Dynamic Event-Triggered Control of Networked Stochastic Systems With Scheduling Protocols

Jin Zhang and Emilia Fridman

***Abstract*—In the present article, we consider network-based control of linear systems with state multiplicative noise. For the sensor-controller network, round-robin, try-once-discard, and independent and identically distributed protocols are proposed to orchestrate the measurement transmission from multiple sensor nodes. By using the time-delay approach to networked control systems, sufficient conditions for the exponential mean-square stability are obtained in terms of linear matrix inequalities. Moreover, a discrete-time dynamic event-trigger is suggested to reduce the number of sent control signals from controller to actuator. An air vehicle system from the literature illustrates the efficiency of the presented approach.**

***Index Terms*—Event-triggered control, networked control system (NCS), scheduling protocol, stochastic system.**

I. Introduction

Networked control systems (NCSs) are systems that are comprised of sensors, controllers, and actuators nodes connected via a shared communication network. It is of great importance to analyze the stability and system performance that take into account the variable sampling intervals, communication delays, and scheduling protocols. NCSs with scheduling protocols were studied by the hybrid system approach [1]–[3] and by the time-delay approach [4]–[10]. Only the latter approach allows large transmission delays (that are larger than the sampling intervals).

Systems with stochastic multiplicative noises are encountered in many areas of applications, e.g., aircraft engineering, process control, and population dynamics [11]–[14]. Multiplicative noise appears due to parameter uncertainties and nonlinearities [12]. Many important results on the control of stochastic systems have been reported in [15]–[18]. However, networked control of stochastic systems with scheduling protocols has not been studied yet.

The objective of the present article is to study network-based control of stochastic systems with scheduling protocols. We consider three protocols to orchestrate the measurement transmission from multiple sensor nodes. The first is a round-robin (RR) protocol with simple implementation, and the second is a try-once-discard (TOD) protocol that may improve results achieved under RR protocol (e.g., a larger domain of attraction under actuator saturation [8]). Note that the existing results for the first two protocols in the presence of large transmission delays assume that there are no collisions [4], [6]. The third is an independent and identically distributed (i.i.d) protocol that allows collisions with a certain probability [7]. Note that the i.i.d protocol includes RR protocol as a particular case under the following assumptions: the zero probability of the collisions, and equal probabilities for access to the network by each sensor node.

We extend the time-delay approach to stochastic NCSs. This approach includes two steps [5]: 1) time-delay modeling of the closed-loop system and 2) a choice of appropriate Lyapunov method. The first step for stochastic NCSs is similar to the deterministic NCSs. Thus, under RR protocol we model the closed-loop system as a system with multiple delays, whereas under TOD and i.i.d protocols we model it as an impulsive system with delayed dynamics and reset conditions. The second step is challenging since state-derivative-dependent Lyapunov functionals, that are used in the deterministic case, are not applicable. This is because the solution of the stochastic system does not have a derivative. We propose novel Lyapunov functionals that depend on the deterministic and stochastic parts. Particularly, we propose novel terms to compensate for the delays in the reset conditions.

To reduce the network workload, an event-trigger (ET) can be employed [19]–[21]. Inspired by [22], we suggest a discrete-time dynamic ET based on the control signals that may further reduce the number of sent control signals compared with the static ET [20], [21]. Note that in the absence of scheduling protocols, a discrete-time ET for stochastic systems was suggested in [21] and [23]. The efficiency of the presented method is illustrated by the air vehicle system from [13]. Some preliminary results for the case of RR and TOD protocols were presented in [24].

Notation: Throughout this article, $\mathbb{R}^n$ represents the n dimensional Euclidean space with Euclidean norm $|\cdot|$, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ dimensional real matrices. $X > 0$ means that X is symmetric and positive definite matrix, $\mathrm{He}\{X\}$ denotes $X^T + X$, $|X|^2_S$ denotes the expression X^TSX where matrix S and vector X have appropriate dimensions and $\mathrm{E}\{x\}$ stands for the expectation of stochastic variable x.

