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Rejection of mismatched disturbances for systems with input delay via a predictive extended state observer

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Summary

The problem of output stabilization and disturbance rejection for input-delayed systems is tackled in this work. First, a suitable transformation is introduced to translate mismatched disturbances into an equivalent input disturbance. Then, an extended state observer is combined with a predictive observer structure to obtain a future estimation of both the state and the disturbance. A disturbance model is assumed to be known but attenuation of unmodeled components is also considered. The stabilization is proved via Lyapunov-Krasovskii functionals, leading to sufficient conditions in terms of linear matrix inequalities for the closed-loop analysis and parameter tuning. The proposed strategy is illustrated through a numerical example.

KEYWORDS

input delay, linear matrix inequality, mismatched disturbance, predictive observer

1 | INTRODUCTION

Time-delay systems have received growing attention from the research community over the past years. They commonly exist in many engineering applications such as chemical or biological processes, oil or gas factories, and networked control.[1] Large delays often lead to closed-loop instability if they are not taken into account and limit the achievable performance of conventional controllers.[2] Among the traditional control objectives, disturbance rejection in time-delay systems deserves special attention because delays impose fundamental limitations no matter what controller is used. Indeed, if a disturbance reaches the input at time t_0, the information lag will cause the system to run in open loop over the time window $t \in [t_0, t_0 + h]$, where h is the time delay.

The most celebrated strategy to compensate delays was proposed in 1957 with the introduction of the Smith predictor (SP), applicable to SISO open-loop stable plants.[3] The disturbance rejection shortcomings of the original SP were early detected and many modifications were proposed to mitigate them, see other works[4-8] and the references therein. A similar idea was extended to MIMO stable/unstable systems with the finite spectrum assignment technique,[9] also known as the reduction-based approach.[10] In contrast to the SP, this strategy was formulated in the time domain with the introduction of a state predictor.

Several works devoted to improving disturbance rejection and robustness of state predictors have been reported recently in the literature. The inverse optimality of a filtered state predictor with respect to a functional involving the disturbance was shown in the work of Krstic.[11] Additional delayed feedback was considered in the work of Léchappé et al[12] to reject constant disturbances. A modified prediction based on a disturbance observer was proposed in the work of Basturk,[13] leading to the rejection of polynomial-in-time disturbances and better attenuation of sufficiently smooth signals. For unknown sinusoidal disturbances, cancelation by means of adaptive control schemes have been also achieved in the works of Basturk.[14,15] Uncertainty observers have been also used in the work of Sanz et al[16] to deal with norm-bounded nonlinearities.

Another handicap of state predictors lies on the fact that their implementation requires the computation of a distributed integral term. This has been a matter of concern for some researchers,[17,18] as the discretization of the integral may lead to instability of the closed-loop. In the work of Zhou et al,[19] a first-order truncated predictor that ignores the infinite-dimensional part of the controller was proposed and extended later to include higher-order terms in the work of Zhou et al.[20] An approach that avoids the use of distributed terms by introducing sequential predictors in observer form was introduced in the work of Besançon et al[21] and further developed in the work of Najafi et al.[22] The advantage of avoiding distributed terms has been further exploited recently in the works of Léchappé et al,[23] Cacace et al,[24] Mazenc and Malisoff.[25]

In this paper, the asymptotic stabilization of linear time-delay systems in the presence of external mismatched disturbances is considered. In order to estimate the disturbance, a structure similar to the one presented in the work of Guo and Chen[26] is adopted, consisting of an extended state observer (ESO) that contains both the plant and disturbance models. The main contribution of the present work lies on extending the applicability of the ESO to input-delayed systems using a predictor in observer form.[22] As a result, a prediction h units of time ahead of both the state and the disturbance is obtained. In addition, attenuation of unmodeled components of the disturbance is considered, which is a departure from the aforementioned work.[26] Furthermore, the proposed strategy is designed to deal with mismatched uncertainties and partial state measurement, in contrast to other works.[11-14] The regulation problem is translated into a conventional H_∞ stabilization problem and sufficient stability conditions in terms of linear matrix inequalities (LMIs) are derived.

The rest of this paper is structured as follows. The problem at hand is stated in Section 2. The proposed control strategy is developed in Section 3, where the problem is translated into that of stabilizing an augmented closed-loop system, sufficient stability conditions are derived and the design of the controller parameters is addressed. In Section 4, the strategy is adapted to track time-varying smooth references. The proposed method is illustrated through simulations in Section 5, and some conclusions are given in Section 6.

