A New H_∞ Filter Design for Linear Time Delay Systems

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***Abstract*—A new delay-dependent H_∞ filtering design is proposed for linear, continuous, time-invariant systems with time delay. The obtained filter is of the Luenberger observer type. The design guarantees that the H_∞-norm of the system, relating the exogenous signals to the estimation error, is less than a prescribed level. The filter is based on the application of a newly derived version of the bounded real lemma for time-delayed systems. This novel approach is compared, via an example, with another solution that appears in the literature.**

***Index Terms*—Bounded real lemma, delay-dependent stability, H_∞-filtering, linear matrix inequalities, time-delay systems.**

I. Introduction

THE H_∞ filtering problem for linear systems with delay-dependent [1]–[3] and (more conservative) delay-independent [4], [5] designs have received a lot of attention recently. The prevailing methods are based on bounded real lemma (BRL) in terms of Riccati algebraic equations or linear matrix inequalities (LMIs), which guarantee a prescribed attenuation level. Unfortunately, these criteria provide only sufficient conditions for the required attenuation, and they may lead, in many cases, to a conservative filter design.

Recently, a new efficient criterion has been introduced for verifying the stability and the H_∞-norm of time delayed systems [6], [7]. We derive below a new delay-dependent version of this criterion, which we apply to the adjoint system in order to solve the filtering problem.

Notation: Throughout the paper, the superscript "T" stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$, for $P\in\mathcal{R}^{n\times n}$, means that P is symmetric and positive definite. The space of functions in $\mathcal{R}^q$ that are square integrable over $[0\ \ \infty)$ is denoted by $\mathcal{L}_2^q[0,\infty)$.

II. New Version of the Bounded Real Lemma

Given the system Σ_1

$$\dot{x}(t) = A_0x(t) + A_1(t-h) + Bw(t) \tag{1a}$$

$$z(t) = Lx(t), \qquad x=0,\ \forall\, t\in[-h\ 0] \tag{1b}$$

where $x(t)\in\mathcal{R}^n$ is the system state vector, $w(t)\in\mathcal{L}_2^q[0,\infty]$ is the exogenous disturbance signal, and $z(t)\in\mathcal{R}^p$ is the state combination to be estimated. The time delay h is assumed to be known, and the matrices A_0, A_1, B, and L are constant matrices of appropriate dimensions. Assuming that this system is asymptotically stable and given $\gamma>0$, we seek a criterion that will ensure that the performance index

$$J(w) = \int_0^\infty \left(z^Tz - \gamma^2 w^Tw\right) d\tau \tag{2}$$

is negative $\forall\, w(t)\in\mathcal{L}_2^q[0,\infty]$.

Representing (1) in the equivalent descriptor form

$$\begin{aligned}\dot{x}(t) &= \overline{y}(t),\\ 0 &= -\overline{y}(t) + (A_0+A_1)x(t)\\ &\quad - A_1\int_{t-h}^t \overline{y}(s)\,ds + Bw(t)\end{aligned} \tag{3}$$

the following Lyapunov–Krasovskii functional has been suggested in [6] and [7] for the system (1):

$$\begin{aligned}V(t) &= \begin{bmatrix} x^T(t) & \overline{y}^T(t)\end{bmatrix} EP \begin{bmatrix} x(t)\\ \overline{y}(t)\end{bmatrix}\\ &\quad + \int_{-h}^0\int_{t+\theta}^t \overline{y}^T(s)R\overline{y}(s)\,ds\,d\theta\end{aligned} \tag{4}$$

where

$$E = \begin{bmatrix} I_n & 0\\ 0 & 0\end{bmatrix},\qquad P = \begin{bmatrix} P_1 & 0\\ P_2 & P_3\end{bmatrix},\qquad P_1>0,\ R>0. \tag{5}$$

Based on this functional, an efficient BRL has been derived in [7], in terms of an LMI, which provides the required criterion.

Lemma 2.1 [7]: Consider the system of (1). For a prescribed $\gamma>0$, the cost function (2) achieves $J(w)<0$ for all nonzero $w\in\mathcal{L}_2^q[0,\infty)$ if there exist $0<P_1$, P_2, P_3, and $R=R^T\in\mathcal{R}^{n\times n}$ that satisfy the LMI in (6), shown at the bottom of the next page.