II. Problem Formulation

Consider a linear Itô stochastic system

$$dx(t) = [Ax(t) + Bu(t)]dt + Dx(t)dw(t) \tag{1}$$

with the state $x(t) \in \mathbb{R}^n$, control input $u(t) \in \mathbb{R}^{n_u}$, one-dimensional Brownian motion $w(t)$ defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ [12], [14], and constant matrices A, $D \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n_u}$. For a notion of a solution to stochastic system and its existence see [12] and [14].

The stochastic system (1) has N sensor nodes, a controller node and an actuator node connected via sensor-controller and controller-actuator networks. The measurements are given by

$$y_i(t) = C_ix(t) \in \mathbb{R}^{n_{y_i}}, \quad i = 1, \ldots, N, N \geq 2 \tag{2}$$

Manuscript received March 26, 2020; accepted February 17, 2021. Date of publication February 23, 2021; date of current version December 3, 2021. This work was supported in part by Israel Science Foundation under Grant 673/19, in part by C., and H. Manderman Chair on System Control at Tel Aviv University, and in part by the Planning, and Budgeting Committee (PBC) Fellowship from the Council for Higher Education, Israel. Recommended by Associate Editor R. Jain. *(Corresponding author: Jin Zhang.)*

The authors are with the School of Electrical Engineering, Tel Aviv University, Tel Aviv 69978, Israel (e-mail: zhangjin1116@126.com; emilia@tauex.tau.ac.il).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TAC.2021.3061668.

Digital Object Identifier 10.1109/TAC.2021.3061668

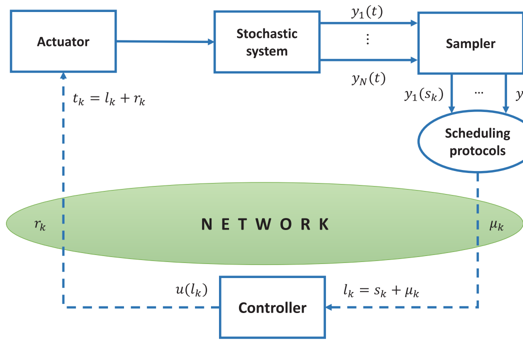

Fig. 1. Networked stochastic systems with scheduling protocols.

with constant matrices $C_i \in \mathbb{R}^{n_{y_i} \times n}$ $(i = 1, \ldots, N)$. For notational brevity, denote $y(t) = [y_1^T(t), \ldots, y_N^T(t)]^T \in \mathbb{R}^{n_y}$, $n_y = \sum_{i=1}^N n_{y_i}$ and $C = [C_1^T, \ldots, C_N^T]^T$.

Let s_k be sampling instants such that

$$s_0 = 0, 0 < s_{k+1} - s_k \leq \text{MATI}, \quad \lim_{k \to +\infty} s_k = +\infty \tag{3}$$

where MATI denotes the maximum allowable transmission interval. We assume that the signal transmissions are subject to network-induced delays μ_k and r_k (see Fig. 1). Thus, the updating instants of controller and actuator are $l_k = s_k + \mu_k$ and $t_k = s_k + \mu_k + r_k$. Notice that μ_k and $\mu_k + r_k$ are not required to be less than the sampling intervals but the sequences $\{l_k\}$ and $\{t_k\}$ should be increasing. Here μ_k and r_k are time-varying bounded by

$$0 \leq \mu_m \leq \mu_k \leq \mu_M, \quad 0 \leq r_m \leq r_k \leq r_M. \tag{4}$$

Denote by $\tilde{y}(s_k) = [\tilde{y}_1^T(s_k), \ldots, \tilde{y}_N^T(s_k)]^T$ the most recently received output information on the controller side. Assume that there exists a matrix $K = [K_1, \ldots, K_N] \in \mathbb{R}^{n_u \times n_y}$ such that $A + BKC$ is Hurwitz. Then a static output-feedback controller takes the form

$$u(l_k) = K\tilde{y}(s_k) \tag{5}$$

and the closed-loop system under this controller has the form

$$\begin{aligned} dx(t) &= [Ax(t) + BK\tilde{y}(s_k)]dt + g(t)dw(t) \\ g(t) &= Dx(t), t \in [t_k, t_{k+1}). \end{aligned} \tag{6}$$

The main objective of this article is to analyze the exponential mean-square stability of stochastic system (6) with scheduling protocols using novel Lyapunov functionals. Additionally, we suggest a discrete-time dynamic ET to reduce the workload of controller-actuator network.

