}] P \dot{x}(t) + \int_{t-h}^{t} y^T(s)Ry(s)ds
\]

\[\leq h\dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right] R^{-1}[0 \begin{array}{c} 0 \\ \bar{A}_1^T \end{array}] P \dot{x}(t) + \int_{t-h}^{t} y^T(s)Ry(s)ds.
\]

Similarly

\[
\eta_1 \leq \delta_1^{-1}\dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right]R^{-1}[0 \begin{array}{c} 0 \\ \bar{A}_1^T \end{array}] P \dot{x}(t) + \delta_11\dot{x}^T(t)\bar{E}_1^T \bar{E}_1x(t),
\]

\[
\eta_2 \leq h\delta_2^{-1}\dot{x}^T(t)P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right]R^{-1}[0 \begin{array}{c} 0 \\ \bar{A}_1^T \end{array}] P \dot{x}(t)
\]

\[+ \delta_2 \int_{t-h}^{t} y^T(s)\bar{E}_1^T \bar{E}_1y(s)ds.
\]

Substituting the right sides of the latter inequalities into (10), we obtain

\[
\frac{dV(t)}{dt} \leq \dot{x}^T(t)\tilde{\Gamma}\dot{x}(t)
\]

where

\[
\tilde{\Gamma} = \Psi + hP\left[0 \begin{array}{c} 0 \\ \bar{A}_1 \end{array} \right] R^{-1}[0 \begin{array}{c} 0 \\ \bar{A}_1^T \end{array}] P
\]

\[+ (\delta_1^{-1} + \delta_2^{-1}) \times P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right][0 \begin{array}{c} 0 \\ \bar{H} \end{array}] P.
\]

Therefore, LMI (8) yields by Schur complements that \(\tilde{\Gamma} < 0\) and hence \(V < 0\), while \(V \geq 0\), and thus (4) is asymptotically stable (Kolmanovskii and Myshkis 1999, Fridman 2002). □

### 2.2. State-feedback stabilization

The results of Lemma 1 can also be used to verify the stability of the closed-loop obtained by applying (3) to the system (1) if we set in (8)

\[
\bar{A}_i = A_i + B_iK, \quad i = 0, 1, \quad \bar{E}_0 = E_0 + E_2K
\]

and verify that the resulting LMI is feasible. We thus obtain the following matrix inequality:

\[
\left[
\Psi \quad hP\left[0 \begin{array}{c} 0 \\ \bar{A}_1 \end{array} \right] R^{-1} \begin{array}{c} 0 \\ hI_n \end{array} \begin{array}{c} \bar{E}_1^T \\ 0 \end{array} \\
* \quad -hR^{-1} \quad 0 \quad 0
\right]
\]

\[
\left[
\begin{array}{c} h \quad \bar{E}_1^T \\ 0 \end{array} \delta_1^{-1}P\left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right] \begin{array}{c} \delta_1^{-1}hP \end{array} \left[0 \begin{array}{c} 0 \\ \bar{H} \end{array} \right]
\right]
\]

\[< 0. \]
Consider the inverse of $P$. It is obvious from the requirement $P_1 > 0$, and the fact that in (8) $-(P_3^2 + P_1)$ must be negative definite, that $P$ is non-singular. Defining

$$P^{-1} = Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix}$$

and $M = \text{diag}(Q, I_{2(n+p+q)})$

we multiply (14) by $M^T$ and $M$, on the left and on the right, respectively. Choosing

$$R^{-1} = Q_1 \varepsilon,$$

where $\varepsilon$ is a positive number, and introducing $\delta_1 = \delta_1^{-1}$ and $\delta_2 = \delta_2^{-1}$, we obtain the LMI

$$\begin{bmatrix}
\Phi & h \\
A_1 Q_1 \varepsilon & Q^T \begin{bmatrix} \delta_1 I_1 & 0 \\ 0 & hI \end{bmatrix} & \tilde{E}_1^T \\
* & -hQ_1 \varepsilon & 0 \\
* & * & -hQ_1 \varepsilon & 0 \\
* & * & * & -\delta_1 I_q \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} \begin{bmatrix}
-\delta_1 I_q \\
* & * & * & * \\
\end{bmatrix} \begin{bmatrix}
0 \\
H \\
\end{bmatrix} \begin{bmatrix} Q \tilde{E}_1^T \\
0 \\
\end{bmatrix} \begin{bmatrix} hQ_1 \varepsilon \\
0 \\
0 \\
0 \\
\end{bmatrix} \begin{bmatrix} \delta_1 I_1 & 0 \\ 0 & hI \end{bmatrix} \begin{bmatrix} 0 \\
H \\
\end{bmatrix}
\end{bmatrix} < 0,$$

where

$$\begin{bmatrix}
0 \\
I_n \\
\end{bmatrix} Q + Q^T \begin{bmatrix} 0 & I_n \\ A_T & -I_n \end{bmatrix}.$$

Substituting (13) into (16) and denoting $Y = KQ_1$, $B_T = B_0 + B_1$, we obtain the following.