The proof is based on the following idea. It is first noted that

$$\begin{bmatrix} x^T & \overline{y}^T\end{bmatrix} EP\begin{bmatrix} x\\ \overline{y}\end{bmatrix} = x^TP_1x.$$

Thus, differentiating of the first term of (4) with respect to t, we get

$$\begin{aligned}\frac{dV(t)}{dt} &= \frac{d}{dt}\left\{\begin{bmatrix} x^T(t) & \overline{y}^T(t)\end{bmatrix} EP\begin{bmatrix} x(t)\\ \overline{y}(t)\end{bmatrix}\right\}\\ &= 2x^T(t)P_1\dot{x}(t)\\ &= 2\begin{bmatrix} x^T(t) & \overline{y}^T(t)\end{bmatrix} P^T\begin{bmatrix}\dot{x}(t)\\ 0\end{bmatrix}.\end{aligned} \tag{7}$$

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Substituting in the right-hand side of (7), the expressions for $\dot{x}$ and 0 in (3) and applying the standard technique for delay-dependent LMI conditions (see e.g., [8]), it is obtained that $J < 0$ if the following LMI holds:

$$\begin{bmatrix} \Psi & P^T\begin{bmatrix}0\\B\end{bmatrix} & hP^T\begin{bmatrix}0\\A_1\end{bmatrix} \\ \begin{bmatrix}0 & B_1^T\end{bmatrix}P & -\gamma^2 I & 0 \\ h\begin{bmatrix}0 & A_1^T\end{bmatrix}P & 0 & -hR \end{bmatrix} < 0 \tag{8}$$

where

$$\Psi = P^T\begin{bmatrix}0 & I\\ (A_0+A_1) & -I\end{bmatrix} + \begin{bmatrix}0 & \left(A_0^T+A_1^T\right)\\ I & -I\end{bmatrix}P + \begin{bmatrix}L^TL & 0\\ 0 & hR\end{bmatrix}. \tag{9}$$

The latter LMI is equivalent to (6).

Unfortunately, the structure of the resulting LMI is not amenable to solving the corresponding filtering problem. We therefore derive another version of the BRL, which is more suitable for the filter design.

We begin by noting that the H_∞-norm of the system Σ_1 of (1) is given by

$$\|\Sigma_1\|_\infty = \sup_{\omega\in\mathcal{R}} \overline{\sigma}\left\{L\left(j\omega I_n - A_0 - A_1e^{-jwh}\right)^{-1}B\right\} \tag{10}$$

where $\overline{\sigma}\{D\}$ denotes the largest singular value of D. Since

$$\overline{\sigma}\{H(j\omega)\} = \overline{\sigma}\left\{H^T(-j\omega)\right\}$$

for all the transfer function matrices $H(s)$ with real coefficients, it follows that the H_∞-norm of Σ_1 is equal to the H_∞-norm of the following system:

$$\begin{aligned} -\dot{\xi}(t) &= A_0^T\xi(t) + A_1^T\xi(t+h) + L^T\tilde{z}(t)\\ \tilde{w}(t) &= B^T\xi(t), \qquad \xi = 0,\ \forall\, t\in[0\ \ h] \end{aligned} \tag{11}$$

where

$$\xi(t)\in\mathcal{R}^n,\quad \tilde{z}(t)\in\mathcal{R}^p \quad\text{and}\quad \tilde{w}(t)\in\mathcal{R}^q.$$

Note that the latter system represents the backward adjoint of Σ_1 [9]. Its forward representation Σ_2 is described by

$$\begin{aligned} \dot{\xi}(\tau) &= A_0^T\xi(\tau) + A_1^T\xi(\tau-h) + L^T\tilde{z}(\tau)\\ \tilde{w}(t) &= B^T\xi(\tau), \qquad \xi = 0,\ \forall\, t\in[-h\ \ 0]. \end{aligned} \tag{12}$$

Since the characteristic equations of Σ_2 and Σ_1 are identical, the former system is asymptotically stable iff Σ_1 is as well.