III. Main Results: Stability Analysis With Scheduling Protocols

For stochastic system (6), we will consider three scheduling protocols defined by RR, TOD, and i.i.d protocols. Under the first two protocols, only one sensor is allowed to obtain access to the network, whereas under i.i.d protocol, all the sensors obtain access to the network with a certain probability. Denote by i_k^* the selected sensor node getting access at instant s_k. The value of i_k^* is chosen by scheduling protocols as follows.

1) RR protocol: The sampled measurements are sent in a periodic manner one after another. Thus i_k^* is calculated as

$$i_k^* = \text{mod}(k, N) + 1 \tag{7}$$

where mod denotes the *modulo operation*.

2) TOD protocol: The sampled measurements are sent with the greatest weighted error $\epsilon_i^T(s_k)Q_i\epsilon_i(s_k)$ $(i = 1, \ldots, N)$, where $\epsilon_i(s_k) = \tilde{y}_i(s_{k-1}) - y_i(s_k)$, and Q_i $(i = 1, \ldots, N)$ are positive weighting matrices to be determined. In other words, i_k^* is chosen as

$$i_k^* \triangleq \arg\max_{i=1,\ldots,N}\{\epsilon_i^T(s_k)Q_i\epsilon_i(s_k)\}. \tag{8}$$

3) i.i.d protocol: The choice of i_k^* is assumed to be i.i.d with the probabilities given by

$$\text{Prob}\{i_k^* = i\} = \beta_i, \quad i = 0, \ldots, N \tag{9}$$

where β_0 is the probability of collision, β_i $(i = 1, \ldots, N)$ are the probabilities of the measurements $y_i(s_k)$ to be transmitted at instants s_k and $\sum_{i=0}^N \beta_i = 1$. If collision occurs, then the measurement is dropped [7].

A. Stability Analysis With RR Protocol

Taking into account RR protocol (7), the last transmitted signal before time instant s_k of node i is expressed by

$$\tilde{y}_i(s_k) = y_i(s_{k-\Delta_{k,i}}), \quad i = 1, \ldots, N \tag{10}$$

with some $\Delta_{k,i} \in \{0, 1, \ldots, N-1\}$. We set $y_i(s_{k-\Delta_{k,i}}) = 0$ if $k - \Delta_{k,i} < 0$. Following the time-delay approach [5], [6], define $\tau_i(t) \triangleq t - s_{k-\Delta_{k,i}}, t \in [t_k, t_{k+1})$ from which we have

$$\begin{aligned} 0 &\leq \eta_m \triangleq \mu_m + r_m \leq \tau_i(t) \leq t_{k+1} - s_{k-\Delta_{k,i}} \leq (\Delta_{k,i} + 1) \\ &\times \text{MATI} + \mu_M + r_M \leq N \times \text{MATI} + \mu_M + r_M \triangleq \eta_M. \end{aligned}$$

Then system (6) under RR protocol (7) is expressed by

$$\begin{aligned} dx(t) &= f_1(t)dt + g(t)dw(t), t \geq t_0, \\ f_1(t) &= Ax(t) + \sum_{i=1}^N BK_iC_ix(t - \tau_i(t)). \end{aligned} \tag{11}$$