Notation 1. The n-dimensional Euclidean space is denoted by $\mathbb{R}^n$, whereas $\mathbb{R}^{n\times m}$ is the space of real $n \times m$ matrices. The standard Euclidean vector norm and its induced matrix norm are represented by $\|\cdot\|$. The following notation $x_t : [-h, 0] \to \mathbb{R}^n$ is used to represent the interval $x_t(\theta) = x(t+\theta), \theta \in [-h, 0]$. The function $\varphi(s) : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is said to belong to $L_2[0, \infty)$ if the norm $\|\varphi(t)\|_2 = \sqrt{\int_0^\infty \varphi^T(s)\varphi(s)\,\mathrm{d}s}$ exists and is finite. The ith time derivative of a function $\varphi(t)$ is written in short as $d^i\varphi/dt^i(t) = \varphi^{(i)}(t)$, being $\varphi^{(0)}(t) = \varphi(t)$.

2 | PROBLEM STATEMENT

The developments presented in this paper consider a class of disturbed single-input time-delay systems given by

$$\dot{\mathcal{X}}(t) = A\mathcal{X}(t) + Bu(t-h) + \Delta_l d(t) \tag{1}$$

$$y(t) = C\mathcal{X}(t) \tag{2}$$

$$z(t) = D\mathcal{X}(t), \tag{3}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^n, C \in \mathbb{R}^{p\times n}$, and $D \in \mathbb{R}^{q\times n}$ are known matrices, $\mathcal{X} \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^p$ is the measured output, $z \in \mathbb{R}^q$ is the regulated variable, $d : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is an unknown external disturbance, and $u \in \mathbb{R}$ the actuator signal, affected by a delay of h units of time. The vector $\Delta_l \in \mathbb{R}^n$ is defined such that its lth entry is equal to one, whereas the rest are zero, being $l \in [1, n]$. The following assumptions are made.

Assumption 1. The pair (A, C) is detectable and the pair (A, B) is controllable.

Assumption 2. The external disturbance can be represented by $d(t) = \nu(t) + \eta(t)$, where

$$\dot{\xi}_d(t) = A_\xi \xi_d(t) \tag{4}$$

$$\nu(t) = C_\xi \xi_d(t), \tag{5}$$

the matrices $A_\xi \in \mathbb{R}^{r\times r}, C_\xi \in \mathbb{R}^{1\times r}$ are known (the so-called exogenous system) and form a completely observable pair, $\xi_d \in \mathbb{R}^r$ is the generator vector with unknown initial condition $\xi_d(0)$, and $\eta : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is an unknown bounded signal that represents the unmodeled disturbance components and satisfies $\eta(t) \in L_2[0, \infty)$.

Assumption 3. The pair $\left(\begin{bmatrix} A & BC_\xi \\ 0 & A_\xi \end{bmatrix}, [C, 0]\right)$ is detectable.

Assumption 4. The matrix $D \in \mathbb{R}^{q\times n}$ has the structure $D = [\bar{D}, 0]$, with $\bar{D} \in \mathbb{R}^{q\times l}$.

The first part of Assumption 1 is necessary for the stabilization of the system via error feedback, whereas the second part is assumed for simplicity.* Assumption 2 is similar to that of the output regulation theory.[28,29] The eigenvalues of the matrix A_ξ usually lie on the imaginary axis, which means that for $\eta(t) = 0$, model (4)- (5) can represent sinusoidal disturbances or piecewise-continuous signals of polynomial growth. Assumption 3 does not imply loss of generality because it can always be fulfilled if (C, A) is detectable, by changing the dimension of the exogenous model.[29] Finally, Assumption 4 simply points out that the effect of mismatched disturbances cannot be completely removed from all states if $l \neq n$.

The goal of this paper is to find a control strategy that, in spite of the input delay, achieves cancelation of mismatched disturbances accurately modeled by (4)-(5), that is, when $\eta(t) = 0$. In addition, some attenuation level characterized by the L_2-gain (denoted by $\gamma > 0$) should be guaranteed when there are unmodeled components in the disturbance, that is, when $\eta(t) \neq 0$. This is cast into an H_∞ problem as follows:

Problem 1. Under Assumptions 1-4, find a dynamic output control law that internally stabilizes (1)-(2) and guarantees $\|z(t)\|_2 \leq \gamma \|\eta(t)\|_2$ for all $0 \neq \eta(t) \in L_2[0, \infty)$ and some $\gamma > 0$, assuming $\mathcal{X}_0 = 0$.

