

Discussion on: “Stabilization of Networked Control Systems with Data Packet Dropout and Transmission Delays”: Continuous-Time Case

Stephen M. Phillips

Department of Electrical Engineering, Arizona State University, Tempe, AZ 85287, USA

Recent interest in networked control systems is motivated, in large part, by the increasingly pervasive low-cost wired and wireless data networks. These networks would seem to be well suited to large scale distributed feedback control systems. However, the most commonly encountered protocols suffer from variable transmission delays and even data loss. These disadvantages are certainly a problem for feedback control and have led to several proprietary network solutions specifically targeted for control applications. These proprietary solutions, however, are relatively expensive and non-pervasive. From the control perspective, most networked control systems work has been focused on addressing the stability and performance of feedback systems with delays and data losses.

The paper by Yu et al. presents a stability analysis for a class of networked control systems. The class considered here consists of a continuous-time plant whose state vector is sensed, sampled, and transmitted to a controller for processing after which it is transmitted for reconstructed via zero-order hold at the actuator. The transmissions are over a network and may involve data losses or delays and the zero-order hold is synchronized with the state sampling. When data is lost the controller holds the previous control until the next sensed state data arrives. There is an assumed upper bound on the maximum latency of any data provided to the actuator. With this model the entire system can be viewed as a linear system with variable delay similar to the analysis in [1,2].

The Lyapunov stability analysis of this paper appears to be novel in that it simultaneously (i) avoids the requirement for a bound on the rate of change of the delay, (ii) is constructive in the sense of providing a set of stabilizing state feedback gains and (iii) provides an explicit linear matrix inequality based optimization problem to determine the loosest latency bound (maximum allowable delay). While this latter fact may help reduce the conservatism of the approach, as shown below the assumed model has some weaknesses that may still lead to poorly performing designs. The extension of the main result to multiple packet transmissions, while straightforward, is cumbersome in terms of notation. As with most control oriented papers on networked control systems a few low order time delay numerical examples are provided.

Much of the effort in networked control systems has been in exploring appropriate models for the actual implementations [3,4]. While the approach here is well motivated from a feedback control perspective there are some weaknesses. First, the model does not explicitly allow the designer to exploit the benefits of time-stamped sensor data [4]. For example, for many networks there is no guarantee that the data will be received in the same order that it is transmitted. For many non-real-time control network applications such as web browsing and file transfer this is not critical since the data can be reassembled in the correct sequence when received. With the model described where this is especially detrimental for the control design since older data may be received after more recent data, greatly degrading the controller

performance and, for the results of this paper, increasing the conservatism of the design due to an unnecessarily large assumption on the upper bound on latency. Of course, a relatively simple state estimator can be implemented to discard the older data or a more sophisticated state estimator based on recently observed data can be used to construct an estimate of the state from output or state measurements [2].

In addition to the incorporation of state estimators into this approach, another direction to explore is the incorporation of control performance along with the stability result. The linear matrix inequality approach used here would seem to be complimentary to such an effort due to the large number of performance criteria which can be accommodated [5]. It would also be useful to compare the conservatism of this approach to that of similar time delay analyses such as [1,2].

For successful widespread deployment of networked control systems it will be necessary to investigate scaling issues. It is unlikely that a single low order plant will justify the investment in a networked approach to feedback control. Large numbers of plants or plants with many inputs and outputs are more likely generate the interest of control practitioners. Investigations of how this and other analysis

techniques can scale to large numbers of plants is needed. One must also realize that the types of pervasive inexpensive wired and wireless networks available today were never designed to be explicitly used for real-time feedback control. There is a real opportunity for significant control performance and robustness improvements by collaborating with researchers studying the details of network congestion control and scheduling algorithms.

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D.M. Tilbury* and J.R. Moyne**

Department of Mechanical Engineering, The University of Michigan, Ann Arbor, MI 48109-2125, USA

As noted in the title, the authors consider in this paper the stabilization problem for a networked control system: a feedback control system where the sensors and actuators are connected over a communication network. The profusion of communication networks has motivated many groups of researchers to consider similar problems in recent years, as noted in the references and in many other papers on the topic (see [1,2]). A handbook on the topic of networked and embedded control systems has recently been published [3].