Applying Lemma 2.1 [LMI (8)] to system Σ_2, we obtain the following LMI:

$$\begin{bmatrix} \overline{\Psi} & P^T\begin{bmatrix}0\\L^T\end{bmatrix} & hP^T\begin{bmatrix}0\\A_1^T\end{bmatrix} \\ [0\ \ L]P & -\gamma^2 I & 0 \\ h[0\ \ A_1]P & 0 & -hR \end{bmatrix} < 0 \tag{13}$$

where

$$\overline{\Psi} = P^T\begin{bmatrix}0 & I\\ \left(A_0^T+A_1^T\right) & -I\end{bmatrix} + \begin{bmatrix}0 & (A_0+A_1)\\ I & -I\end{bmatrix}P + \begin{bmatrix}BB^T & 0\\ 0 & hR\end{bmatrix}. \tag{14}$$

It is obvious from the requirement of $0 < P_1$, and the fact that in (6) $-(P_3+P_3^T)$ must be negative definite, that P is nonsingular. Defining

$$P^{-1} = Q = \begin{bmatrix}Q_1 & 0\\ Q_2 & Q_3\end{bmatrix} \tag{15a}$$

$$\Delta = \mathrm{diag}\{Q,\ I_{q+n}\} \tag{15b}$$

we multiply (13) by Δ^T and Δ, on the left and on the right, respectively. Applying Schur formula (see, e.g., [10]) to the quadratic term in Q and to hR, we find the following inequality:

$$\begin{bmatrix} \Phi & \begin{bmatrix}0\\L^T\end{bmatrix}Q & h\begin{bmatrix}0\\A_1^T\end{bmatrix} & Q\begin{bmatrix}B\\0\end{bmatrix} & hQ^T\begin{bmatrix}0\\I\end{bmatrix} \\ [0\ \ L] & -\gamma^2 I & 0 & 0 & 0 \\ h[0\ \ A_1] & 0 & -hR & 0 & 0 \\ \begin{bmatrix}B^T & 0\end{bmatrix}Q & 0 & 0 & -I_q & 0 \\ [0\ \ I]Q & 0 & 0 & 0 & -hR^{-1} \end{bmatrix} < 0 \tag{16}$$

where

$$\Phi = \begin{bmatrix}0 & I\\ \left(A_0^T+A_1^T\right) & -I\end{bmatrix}Q + Q^T\begin{bmatrix}0 & (A_0+A_1)\\ I & -I\end{bmatrix}.$$

We thus obtain the following.

Lemma 2.2: Consider the system of (1) and the cost function of (2). For a prescribed $0 < \gamma$, $J(w) < 0$ for all nonzero $w \in \mathcal{L}_2^q[0,\ \infty)$ if there exist $Q_1 > 0, \overline{R} = \overline{R}^T = R^{-1}, Q_2, Q_3$, all in $\mathcal{R}^{n\times n}$, that satisfy the LMI in (17), shown at the bottom of the next page. We note that if the latter LMI possesses a solution for $h = \overline{h} > 0$, then because of the special dependence of the matrix entries of the LMI on the delay length h, it will also posses a solution for all $h < \overline{h}$.

Lemma 2.2 can readily be applied to solving the filtering problem.

$$\begin{bmatrix} \left(A_0^T+A_1^T\right)P_2 + P_2^T\left(A_0+A_1\right) + L^TL & P_1 - P_2^T + \left(A_0^T+A_1^T\right)P_3 & P_2^TB & h_1P_2^TA_1 \\ P_1 - P_2 + P_3^T(A_0+A_1) & -P_3 - P_3^T + hR & P_3^TB & hP_3^TA_1 \\ B^TP_2 & B^TP_3 & -\gamma^2 I_q & 0 \\ hA_1^TP_2 & hA_1^TP_3 & 0 & -hR \end{bmatrix} < 0 \tag{6}$$

III. Filtering Problem

We consider the system of (1a), with the measurement law of

$$y(t) = \mathrm{col}\{C_0 x(t),\, C_1 x(t-h)\} + D_{2,1} w \tag{18}$$

where $y(t) \in \mathcal{R}^r$ is the measurement vector, and the matrices $C_0 \in \mathcal{R}^{r_1 \times n}$, $C_1 \in \mathcal{R}^{r_2 \times n}$, and $D_{2,1} \in \mathcal{R}^{r \times q}$ are constant matrices. We seek a filter of the following observer form:

$$\dot{\hat{x}}(t) = A_0 \hat{x}(t) + A_1 \hat{x}(t-h) + K_0(y(t) - C_0 \hat{x}(t)) \tag{19}$$

such that the H_∞-norm of the resulting transference between the exogenous signal w and the estimation error z is less than a prescribed value γ, where

$$z(t) \triangleq L(x(t) - \hat{x}(t)). \tag{20}$$

We begin solving the problem for the simpler case where $C_1 = 0$. In this case, it follows from (1a), (18) and (19) that the estimation error is described by the following model:

$$\begin{aligned}\dot{e}(t) =& (A_0 - K_0 C_0) e(t) + A_1 e(t-h)\\ &+ (B - K_0 D_{2,1}) w \end{aligned}\tag{21a}$$

$$z(t) = Le(t) \tag{21b}$$

where $e(t) \triangleq x(t) - \hat{x}(t)$. The problem then becomes one of finding the filter gain K_0 such that the H_∞-norm of the system of (21) will be less than a prescribed value. Applying Lemma 2.2 and denoting $Y = Q_1 K_0$ we obtain the following.