Before introducing the proposed strategy to solve this problem, system (1)-(3) is reformulated in a more convenient form. By virtue of Assumption 1 and without loss of generality, let us consider the pair (A, B) to be given in the canonical controllable form, that is, with

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix}. \tag{6}$$

The disturbance $d(t)$ in (1) can be mismatched (if $l \neq n$), meaning that it affects the state through channels in which the input has no direct influence. In order to obtain an equivalent input disturbance, let us consider the following change of variable[30]:

$$x_j(t) = \mathcal{X}_j(t), \forall j \in \{1, \ldots, l\}, \qquad x_j(t) = \mathcal{X}_j(t) + v^{(j-l-1)}(t), \forall j \in \{l+1, \ldots, n\}, \tag{7}$$

which can be used to transform system (1)-(3) into

$$\dot{x}(t) = Ax(t) + B[u(t-h) + w(t)] + \Delta_l \eta(t), \tag{8}$$

$$y(t) = Cx(t), \tag{9}$$

$$z(t) = Dx(t), \tag{10}$$

where

$$w(t) = \frac{1}{b}\left(v^{(n-l)}(t) - \sum_{j=l+1}^{n} a_j v^{(j-l-1)}(t)\right). \tag{11}$$

Proposition 1. *The exogenous model (4)- (5) is also a generator of the equivalent input disturbance defined in (11), ie, it can be represented by*

$$\dot{\xi}_w(t) = A_\xi \xi_w(t), \tag{12}$$

$$w(t) = C_\xi \xi_w(t), \tag{13}$$

where $\xi_w \in \mathbb{R}^r$ is a generator vector with unknown initial condition $\xi_w(0)$.

*The proposed strategy can be also applied to systems that are only stabilizable as long as the disturbance does not affect the uncontrollable states. For that purpose, one should simply consider $\mathcal{X}$ in (1) to contain only the controllable states, which can be achieved using a suitable transformation.[27]

Proof. See Appendix A. □

The term $w \in \mathbb{R}$ should be understood as an equivalent input disturbance. Note that the triplet (A, B, C) is not modified by this transformation and thus controllability and detectability are preserved. Intuitively, the components of the disturbance are pushed through the chain of integrators by considering their higher derivatives. It should be remarked that the change of variable (7) is only used for analysis purposes, and it is not needed for the implementation of the proposed control strategy. In what follows, the equivalent system (8)-(10) along with the generator model (12)- (13) are considered.

Remark 1. The transformation (7) is not well defined if $l = n$ because, in such case, model (1)-(3) is already in the form of (8)-(10), and the subsequent analysis can be directly applied.

Remark 2. Although the generalization to MIMO systems seems feasible, it cannot be derived in an easy way from the proposed strategy. On one hand, having multiple inputs usually implies having multiple time delays as well. In that case, the extension of the predictor-observer introduced in the next section is not trivial. On the other hand, the derivation of the transformation (7) is not straightforward for the general MIMO case, as it would require using the concept of the normal form and vector relative degree.[31]

3 | PROPOSED CONTROL STRATEGY

The proposed solution to Problem 1 is given in this section. The observer-based controller structure is introduced, and the closed-loop equations are derived. Then, sufficient stability conditions are given in terms of LMIs.

3.1 | Observer-based predictive controller

Let us represent the system dynamics by defining an augmented state $\zeta(t) = [x^T(t), \xi_w(t)]^T$, which includes the exosystem model. Using (8)-(9) and (12)-(13), the dynamics in terms of $\zeta(t)$ is derived as

$$\dot{\zeta}(t) = \underbrace{\begin{bmatrix} A & BC_\xi \\ 0 & A_\xi \end{bmatrix}}_{A_z}\zeta(t) + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_z} u(t-h) + \underbrace{\begin{bmatrix} \Delta_l \\ 0 \end{bmatrix}}_{B_\eta} \eta(t), \tag{14}$$

$$y(t) = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_z}\zeta(t), \tag{15}$$

where $A_z \in \mathbb{R}^{(n+r)\times(n+r)}$, $B_z \in \mathbb{R}^{(n+r)}$, and $C_z \in \mathbb{R}^{q\times(n+r)}$. The main idea introduced in this paper is to construct an observer to obtain an estimation of the augmented state h units of time ahead $\zeta(t+h)$, denoted by $\bar{\zeta}(t) \triangleq [\bar{x}^T(t), \bar{\xi}_w(t)]^T$. In this way, the observer forecasts both the state and the disturbance, which can be computed as $\bar{w}(t) = C_\xi\bar{\xi}_w(t)$. Note that, because of the input delay, the latter is needed to effectively counteract the disturbance, as pointed out in the work of Sanz et al.[13] Following the ideas in the work of Najafi et al,[22] a plausible observer is given by

$$\dot{\bar{\zeta}}(t) = A_z\bar{\zeta}(t) + B_z u(t) + L\left(y(t) - C_z\bar{\zeta}(t-h)\right), \qquad \bar{\zeta}_0 = 0. \tag{16}$$