Networked control systems do have many advantages over traditional point-to-point wired systems.

For systems with many inputs and outputs, the reduction in wiring (and consequent ease of set-up, diagnostics, maintenance, and debugging) results in a significant cost savings. Wireless networks reduce some of these costs even further, but bring their own set of challenges regarding channel quality, interference, contention, and reliability. A communication network in the feedback loop does increase the total delay in the system, which typically (but not always) decreases the performance. However, the increasing speed of commercially available networks (e.g. 10-Gbps Ethernet, 100-Mbps wireless) is making the network delay less and less relevant.

The authors of the current paper focus their attention on a single feedback loop, with a continuous-time plant model and full-state feedback. All of the plant

*E-mail: tilbury@umich.edu

**E-mail: moyne@umich.edu

states are sampled simultaneously, encoded into a single packet, and sent over the network. In the absence of packet loss or network delay, the problem reduces to the standard digital control problem with a uniform sampling time on the outputs and a zero-order hold on the actuator signal. The authors then add to this model progressively: packet loss (but no delay), packet loss (with delay), and split packets (but without packet loss or delay). For each case, a set of sufficient conditions for stability of the closed-loop system is derived using an LMI approach. In all cases, a maximum bound on the total delay, including packet loss and delay, is assumed.

The results of the paper are fairly theoretical, giving sufficient conditions under which a stabilizing controller can be found. In this discussion piece, we will highlight some of the practical issues that are commonly found in networked control systems, and relate them to the results obtained by the authors.

Most commercially available communication networks can be characterized as one of three types: bit-synchronized (e.g. CAN), token-passing, or Ethernet-based [4]. All types of networks have delays associated with waiting to gain access to the network, but the waiting time has a different characteristic depending on the type of network. In a typical control system where each node sends one packet of data every sample period, a bit-synchronized network will exhibit constant waiting times (with the highest priority node having the shortest delay), a token-passing network will have periodic waiting time delays, and an Ethernet-based network can demonstrate random delays. In a bit-synchronized or token-passing network, there are no packet losses unless the network is saturated and cannot physically carry all of the data that the nodes attempt to send. In an Ethernet-based network, however, packet losses can occur occasionally even at low network loads due to the randomness inherent in the protocol.

Due to the increasing speed of most network protocols, the largest portion of the delay in practice is usually due to the device delay: the delay in sampling the variable of interest from the environment, encoding that data into a packet, and then transmitting that data across the network. Experimental investigations show that the device delay can be nearly constant or highly variable, and can be long enough to dominate any network-induced delay [5]. The control design method described in the paper can be used to accommodate for both waiting time and device delays, provided that they can be bounded *a priori*. Bounds on the waiting times can be computed based on the network protocol and the amount of data that needs to be sent over the network [4], but device delays (for

network devices) need to be measured experimentally, as this information is not typically included in the specification sheet from the manufacturer.

Networks are most advantageous when there are many sensors and actuators. The architecture considered in the paper, in which all of the sensors are connected at a single node, would require hard-wiring between each sensor and the sensor-network interface. This architecture would be approximately as complex as directly wiring each sensor to the controller, and thus there is no clear advantage to using a network in this fashion. A more practical (but more complex) model includes a different delay for each sensor output as in [6]. Synchronization of all nodes on a network (as assumed by the framework in this paper) is also a difficult problem; the IEEE 1588 standard addresses this issue.

In the case that the sensors are all collected at a single node, it is unlikely that their information would need to be split into multiple packets. Most control networks have a minimum packet size so that small packets don't get "lost" on the network. For example, a DeviceNet packet can have between 0 and 8 bytes of data and an Ethernet packet can have between 46 and 1500 bytes of data, in addition to the headers, addressing information, and other overhead that must be included to ensure correct delivery of the packet. Padding is added to Ethernet packets when less than 46 bytes of data need to be sent. Since most D/A and A/D cards use either 12 or 16 bits (2 bytes) to encode sensor/actuator data, several channels of data can easily be transmitted in a single packet.