**Theorem 1:** The control law of (3) asymptotically stabilizes (1) if, for some positive number $\varepsilon$, there exist scalars $\delta_1 > 0$, $\delta_2 > 0$ and matrices $0 < Q_1$, $Q_2$, $Q_3$, $\in \mathbb{R}^{n \times n}$ $Y \in \mathbb{R}^{m \times n}$ that satisfy the following LMI:

$$\begin{bmatrix}
Q_2 + Q_2^T & Q_1 A_1^T + Y^T B_1^2 - Q_3^2 + Q_3 & 0 & hQ_2^T \\
* & -Q_3 - Q_3^T & h\varepsilon (A_1 Q_1 + B_1 Y) & hQ_3^T \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} < 0$$

$$Q_1 E_1^T + Y^T E_2^T hQ_1^T E_1^T 0 0 0 \\
0 hQ_1^T E_1^T \delta_1 H h\varepsilon 2H \\
0 0 0 0 \\
0 -\delta_1 I_q 0 0 \\
* -\delta_2 I_q 0 0 \\
* * -\delta_1 I_p 0 \\
* * * -\delta_2 I_p 0 \\
\end{bmatrix} < 0,$$

where $K = YQ_1^{-1}$.

3. **Sliding mode controller**

This section focuses on time-delay systems that can be represented, possibly, after a change of state coordinates and input, in the following regular form (Gouaisbaut et al. 2002, Perruquetti and Barbot 2002):

$$\frac{dz_1(t)}{dt} = (A_{11} + H \Delta(t)E_0)z_1(t) + (A_{12} + H \Delta(t)E_2)z_2(t) + A_{11}z_2(t - \tau(t))$$

$$+ A_{12}z_2(t - \tau(t)) + Du(t) + f(t, z(t))$$

$$\frac{dz_2(t)}{dt} = \sum_{i=1}^{2} (A_{21}z_1(t) + A_{22}z_2(t - \tau(t))) + Du(t) + f(t, z_1)$$

$$z(t) = \phi(t) \text{ for } t \in [-h, 0]$$

(19)
Theorem 2: Assume A1–A3. If, for some positive number \( \varepsilon \), there exist positive numbers \( \tilde{\delta}_1 \), \( \tilde{\delta}_2 \) and matrices

\[
0 < Q_1, Q_2, Q_3 \in \mathbb{R}^{(n-m) \times (n-m)}, \quad Y \in \mathbb{R}^{m \times (n-m)} \quad \text{that satisfy the following LMI:}
\]

\[
\begin{bmatrix}
Q_2 + Q_1^T & X_{12} & 0 & hQ_1^T \\
* & -Q_3 - Q_1^T & h\varepsilon(A_{d11}Q_1 + A_{d12}Y) & hQ_1^T \\
* & * & -h\varepsilon Q_1 & 0 \\
* & * & * & -h\varepsilon Q_1 \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} < 0,
\]

where

\[
Q_1E_1^T + Y^T E_2^T hQ_1^T E_1^T 0 0 \\
0 hQ_3^T E_1^T \tilde{\delta}_1 H h\tilde{\delta}_2 H \\
0 0 0 0 \\
0 0 0 0 \\
-\tilde{\delta}_1 I 0 0 0 \\
* -h\tilde{\delta}_2 I 0 0 \\
* * -\tilde{\delta}_1 I 0 \\
* * * -\tilde{\delta}_2 hI
\]

then the sliding mode control law

\[
u(t) = -D^{-1} \left[ \Omega(z(t)) + (F_M(z, t) + D_M(z)) s(z(t)) + \|KH\| \|\Theta(z(t))\| + M \frac{s(z(t))}{\|s(z(t))\|} \right],
\]

where \( K = YQ_1^{-1}, \ M > 0 \) and \( s, \Omega, \Theta, D_M \) are defined in (20–22), asymptotically stabilizes system (19) for any delay function \( \tau(t) \leq h \).

Proof: The proof is divided into two parts. The first is dedicated to the proof of the existence of an ideal sliding motion on the surface \( s(z) = 0 \); the second is dedicated to the proof of the stability of the reduced system.