The estimation error is defined by[†]

$$e(t) \triangleq \left[e_x^T(t), e_\xi^T(t)\right]^T \triangleq \zeta(t) - \bar{\zeta}(t-h), \tag{17}$$

where $e_x \in \mathbb{R}^n$ and $e_\xi \in \mathbb{R}^r$. Differentiating (17) and using (14)- (15), the error dynamics is given by

$$\dot{e}(t) = A_z e(t) - LC_z e(t-h) + B_\eta\eta(t). \tag{18}$$

[†]Intuitively, the estimation error should be defined as $\zeta(t+h) - \bar{\zeta}(t)$, provided that $\bar{\zeta}(t)$ is supposed to be a future estimation of $\zeta(t)$. However, the definition (17) is arbitrarily chosen to avoid noncausal terms in subsequent derivation.

Assuming that L is chosen such that (18) is stable, the control law can be selected analogous to that of conventional controllers compensating for matched uncertainties[32]

$$u(t) = -K\bar{x}(t) - \bar{w}(t) = -K\bar{x}(t) - C_\xi \bar{\xi}_w(t) = -\begin{bmatrix} K, C_\xi \end{bmatrix} \bar{\zeta}(t). \tag{19}$$

Plugging (19) into (8) and using (13), (17) leads to

$$\dot{x}(t) = (A - BK)x(t) + \begin{bmatrix} BK, BC_\xi \end{bmatrix} e(t). \tag{20}$$

For convenience, let us define $\mu(t) \triangleq [x^T(t), e^T(t)]^T$ and rewrite the dynamics (18) and (20) along with the regulated variable as

$$\dot{\mu}(t) = \underbrace{\begin{bmatrix} A - BK & \begin{bmatrix} BK, BC_\xi \end{bmatrix} \\ 0 & A_z \end{bmatrix}}_{A_0} \mu(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -LC_z \end{bmatrix}}_{A_1} \mu(t-h) + \underbrace{\begin{bmatrix} \Delta_l \\ B_\eta \end{bmatrix}}_{B_0} \eta(t), \tag{21}$$

$$z(t) = \underbrace{\begin{bmatrix} D & 0 \end{bmatrix}}_{D_0} \mu(t). \tag{22}$$

The original problem posed in this paper has been then translated into the H_∞ stabilization of the closed-loop defined by (21)- (22), which is tackled next.

3.2 | Closed-loop stability

Lemma 1. *Given gains K, L and $\gamma > 0$, $\bar{h} > 0$, let there exist symmetric positive definite matrices $P, Q, R \in \mathbb{R}^{(2n+r)\times(2n+r)}$ and matrices $P_2, P_3 \in \mathbb{R}^{(2n+r)\times(2n+r)}$ that satisfy the LMI*

$$\Phi_1 = \begin{bmatrix} \Phi_1(1,1) & P - P_2^T + A_0^T P_3 & Re^{-2\delta\bar{h}} + P_2^T A_1 & P_2^T B_0 & D_0^T \\ (*) & -P_3 - P_3^T + \bar{h}^2 R & -P_3^T A_1 & P_3^T B_0 & 0 \\ (*) & (*) & -(S+R)e^{-2\delta\bar{h}} & 0 & 0 \\ (*) & (*) & (*) & -\gamma^2 I & 0 \\ (*) & (*) & (*) & (*) & -I \end{bmatrix} < 0, \tag{23}$$

where $\Phi_1(1,1) = A_0^T P_2 + P_2^T A_0 + 2\delta P + Q - Re^{-2\delta\bar{h}}$. Then, system (21)- (22) is exponentially stable with a decay rate $\delta > 0$ for any delay $0 \le h \le \bar{h}$ and achieves $\|z(t)\|_2 \le \gamma \|\eta(t)\|_2$ for any $0 \ne \eta(t) \in L_2[0, \infty)$.