Theorem 3.1: Consider the system of (1a), (18), and (19) with $C_1 = 0$ and the cost function (2), where z is defined in (21b). For a prescribed $0 < \gamma$, $J(w) < 0$ for all nonzero $w \in \mathcal{L}_2^q[0, \infty)$ if there exist $Q_1 > 0$, $\overline{R} = \overline{R}^T$, Q_2, Q_3, all in $\mathcal{R}^{n \times n}$, and $Y \in \mathcal{R}^{n \times r}$ that satisfy the LMI in (22), shown at the bottom of the page. If the existence of the matrices Q_1, $\overline{R}$, Q_2, Q_3, and Y is affirmative, the filter gain is given by

$$K_0 = Q_1^{-1} Y. \tag{23}$$

The result of the Theorem 3.1 is applied to the following example.

Example 1: We consider the same system as found in [8] to which a state-feedback has been applied. We assume that the measurement equation is the same as (18), with $r_2 = 0$, and we seek an optimal observer that achieves a minimum estimation level. The matrices in (1) and (18) are as follows:

$$A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \overline{A}_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad L = [1 \quad 0]$$
$$C_0 = [0 \quad 1], \quad C_1 = 0, \quad D_{21} = 0.01, \quad h = 0.999 \text{ s}.$$

We note that the system is unstable. By Theorem 3.1, we obtain a minimum value of $\gamma = 22.8784$ with a filter gain matrix $K = [4790 \quad 18\,139]$.

The above results were obtained for the case where no delay is encountered in the measurement. In case the measurement also includes a delayed state information [C_1 in (18) is not zero], we add an additional component, in series with the delayed component of y. The state space model of this component is given by

$$\dot{\eta}(t) = -\rho I_{r_2} \eta(t) + [0 \quad \rho I_{r_2}] y(t) \tag{24}$$

for $1 \ll \rho$. Denoting the augmented state vector by $\xi(t) = \mathrm{col}\{x(t),\, \eta(t)\}$, the augmented system is then described by

$$\dot{\xi}(t) = \tilde{A}_0 \xi(t) + \tilde{A}_1 \xi(t-h) + \tilde{B} w \tag{25}$$

where

$$\tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & -\rho I_{r_2} \end{bmatrix}, \quad \tilde{A}_1 = \begin{bmatrix} A_1 & 0 \\ \rho C_1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} Q_2 + Q_2^T & Q_3 - Q_2^T + Q_1(A_0 + A_1) & 0 & 0 & Q_1 B & hQ_2^T \\ Q_3^T - Q_2 + \left(A_0^T + A_1^T\right) Q_1 & -Q_3 - Q_3^T & L^T & hA_1^T \overline{R} & 0 & hQ_3^T \\ 0 & L & -\gamma^2 I_p & 0 & \cdot & 0 \\ 0 & h\overline{R} A_1 & 0 & -h\overline{R} & \cdot & \cdot \\ B^T Q_1 & 0 & \cdot & \cdot & -I_q & 0 \\ hQ_2 & hQ_3 & 0 & \cdot & 0 & -h\overline{R} \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} Q_2 + Q_2^T & Q_3 - Q_2^T + Q_1(A_0 + A_1) - YC_0 & 0 & 0 & Q_1 B - YD_{2,1} & hQ_2^T \\ Q_3^T - Q_2 + \left(A_0^T + A_1^T\right) Q_1 - C_0^T Y^T & -Q_3 - Q_3^T & L^T & hA_1^T \overline{R} & 0 & hQ_3^T \\ 0 & L & -\gamma^2 I_p & 0 & \cdot & 0 \\ 0 & h\overline{R} A_1 & 0 & -h\overline{R} & \cdot & \cdot \\ B^T Q_1 - D_{2,1}^T Y^T & 0 & \cdot & \cdot & -I_q & 0 \\ hQ_2 & hQ_3 & 0 & \cdot & 0 & -h\overline{R} \end{bmatrix} < 0 \tag{22}$$