Attractivity of the manifold:

Consider the Lyapunov–Krasovskii functional:

\[
V(t) = s^T(z(t))s(z(t)) = \|s(z(t))\|^2.
\]
Differentiating (25) on the trajectories of the closed-loop system gives:

\[
\dot{V}(t) = 2z^T(t)(\Omega(z(t)) + \sum_{i=1}^{2} [A_{d2i} - KA_{d1i}]z_i(t - \tau) + Du(t) \\
+ f(t, z_i) - KH\Delta(t)[\Theta(z(t)) + E_1z_1(t - \tau(t))] \].
\]

Using the expression of the control law (24), we get

\[
\dot{V}(t) = 2z^T(t)\left(\sum_{i=1}^{2} (A_{d2i} - KA_{d1i})z_i(t - \tau) + f(t, z_i) \\
- KH\Delta(t)[\Theta(z(t)) + E_1z_1(t - \tau(t))] \\
- [F_M(t, z_i) + D_M(z_i) + \|KH\|\|z(t)\| + M] \right) \|z\| \leq -2MV(t)^{1/2}.
\]

This last inequality is known to prove the finite-time convergence of the system (19) into the surface \(s = 0\) (Perruquetti and Barbot 2002).

**Stability of the reduced system:**

On the sliding manifold \(s(z) = 0\), the system is driven by the following reduced system:

\[
\frac{dz_1(t)}{dt} = (A_{111} + A_{121}K + H\Delta(t)(E_0 + E_2K))z_1(t) \\
+ (A_{d11} + A_{d12}K + H\Delta(t)E_1)z_1(t - \tau(t))
\]  

(26)

According to Theorem 1, this system is asymptotically stable for any delay law \(\tau(t) \leq h\) if, for some positive number \(\varepsilon\), there exist positive numbers \(\tilde{\delta}_1, \tilde{\delta}_2\) and matrices \(0 < Q_1, Q_2, Q_3, Y \in \mathbb{R}^{m \times (n-m)}\) that satisfy the LMI (24).

**Remark 1:** Note that the explicit knowledge of the time-dependance of the delay is not required in the expression of the control law \(u(t)\), all is needed is the knowledge of an upper bound \(h\).

4. **Example**

We demonstrate the applicability of the above theory by solving the example from (Gouaisbaut et al. 2002) for a system without uncertainty. Consider system

\[
\dot{x}(t) = Ax(t) + A_dx(t - \tau) + B[u(t) + f(x, t)],
\]

(27)

with a time-varying delay, where

\[
A = \begin{bmatrix} 2 & 0 \\ 1.75 & 0.25 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -0.1 & -0.25 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
\]

(28)

By an appropriate change of variables, this system is equivalent to:

\[
\dot{z}(t) = \tilde{A}z(t) + \tilde{A}_dz(t - \tau) + \tilde{B}[u(t) + f(x, t)],
\]

where

\[
\tilde{A} = \begin{bmatrix} 0.25 & 0 \\ 1.75 & 2 \end{bmatrix}, \quad \tilde{A}_d = \begin{bmatrix} -0.9 & -0.65 \\ -0.1 & -0.35 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]

(29)

As the pair \((\tilde{A}_{11}, \tilde{A}_{12})\) is not controllable, the system cannot be stabilized independently of the delay.

For this system, previous published works give the following results:

- In the case of a constant delay and \(f = 0\), the system may be stabilized using a linear memoryless controller \(u(t) = Kx(t)\) for the following maximum values of \(h\): \(h = 0.51\) by Li and de Souza (1997), \(h = 0.984\) by Fu et al. (1997) and \(h = 1.46\) by Ivancevic (2000). By sliding mode control for the case of constant delay and \(f \neq 0\) the maximum value found for \(h\) is 1.65.

- Applying Theorem 2 in the case of a time-varying delay and \(f \neq 0\), the corresponding value of \(h = 3.999\) is achieved.

This is summarized in table 1.

5. **Conclusions**

The problem of finding a sliding mode controller that asymptotically stabilizes a system with time-varying delay and norm-bounded uncertainty has been solved. A delay-dependent solution has been derived using a special Lyapunov–Krasovskii functional. The result is based on a sufficient condition and it thus entails an overdesign. This overdesign is considerably reduced due to the fact that the method is based on the descriptor representation. As a byproduct, for the first time on the basis of the descriptor model transformation, the solution to the stabilization problem by the feedback,

<table>
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<th>Table 1. Comparison of results for example (26–27).</th>
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which depends on both non-delayed and delayed state, is solved. Finally, a numerical example shows the effectiveness of the combined method: sliding mode and descriptor representation.

References