Assuming for simplicity $x(t) \equiv x(t_0)$ for $t \leq t_0$, consider the following Lyapunov functional

$$\begin{aligned} V_1 &= V_P + V_{S_1} + V_{S_2} + \eta_m V_{R_1} + V_{F_1} \\ &\quad + \sum_{i=1}^N [(\eta_M - \eta_m)V_{R_{2i}} + V_{F_{2i}}], t \geq t_0 \end{aligned} \tag{12}$$

where

$$\begin{array}{ll} V_P = x^T(t)Px(t) & P > 0 \\ V_{S_1} = \int_{t-\eta_m}^t e^{2\alpha(s-t)}|x(s)|_{S_1}^2 ds & S_1 > 0 \\ V_{S_2} = \int_{t-\eta_M}^{t-\eta_m} e^{2\alpha(s-t)}|x(s)|_{S_2}^2 ds & S_2 > 0 \\ V_{R_1} = \int_{-\eta_m}^0 \int_{t+\theta}^t e^{2\alpha(s-t)}|f_1(s)|_{R_1}^2 ds d\theta & R_1 > 0 \\ V_{R_{2i}} = \int_{-\eta_M}^{-\eta_m} \int_{t+\theta}^t e^{2\alpha(s-t)}|f_1(s)|_{R_{2i}}^2 ds d\theta & R_{2i} > 0 \\ V_{F_1} = \int_{-\eta_m}^0 \int_{t+\theta}^t e^{2\alpha(s-t)} g(s)|_{F_1}^2 ds d\theta & F_1 > 0 \\ V_{F_{2i}} = \int_{-\eta_M}^{-\eta_m} \int_{t+\theta}^t e^{2\alpha(s-t)}|g(s)|_{F_{2i}}^2 ds d\theta & F_{2i} > 0 \end{array}$$

and $\alpha > 0$. Functional V_1 is stochastic extension of the Lyapunov functional for the exponential stability with a decay rate α of systems with interval delays (see [5, p. 101]). The terms V_P, V_{S_1} and V_{S_2} have the same form as in the deterministic case. The terms V_{R_1} and $V_{R_{2i}}$ are stochastic extensions of the state-derivative-dependent double integral terms, whereas the terms V_{F_1} and $V_{F_{2i}}$ compensate the stochastic part of (11) [14], [16]. For functional V_1, we use generator $\mathcal{L}$ [11], [14]

$$\mathcal{L}V_1 = \frac{\partial V_1}{\partial t} + \left(\frac{\partial V_1}{\partial x_1}, \dots, \frac{\partial V_1}{\partial x_n}\right) f_1(t) + \frac{1}{2}\text{trace}\{g^T(t)(\frac{\partial^2 V_1}{\partial x_i \partial x_j})_{n\times n} g(t)\}. \quad (13)$$

We arrive at the following result (proved in Appendix A).

Theorem 1: Given scalars $0 \le \eta_m < \eta_M$, $\alpha > 0$ and $N \ge 2$, let there exist $n \times n$ matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_{2i} > 0$, $F_1 > 0$, $F_{2i} > 0$ and W_i $(i = 1, \dots, N)$ that satisfy

$$\Theta = \Theta_1 + \sum_{i=1}^{N} \Theta_2^i + \Theta_3 < 0$$

$$\Xi_i = \begin{bmatrix} R_{2i} & W_i \\ \star & R_{2i} \end{bmatrix} \ge 0, i = 1, \dots, N \quad (14)$$