and

$$\tilde{B} = \begin{bmatrix} B \\ \rho[0 \;\; I_{r_2}]D_{2,1} \end{bmatrix}.$$

We consider the following augmented filter:

$$\begin{aligned}\dot{\hat{\xi}}(t) = &\tilde{A}_0\hat{\xi}(t) + \tilde{A}_1\hat{\xi}(t-h)\\ &+ \tilde{K}\left(\mathrm{col}\,\{[I_{r_1} \;\; 0]y(t),\, \eta(t)\} - \tilde{C}\hat{\xi}(t)\right)\end{aligned} \tag{26}$$

where

$$\tilde{C} = \begin{bmatrix} C_0 & 0 \\ 0 & I_{r_2} \end{bmatrix}.$$

The resulting estimation error vector is denoted by $\tilde{e}(t) = \xi(t) - \hat{\xi}(t)$, and we obtain the following state space representation for this error vector:

$$\begin{aligned}\dot{\tilde{e}}(t) = &\left(\tilde{A}_0 - \tilde{K}\tilde{C}\right)\tilde{e}(t) + \tilde{A}_1\tilde{e}(t-h)\\ &+ \left(\tilde{B} - \tilde{K}\,[I_{r_1} \;\; 0]\,D_{2,1}\right)w(t).\end{aligned} \tag{27}$$

Letting $\tilde{z} = [L \;\; 0]\tilde{e}$ and considering

$$J_1 = \int_0^\infty \left(\tilde{z}^T\tilde{z} - \gamma^2 w^T w\right) d\tau \tag{28}$$

we apply Lemma 2.2 and obtain the following:

Theorem 3.2: Consider the system of (25), (18), and (26) and the cost function (28). For a prescribed $0 < \gamma$ and for $\rho \gg 1$, $J_1 < 0$ for all nonzero $w \in \mathcal{L}_2^q[0, \infty)$ if there exist $Q_1 > 0$, $\overline{R} = \overline{R}^T$, Q_2, Q_3, all in $\mathcal{R}^{(n+r)\times(n+r)}$, and $Y \in \mathcal{R}^{(n+r)\times r}$ that satisfy the LMI in (29), shown at the bottom of the page. If the existence of the matrices Q_1, $\overline{R}$, Q_2, Q_3, and Y is affirmative, the filter gain is given by

$$\tilde{K} = Q_1^{-1}Y. \tag{30}$$

The existence of a solution to (22) guarantees that the filter that is built from the series connection of (24) and (26) will achieve the required performance as long as $0 < \rho$. Considering, however, $1 \ll \rho$ and denoting

$$\tilde{K} = \begin{bmatrix} K_{0,0} & K_{0,1} \\ K_{1,0} & K_{1,1} \end{bmatrix}$$

it follows from Theorem 3.2 that if there exists a solution to the LMI of (22), then the estimate of $x(t)$ is given by

$$\begin{aligned}\dot{\hat{x}}(t) = &A_0\hat{x}(t) + A_1\hat{x}(t-h) + K_{0,0}([I_{r_1} \;\; 0]y(t) - C_0\hat{x})\\ &+ K_{0,1}(\eta - \hat{\eta}).\end{aligned} \tag{31}$$

When $1 \ll \rho$ and $r_1 = 0$ (namely, when all the measurements are delayed), the latter equation, together with the one obtained from (26) for $\hat{\eta}$, leads to the following filter:

$$\begin{aligned}\dot{\hat{x}}(t) = &A_0\hat{x}(t) + A_1\hat{x}(t-h)\\ &+ \overline{K}(y(t-h) - C_1\hat{x}(t-h)) + \Delta\end{aligned} \tag{32a}$$

where

$$\overline{K} = (\rho I_r + K_{11})^{-1}\rho K_{01} + O(\rho^{-1}) \tag{32b}$$

and where $\Delta = O(\rho^{-1}, s)$. The latter filter, with $\Delta = 0$, will achieve the required estimation accuracy if ρ is chosen large enough.

We demonstrate the use of the results of Theorem 3.2 in the following example.

Example 2: We consider the system of Example 1, with the measurement delayed by 0.9 s. The matrices of the state space model are given in Example 1, with the difference being that

$$C_0 = 0, \quad C_1 = [0 \quad 1] \quad \text{and} \quad h = 0.9 \text{ s}.$$

By Theorem 3.2, we obtain, using $\rho = 10^{10}$, a minimum value of $\gamma = 128.406$ for the gain matrix $\overline{K} = [-0.8450 \;\; 0.2045]$.