Although this paper and many others have focused on the stabilization problem, and it is an important theoretical topic, any practical networked control system that is operating near the stability boundary will most likely be replaced with a hardwired architecture. Thus, the performance of a networked control system is also an important problem to consider: What is the degradation in performance that can be expected with a networked system (due to unavoidable delays, including the device delays), and how does this trade off with the advantages of networked control systems mentioned above? A few studies on performance of networked control systems can be found in [5,7,8], but many more interesting questions in this domain remain to be answered.

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Emilia Fridman

Department of Electrical Engineering-Systems, Tel-Aviv University, Tel-Aviv 69978, Israel

The authors introduce a new model for continuous-time networked control systems (NCSs), i.e. for systems where feedback control loops are closed through a real-time network. This is a linear system with time-varying input delay, which appears due to sampling, data packet dropouts and transmission delays. The closed-loop system has the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BFx(t_k - \tau_{ca} - \tau_{sc} - d(k)h), \\ t &\in [t_k, t_{k+1}), \quad t_{k+1} - t_k = h, \quad k = 0, 1, \dots, \end{aligned} \quad (1)$$

where $x(t) \in \mathcal{R}^n$, τ_{ca} is a controller-to-actuator delay, τ_{sc} is a sensor-to-controller delay, $d(k) \in Z^+$ is a packet dropout at time t_k and $0 \leq d(k) \leq \bar{d} < \infty$. Equation (1) is further represented as a continuous time system

$$\dot{x}(t) = Ax(t) + BFx(t - \tau(t)), \quad (2)$$

with time-varying delay $\tau(t) = t - t_k + \tau_{ca} + \tau_{sc} + d(k)h$. The delay is treated as a bounded one from the given segment $0 \leq \tau(t) \leq (\bar{d} + 1)h + \tau_{ca} + \tau_{sc} = \bar{\tau}$. Since $\dot{\tau}(t) = 1$, $t \neq t_k$ and since Lyapunov–Krasovskii method is usually applied when $\dot{\tau}(t) \leq d < 1$, the delay-dependent (asymptotic) stability of (2) is analyzed via Lyapunov–Razumikhin approach. Linear matrix inequality (LMI) is derived for finding a stabilizing gain F .

The purpose of this discussion is to point out some directions for reducing the conservatism of the stability and stabilization criteria. As it is noticed by the authors, the case of fast varying delay, where the condition $\dot{\tau}(t) \leq d < 1$ does not hold, can be treated via Lyapunov–Krasovskii functionals (LKFs). For the first time such functionals V were introduced in [1] via descriptor model transformation. The novelty of such V was in the form of \dot{V} , which depended on both, $x(t)$ and $\dot{x}(t)$. LKFs lead usually to less restrictive results.

Another source for the conservatism in the case of transmission delays may occur due to treatment of the delay as a fast-varying one from the segment $[0, \bar{\tau}]$. In fact, the delay can be represented in the following form:

$$\begin{aligned} \tau(t) &= g + \eta(t), \quad g = \tau_{ca} + \tau_{sc}, \quad \eta(t) = t - t_k + d(k)h, \\ t &\in [t_k, t_{k+1}], \end{aligned} \quad (3)$$

where $g > 0$ is a constant value and $\eta(t) \in [0, (\bar{d} + 1)h]$ is a fast-varying delay uncertainty. Such delay $\tau(t)$ with non-zero nominal value g and bounded delay uncertainty $\eta(t)$ was defined in [2] as a ‘non-small’ delay (differently from the ‘small’ delay with the zero nominal value $g = 0$ and bounded delay uncertainty). Stability criteria for ‘non-small’ delay can be found in [2,3,5,6].