Proof. See Appendix B. □

Theorem 1. *Given $\gamma > 0$, $\bar{h} > 0$ and tuning parameters $\alpha > 0$, $\delta > 0$, $\varepsilon > 0$, let there exist symmetric positive definite matrices $P, Q, R \in \mathbb{R}^{(2n+r)\times(2n+r)}$, $S \in \mathbb{R}^{n\times n}$ and matrices $P_{21} \in \mathbb{R}^{n\times n}$, $P_{22} \in \mathbb{R}^{(n+r)\times(n+r)}$, $X \in \mathbb{R}^{1\times n}$ that satisfy the following LMIs:*

$$\Phi_2 = \begin{bmatrix} \Phi_2(1,1) & P - P_2^T + A_0^T \varepsilon P_2 & Re^{-2\delta\bar{h}} + \begin{bmatrix} 0 & 0 \\ 0 & YC_z \end{bmatrix} & P_2^T B_0 & D_0^T \\ (*) & -\varepsilon P_2 - \varepsilon P_2^T + \bar{h}^2 R & -\varepsilon \begin{bmatrix} 0 & 0 \\ 0 & YC_z \end{bmatrix} & \varepsilon P_2^T B_0 & 0 \\ (*) & (*) & -(S+R)e^{-2\delta\bar{h}} & 0 & 0 \\ (*) & (*) & (*) & -\gamma^2 I & 0 \\ (*) & (*) & (*) & (*) & -I \end{bmatrix} < 0, \tag{24}$$

$$SA^T + X^T B^T + AS + BX + 2\alpha S < 0, \tag{25}$$

where $\Phi_2(1,1) = A_0^T P_2 + P_2^T A_0 + 2\delta P + Q - Re^{-2\delta\bar{h}}$ and $P_2 = \text{diag}\{P_{21}, P_{22}\}$. Then, the control law (19) computed by means of the observer (16) with $K = XS^{-1}$ and $L = (P_{22}^T)^{-1} Y$, solves Problem 1.

Proof. See Appendix C. □

Remark 3. The problem posed in Theorem 1 has to be solved sequentially, obtaining first K from (25) and then L from (24). The parameter $\alpha > 0$ is user-supplied and determines how aggressive the resulting controller will be. The value of γ can be also user-supplied or, alternatively, defining $\beta = \gamma^2$, the problem can be cast into minimizing β subject to (24). The parameter $\epsilon > 0$ needs to be supplied, and it should be iteratively adjusted to reach the best value of β in the minimization problem just described (there is a convex behavior of β with respect to ϵ as explained in the work of Fridman and Shaked[33]).

4 | TRAJECTORY TRACKING

In this section, it is shown how the proposed method can be easily adapted to solve the problem of trajectory tracking. First, the following assumption is made:

Assumption 5. The desired trajectory $r(t)$ is bounded and sufficiently smooth so that $r(t) \in C^n$.

Problem 2. Under Assumptions 1-5, find a dynamic output control law that internally stabilizes (1)-(2) and guarantees $\|z(t) - r(t)\|_2 \leq \gamma \|\eta(t)\|_2$ for all $0 \neq \eta(t) \in L_2[0, \infty)$, assuming $\mathcal{X}_0 = 0$.

In what follows, the tracking problem is reduced to a stabilization problem so that the methodology described in Section 3 can be directly applied. To that end, let us introduce an auxiliary reference system

$$\dot{x}_r(t) = Ax_r(t) + Bu_r(t), \qquad y_r(t) = Cx_r(t-h), \qquad z_r(t) = Dx_r(t-h), \tag{26}$$

where the auxiliary state $x_r \in \mathbb{R}^n$ has zero initial condition $x_{r_0} = 0$, the matrices A, B, D are the same as in (1)-(2) and $u_r \in \mathbb{R}$ is to be designed such that the auxiliary system is internally stable and $\lim_{t\to\infty}(z_r(t) - r(t)) = 0$. It can be easily verified, because of the canonical structure of (A, B) that the control signal

$$u_r(t) = -\frac{1}{b}\sum_{j=1}^{n} a_j x_{r_j} + \frac{1}{b}\sum_{j=1}^{n} \left(k_{r_j}\left(r^{(j-1)}(t) - x_j(t)\right) + r^{(n)}(t)\right) \tag{27}$$

achieves that goal for any set of gains $k_{r_j} > 0$. Now, let us define the following variables:

$$\tilde{x}(t) \triangleq x(t) - x_r(t-h), \qquad \tilde{y}(t) \triangleq y(t) - y_r(t), \qquad \tilde{z}(t) \triangleq z(t) - z_r(t). \tag{28}$$

Differentiating $\tilde{x}(t)$ and using (8)-(9), (26) and (28) leads to

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B[\tilde{u}(t-h) + w(t)] + \Delta_l \eta(t), \tag{29}$$

$$\tilde{y}(t) = C\tilde{x}(t), \qquad \tilde{z}(t) = D\tilde{x}(t), \tag{30}$$

where the variable $\tilde{u}(t) \triangleq u(t) - u_r(t)$ has been defined. After this transformation, the new control objective is to drive $\tilde{z}(t)$ to zero. Note that (29)-(30) has the same structure as (8)-(9). Therefore, the tracking problem has been reduced to the stabilization problem solved in Section 3. This result is summarized in the following theorem, which is given without proof:

Theorem 2. *Let K and L be computed according to Theorem 1. Then, given any set of positive gains* $k_{r_j} > 0$, $j = 1, \dots, n$, *the control law*

$$u(t) = \tilde{u}(t) + u_r(t) = -\left[K, C_{\xi}\right]\bar{\zeta}(t) + u_r(t), \tag{31}$$

with $u_r(t)$ *given by (27) and* $\bar{\zeta}(t)$ *computed by means of the observer*

$$\dot{\bar{\zeta}}(t) = A_z\bar{\zeta}(t) + B_z\tilde{u}(t) + L\left(\tilde{y}(t) - C_z\bar{\zeta}(t-h)\right), \qquad \bar{\zeta}_0 = 0, \tag{32}$$

solves Problem 2.

Remark 4. The auxiliary control signal $u_r(t)$ can be chosen arbitrarily as long as $\lim_{t\to\infty}(z_r(t) - r(t)) = 0$. Therefore, alternative expressions to (27) are plausible. The tuning of the resulting strategy is intuitive because the tracking performance is decoupled from the stability, as it happens with a conventional two degrees of freedom PID controller.

This can be seen from the control law (31), where the feedback term depends only on K, and u_r is a feed-forward term generated by the auxiliary system (26), which has no influence on the stability. The gains k_{r_j} can be thus arbitrarily adjusted without jeopardizing the stability.

5 | SIMULATIONS

Let us consider an electromechanical system described by

$$\dot{\mathcal{X}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -k \end{bmatrix} \mathcal{X}(t) + \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} u(t-h) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} d(t), \tag{33}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathcal{X}(t), \qquad z(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathcal{X}(t), \tag{34}$$

where $\mathcal{X}_1$ and $\mathcal{X}_2$ are the position and the velocity, $\mathcal{X}_3$ represents the actuator dynamics, u is the control input, and d can be generated by an external force or torque. In this example, the delay is taken as $h = 0.1$ s, and it is assumed that $d(t)$ is a biased sinusoidal disturbance with known frequency, $\omega = 0.8$ rad/s. The controller is designed, according to Theorem 1. Choosing $\alpha = 1$ and solving (25) yields

$$S = \begin{bmatrix} 0.47 & -0.55 & 0.31 \\ -0.55 & 0.89 & -1.06 \\ 0.31 & -1.06 & 3.15 \end{bmatrix}, X^T = \begin{bmatrix} -0.75 \\ 2.10 \\ 0.50 \end{bmatrix} \implies K = XS^{-1} = \begin{bmatrix} 15.66 & 17.18 & 4.37 \end{bmatrix}. \tag{35}$$

Choosing $\delta = 0.8$, the LMI (24) is found to be feasible with a minimum $\gamma = 11.5$ for $\epsilon = 0.3$. In this configuration, the observer gain is given by

$$L^T = \begin{bmatrix} 9.14 & 36.83 & 76.42 & 53.89 & 89.20 & 44.12 \end{bmatrix}. \tag{36}$$

The results of the first simulation are shown in Figure 1. It can be seen in the top plot how the proposed strategy achieves cancellation of the disturbance effect in the output, as stated in Theorem 1. The disturbance signal is given by $\{d(t) = 0, \forall t \in [0, 10);\ d(t) = 1, \forall t \in [10, 20);\ d(t) = 1 + \sin 0.8t, \forall t \geq 20\}$. One can see in the bottom plot that the equivalent input disturbance (dashed black) is accurately predicted by the observer (blue).

Remark 5. As mentioned above, the linearized LMI in Lemma 1 leads to a very conservative value of γ. If the resulting system is analyzed using Lemma 1 with K and L given by (35)- (36), a tighter value $\gamma = 0.74$ is obtained. The exact minimum can be obtained by inspecting the magnitude plot of the transfer function $T_{\eta\to z}(s) \triangleq D_0(sI - A_0 - A_1 e^{-sh})^{-1} B_0$, which reveals that $|T_{\eta\to z}(s)|_\infty = 0.63$. The system is thus contractive, ie, unmodeled components are attenuated at all frequencies.