IV. Conclusions

The problem of H_∞ filtering for linear, continuous, time-invariant systems with time delay has been solved. The solution procedure is based on applying an observer type filter and provides a sufficient condition for achieving a prescribed estimation accuracy. Since our results are only sufficient, the question arises as to how big an overdesign is entailed in our method and whether or not it is smaller than the one encountered in other designs appearing in the literature. To answer this question, one has to bear in mind that the filter designs are based, one way or another, on a related bounded real lemma (BRL) that provides the sufficient condition for a system with delay to possess an H_∞-norm that is less than a prescribed value. The overdesign of the corresponding filter design will strongly depend on the conservatism of the BRL used. In this paper, we have used the

$$\begin{bmatrix} Q_2 + Q_2^T & Q_3 - Q_2^T + Q_1\left(\tilde{A}_0 + \tilde{A}_1\right) - Y\tilde{C} & 0 & 0 & Q_1\tilde{B} - Y[I_{r_1} \;\; 0]D_{2,1} & hQ_2^T \\ Q_3^T - Q_2 + \left(\tilde{A}_0^T + \tilde{A}_1^T\right)Q_1 - \tilde{C}^T Y^T & -Q_3 - Q_3^T & [L \;\; 0]^T & h\tilde{A}_1^T\overline{R} & 0 & hQ_3^T \\ 0 & [L \;\; 0] & -\gamma^2 I_p & 0 & \cdot & 0 \\ 0 & h\overline{R}\tilde{A}_1 & 0 & -h\overline{R} & \cdot & \cdot \\ \tilde{B}^T Q_1 - D_{2,1}^T[I_{r_1} \;\; 0]^T Y^T & 0 & \cdot & \cdot & -I_q & 0 \\ hQ_2 & hQ_3 & 0 & \cdot & 0 & -h\overline{R} \end{bmatrix} < 0 \tag{29}$$

BRL that is less conservative than all the other BRLs and therefore obtained the best filtering solution.

The solution method in this note is based on achieving a state space model for the estimation error of order equal to the order of the process to be filtered. In spite of the fact that the LMIs of (22) and (29) are affine in the system matrices, the constraint on the order of the estimation model prevents the application of our results to the case where the system parameters are uncertain.

REFERENCES

[1] A. Fattou, O. Sename, and J. Dion, "H_∞ observer design for time-delay systems," in *Proc. 37th IEEE Conf. Decision Contr.*, Tampa, FL, Dec. 1998, pp. 4545–4546.

[2] A. Pila, U. Shaked, and C. de Souza, "H_∞ filtering for continuous-time linear systems with delay," *IEEE Trans. Automat. Contr.*, vol. 44, pp. 1412–1417, Oct. 1999.

[3] R. Palhares, P. Peres, and C. de Souza, "Robust H_∞ filtering for linear continuous-time uncertain systems with multiple delays: An LMI approach," in *Proc. 3rd IFAC Conf. Robust Contr. Design*, Prague, Czech Republic, July 2000.

[4] J. Ge, P. Frank, and C. Lin, "H_∞ control via output feedback for state delayed systems," *Int. J. Contr.*, vol. 64, pp. 1–7, 1996.

[5] M. Mahmoud, *Robust Control and Filtering for Time-Delay Systems*. New York: Marcel Deccer, 2000.

[6] E. Fridman, "New Lyapunov–Krasovskii functionals for stability of linear retarded and neutral type systems," *Syst. Contr. Lett.*, vol. 43, no. 4, pp. 309–319, 2001.

[7] E. Fridman and U. Shaked, "New bounded real lemma representations for time-delay systems and their applications," *IEEE Trans. Automat. Contr.*, to be published.

[8] C. E. de Souza and X. Li, "Delay-dependent robust H_∞ control of uncertain linear state-delayed systems," *Automatica*, vol. 35, pp. 1313–1321, 1999.

[9] A. Bensoussan, G. Do Prato, M. C. Delfour, and S. K. Mitter, *Representation and Control of Infinite Dimensional Systems*. Boston, MA: Birkhäser, 1992, vol. 1.

[10] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequality in Systems and Control Theory*. Philadelphia, PA: SIAM, Apr. 1994.

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