We shall illustrate the above remarks by applying the stability criterion of [2]. The method of [2] is based

on the following construction of the LKF: the nominal LKF V_n , which guarantees the stability of the nominal system (2) (i.e. of (2) with $\eta(t) = 0$), is added by terms that compensate the delay uncertainty. The following stability criterion follows from [2], where V_n is chosen to be the descriptor one:

Lemma 1. Given a gain matrix F , the system (2) with τ given by (3) is stable for all $0 \leq g \leq \bar{g}$ and $\eta(t) \in [0, (\bar{d}+1)h]$, if there exist $n \times n$ matrices $0 < P_1, P_2, P_3, S, Y_1, Y_2, R$ and R_a that satisfy

$$\begin{bmatrix} \Psi_n & P^T \begin{bmatrix} 0 \\ BF \end{bmatrix} - Y^T & (\bar{d}+1)hP^T \begin{bmatrix} 0 \\ BF \end{bmatrix} & \begin{bmatrix} 0 \\ (\bar{d}+1)hR_a \end{bmatrix} & \bar{g}Y^T \\ * & -S & 0 & 0 & 0 \\ * & * & -(\bar{d}+1)hR_a & 0 & 0 \\ * & * & * & -(\bar{d}+1)hR_a & 0 \\ * & * & * & * & -\bar{g}R \end{bmatrix} < 0, \quad (4)$$

where Y, Z and $\bar{\Psi}_n$ are given by

$$\begin{aligned} \bar{\Psi}_n = P^T & \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}^T P + \begin{bmatrix} S & 0 \\ 0 & \bar{g}R \end{bmatrix} \\ & + \begin{bmatrix} Y \\ 0 \end{bmatrix} + \begin{bmatrix} Y \\ 0 \end{bmatrix}^T, \quad Y = [Y_1 \quad Y_2]. \end{aligned}$$

Following [7] we choose $P_3 = \varepsilon P_2, \varepsilon \in \mathcal{R}$, where ε is a tuning scalar parameter. Note that P_2 is non-singular due to the fact that the only matrix which can be negative definite in the second block on the diagonal of (4) is $-\varepsilon(P_2 + P_2^T)$. Defining:

$$\begin{aligned} \bar{P} &= P_2^{-1}, [\bar{P}_1 \quad \bar{Y}_i \quad \bar{S} \quad \bar{R} \quad \bar{R}_a] \\ &= \bar{P}^T [P_1 \bar{P} \quad Y_i \bar{P} \quad S_i \bar{P} \quad R \bar{P} \quad R_a \bar{P}], \quad i = 1, 2. \end{aligned}$$

and $W = K\bar{P}$, multiplying (4) by $\text{diag}\{\bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}\}$ and its transpose, from the right and the left, respectively, we obtain:

Theorem 2. Equation (2) with τ given by (3) is asymptotically stable for all $0 \leq g \leq \bar{g}$ and $\eta(t) \in [0, (\bar{d}+1)h]$ if for some tuning scalar parameter ε there exist $n \times n$ matrices $0 < \bar{P}_1, \bar{P}, \bar{S}, \bar{Y}_i, \bar{R}, \bar{R}_a$, $i = 1, 2$, $W \in \mathcal{R}^{m \times n}$ that satisfy the following LMI:

$$\begin{bmatrix} \bar{P}^T A^T + A\bar{P} + \bar{Y}_1 + \bar{Y}_1^T + \bar{S} & \bar{P}_1 - \bar{P} + \varepsilon \bar{P}^T A^T + \bar{Y}_2 & BW - \bar{Y}_1^T & BW & 0 & \bar{g} \bar{Y}_1^T \\ * & -\varepsilon(\bar{P}^T + \bar{P}) + \bar{g} \bar{R} & \varepsilon BW - \bar{Y}_2^T & \varepsilon(\bar{d}+1)hBW & (\bar{d}+1)hR_a & \bar{g} \bar{Y}_2^T \\ * & * & -\bar{S} & 0 & 0 & 0 \\ * & * & * & -(\bar{d}+1)h\bar{R}_a & 0 & 0 \\ * & * & * & * & -(\bar{d}+1)h\bar{R}_a & 0 \\ * & * & * & * & * & -\bar{g} \bar{R} \end{bmatrix} < 0. \quad (5)$$

The state-feedback gain is then given by $F = W\bar{P}^{-1}$.