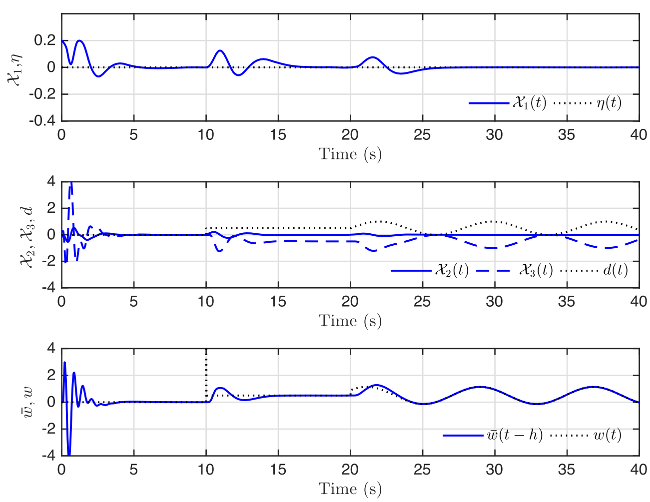

FIGURE 1 Simulation with accurate disturbance model: output and unmodeled component (top), internal states and mismatched disturbance (center), and equivalent input disturbance and its delayed prediction (bottom) [Colour figure can be viewed at wileyonlinelibrary.com]

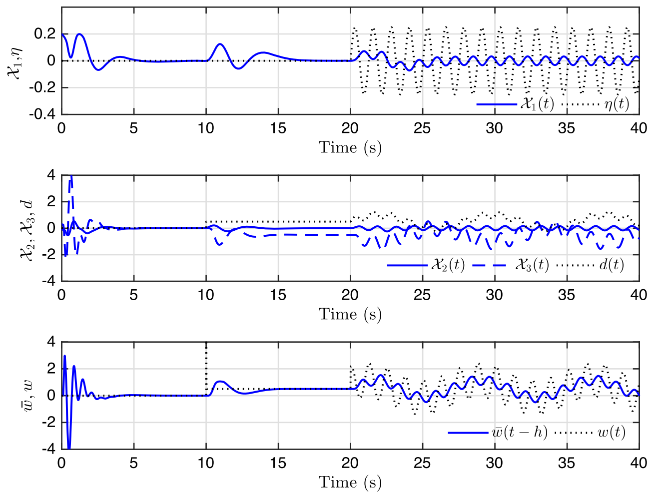

FIGURE 2 Simulation with inaccurate disturbance model: output and unmodeled component (top), internal states and mismatched disturbance (center), and equivalent input disturbance and its delayed prediction (bottom) [Colour figure can be viewed at wileyonlinelibrary.com]

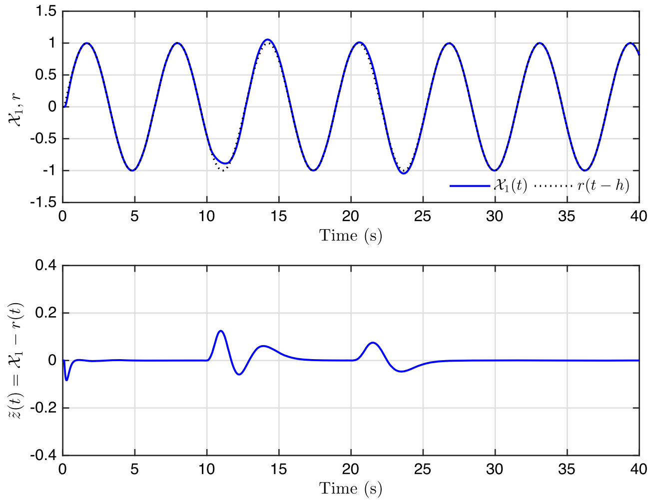

FIGURE 3 Simulation with trajectory tracking: output and reference (top) and tracking error (bottom) [Colour figure can be viewed at wileyonlinelibrary.com]

The second simulation shows the effect of adding an unmodeled disturbance component. In this case, a sinusoidal of higher frequency and smaller amplitude, $\eta(t) = 0.5 \sin 5t, \forall t \geq 30$, is added to the previous disturbance signal. The simulation results are shown in Figure 2, where it can be seen that the known components of the disturbance are still canceled out while the unmodeled component is attenuated by a factor of $|T_{\eta \to z}(5i)| = 0.13$. In this case, the equivalent input disturbance cannot be exactly predicted, as expected.

Finally, the third simulation demonstrates the trajectory tracking capabilities of the proposed strategy. The signal u_r is computed using (27), with $k_{r_1} = \omega_r^3/b$, $k_{r_2} = 3\omega_r^2/b$, $k_{r_3} = 3\omega_r/b$, and $\omega_r = 10$ rad/s. The tracking signal is chosen as $r(t) = \sin t$. The results are shown in Figure 3, where the system starts from the origin and the same disturbance signal as in the first simulation, depicted in Figure 1, is used. One can see how the output of the system tracks the reference in spite of the disturbance.