To illustrate the efficiency of the method of Theorem 2 we consider the authors' example, where A and B are given to be

$$A = \begin{bmatrix} -1 & -0.01 \\ 1 & 0.02 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}.$$

The authors found that the system is stabilizable for $0 \leq \tau(t) \leq 0.595$.

For simplicity we apply Theorem 2 with $\varepsilon = 1$. Choosing $\bar{g} = 0.1$ and $(\bar{d}+1)h = 9.8$, we find that LMI (5) is feasible. Hence, the system is stabilizable for τ from essentially greater segment $g \leq \tau(t) \leq g + 9.8$, where $0 \leq g \leq 0.1$. The corresponding gain is $F = [-0.0399 \quad -0.0403]$. Application of optimizing search for ε (via 'fminsearch' of Matlab) should lead to a larger stability domain.

Choosing the constant 'nominal' value in the middle of the segment $[\bar{g}, \bar{g} + (\bar{d}+1)h]$ and considering the sign-varying delay uncertainty from $[-(\bar{d}+1)h/2, (\bar{d}+1)h/2]$ may improve the result of Theorem 2. Further improvement may be achieved by application of the input-output approach [3,4]. Finally, the development of the discretized LKF method [3] to fast-varying delays should lead to effective criterion in the case of transmission delays.

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Final comments by the authors

**M. Yu, L. Wang,
T. Chu and F. Hao**

The recently emerged study of networked control systems (NCSs) may be regarded in certain sense as a development in studies of distributed control and remote control problems under the new technical environment of advanced communication/data networks, where information of sensing and controlling is transmitted through a communication channel with finite bandwidth and shared by multiple users. In spite of the possible diversity in modes, protocols, and architectures, from a system-theoretic viewpoint, some basic underlying issues are of common interest and fundamental importance in analysis and design of NCSs, such as transmission delays, asynchrony, data losing, disordering, etc.

The effect of time delays (whatever the cause of them) on control systems has long been studied and new approaches, criteria and improvements have still been emerging from time to time. On the other hand, the network induced data loss and disorder appear to be new issues and require elaborated study. A major

concern in our paper is modeling data loss effect in a NCS. We view data losses as one source of network-induced delays and model the NCS in the presence of data losses by delay differential equations. In this way, the effect of data losses is tantamount to the effect of time delays in the system. This seems intuitively simple and straightforward, and yet allows for the use of the existing valuable results and methods for delay systems in analysis and design of NCSs.

Although we have not explicitly considered the use of time stamp in data packets, our results can be readily accommodated to exploit such information. Indeed, by reading out the time stamp of a data packet, one can decide whether to process the data (if the time stamp is newer than that of the last proceeded data) or just to discard the data (if the time stamp is older than that of the last proceeded data). In this manner, our results can also be used to deal with data disorder to certain extent, as long as the discarded disordering data do not result in a delay exceeding the bound of stability determined by our results.

We have also presented some preliminary results on multiple data packet transmission in our paper. The need of dividing and transmitting data in multiple packets is a natural and common practice in many control systems with networked architectures. For example, the state of aerodynamics on a wing profile of an aircraft can be obtained by assembling the data from a group of sensors distributed on the profile. Even for a single plant to be controlled through a data network shared by multiple users, sometimes, it is still quite necessary to exchange data between sensors and controller in multiple packets because of the finite bandwidth of the communication channel. Perhaps the real challenge in multiple packet transmission in an NCS is the data disordering and data losing of some subpackets in an entire data group. To handle such an issue, new types of state observer or estimator should be developed to reconstruct state from partially available information of the state.