6 | CONCLUSIONS AND FUTURE WORK

A simple solution to the trajectory tracking for linear time-delay systems has been proposed in this paper and illustrated with a numerical example. The solution is based on the combination of an ESO and a predictor in observer form. The implementation of this strategy is as simple as that of a conventional observer-based feedback controller, thus avoiding the implementation issues of the distributed integral terms in conventional predictive controllers. Furthermore, the sufficient stability conditions presented allow an easy computation of stabilizing gains, as it has been illustrated through simulations. Future work may include the extension of the proposed strategy to the case of multiple-input systems with multiple time delays.

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APPENDIX A

Proof of Proposition 1

Let us rewrite (11) as $w(t) = \sum_{k=0}^{\bar{k}} c_j v^{(k)}(t)$, where $j = k + l + 1$ being k a new summation index with $\bar{k} = n - l$, and the coefficients $c_j = -a_j/b, \forall j \in [l+1, n]$, $c_{n+1} = 1/b$ have been defined for convenience. From (4)-(5), the following identities hold:

$$w(t) = \sum_{k=0}^{\bar{k}} c_j v^{(k)}(t) = \sum_{k=0}^{\bar{k}} c_j C_\xi A_\xi^k \xi_d(t) = C_\xi \xi_w(t),$$

where the definition $\xi_w(t) \triangleq \sum_{k=0}^{\bar{k}} c_j A_\xi^k \xi_d(t)$ has been used in the last equality. Differentiating $\xi_w(t)$ and using (4), it is easy to see that (12) holds, which completes the proof.

APPENDIX B

Proof of Lemma 1

The proof is derived using the Lyapunov-Krasovskii functional

$$V(\mu_t, \dot{\mu}_t) = \mu^T(t)P\mu(t) + h\int_{t-h}^{t} e^{2\delta(s-t)}\mu^T(s)S\mu(s)\,\mathrm{d}s + h\int_{-h}^{0}\int_{t+\theta}^{t} e^{2\delta(s-t)}\dot{\mu}^T(s)R\dot{\mu}(s)\,\mathrm{d}s\mathrm{d}\theta, \tag{B1}$$

which is a slightly simplified version of the one presented in the work of Fridman.[34] The statement of Lemma 1 holds true if it can be shown that[2]

$$\dot{V}(\mu_t, \dot{\mu}_t) + 2\delta V(\mu_t, \dot{\mu}_t) + z^T(t)z(t) - \gamma^2|\eta(t)|^2 \leq 0. \tag{B2}$$

Differentiating (B1), one finds

$$\begin{aligned}\dot{V}(\mu_t, \dot{\mu}_t) + 2\delta V(\mu_t, \dot{\mu}_t) \leq{}& 2\mu^T(t)P\dot{\mu}(t) + 2\delta\mu^T(t)P\mu(t) + h^2\dot{\mu}^T(t)R\dot{\mu}(t)\\ &- he^{-2\delta h}\int_{t-h}^{t}\dot{\mu}^T(s)R\dot{\mu}(s)\,\mathrm{d}s + \mu^T(t)S\mu(t)\\ &- e^{-2\delta h}\mu^T(t-h)S\mu(t-h)\\ &+ 2\left[\mu^T(t)P_2^T + \dot{\mu}^T(t)P_3^T\right]\cdot[\text{RHS of}(21) - \dot{\mu}(t)].\end{aligned} \tag{B3}$$

The last term in (B3), which is identically zero, follows from the application of the descriptor method.[2] The Jensen's inequality is employed to bound

$$-h\int_{t-h}^{t}\dot{\mu}^T(s)R\dot{\mu}(s) \leq -\left[\mu(t) - \mu(t-h)\right]^T R[\mu(t) - \mu(t-h)]. \tag{B4}$$

Let us define $q(t) = [\mu^T(t), \dot{\mu}^T(t), \mu^T(t-h), w(t)]^T$. Using (B3)-(B4), it follows that (B2) holds if (23) is satisfied, completing the proof.

APPENDIX C

Proof of Theorem 1

In order to partially linearize the LMI (23), let us assume $P_2 = \text{diag}\{P_{21}, P_{22}\}$ and, following the work of Fridman,[2] $P_3 = \varepsilon P_2$. Defining $Y = P_{22}^T L$, and after some straightforward calculations, the LMI (23) is transformed into (24). From the triangular structure of the state matrices in (21), $A - BK$ needs to be Hurwitz to ensure the stability of the overall system, which is guaranteed by (25).