where

$$\begin{aligned}
\Theta_1 &= \eta_m |D\ell_1|^2_{F_1} + \rho_m [\text{He}\{\ell_1^T R_1 \ell_2\} - |\ell_1 - \ell_3|^2_{R_1} \\
&\quad - |\ell_2|^2_{R_1+F_1}] \\
\Theta_2^i &= (\eta_M - \eta_m)|D\ell_1|^2_{F_{2i}} + \rho_M [\text{He}\{\ell_3^T (R_{2i}\ell_{3i+1} \\
&\quad + W_i \ell_{3i+3}) + \ell_{3i+2}^T R_{2i} \ell_{3i+3}\} - \vartheta_{1i} - \vartheta_{2i}] \\
\Theta_3 &= \text{He}\{\ell_1^T P \chi\} + 2\alpha |\ell_1|^2_P + |\ell_1|^2_{S_1} + |D\ell_1|^2_P \\
&\quad - \rho_m |\ell_3|^2_{S_1 - S_2} - \rho_M |\ell_{3N+4}|^2_{S_2} \\
&\quad + |\eta_m \chi|^2_{R_1} + \sum_{i=1}^{N} |(\eta_M - \eta_m)\chi|^2_{R_{2i}} \\
\vartheta_{1i} &= \begin{bmatrix} \ell_3 - \ell_{3i+2} \\ \ell_{3i+2} - \ell_{3N+4} \end{bmatrix}^T \Xi_i \begin{bmatrix} \ell_3 - \ell_{3i+2} \\ \ell_{3i+2} - \ell_{3N+4} \end{bmatrix} \\
\vartheta_{2i} &= \begin{bmatrix} \ell_{3i+1} \\ \ell_{3i+3} \end{bmatrix}^T \begin{bmatrix} R_{2i} + F_{2i} & W_i + F_{2i} \\ \star & R_{2i} + F_{2i} \end{bmatrix} \begin{bmatrix} \ell_{3i+1} \\ \ell_{3i+3} \end{bmatrix} \\
\chi &= A\ell_1 + \sum_{i=1}^{N} BK_i C_i \ell_{3i+2} \\
\ell_i &= [0_{n\times(i-1)n}, I_n, 0_{n\times(3N+4-i)n}], i = 1, \dots, 3N+4
\end{aligned}$$

with

$$\rho_m = e^{-2\alpha\eta_m}, \rho_M = e^{-2\alpha\eta_M}. \quad (15)$$

Then system (11) is exponentially mean-square stable with a decay rate $\alpha > 0$.

Remark 1: Note that LMIs in Theorem 1 are obtained via Jensen's inequality [5], [25], and reciprocally convex approach [26]. We have avoided using the free-weighting matrix technique [17], [21] that introduces slack decision variables and increases the complexity of LMIs.

B. Stability Analysis With TOD Protocol

Following [6], a piecewise-continuous error is defined as $e_i(t) = \epsilon_i(s_k)$, $t \in [t_k, t_{k+1})$, and some indicator functions are introduced as follows:

$$\pi_{\{i_k^*, i\}} = \begin{cases} 1, & i_k^* = i, \\ 0, & i_k^* \ne i. \end{cases} \quad i = 1, \dots, N \quad (16)$$

Therefore, the reset condition of $e_i(t)$ at $t = t_{k+1}$ is given by

$$e_i(t_{k+1}) = (1 - \pi_{\{i_k^*, i\}})e_i(t_k) + C_i[x(s_k) - x(s_{k+1})] \quad (17)$$

where we assume $x_i(s_{-1}) = 0$ implying $e_i(t_0) = -y_i(s_0) = -C_i x(s_0)$. Define $\tau(t) = t - s_k$ for $t \in [t_k, t_{k+1})$ that yields

$$0 \le \eta_m \le \tau(t) \le t_{k+1} - s_k \le \text{MATI} + \mu_M + r_M \triangleq \tau_M.$$

Then system (6) under TOD protocol (8) is expressed by

$$\begin{aligned}
dx(t) &= f_2(t)dt + g(t)dw(t), t \ge t_0 \\
f_2(t) &= Ax(t) + BKCx(t - \tau(t)) + \sum_{i=1, i\ne i_k^*}^{N} BK_i e_i(t). \quad (18)
\end{aligned}$$

We consider the Lyapunov functional

$$V_2 = \tilde{V}_1 + (\tau_M - \eta_m)V_G + V_Y + V_U + \sum_{i=1}^{N} |e_i(t)|^2_{Q_i}, t \in [t_k, t_{k+1}) \quad (19)$$

where $\tilde{V}_1$ is obtained from V_1 given by (12) with f_1, $\sum_{i=1}^{N} R_{2i}$, $\sum_{i=1}^{N} F_{2i}$, η_M changed by f_2, R_2, F_2, τ_M, and

$$\begin{aligned}
V_G &= \sum_{i=1}^{N} \int_{s_k}^{t} e^{2\alpha(s-t)} |C_i f_2(s)|^2_{G_i} ds \\
V_Y &= \sum_{i=1}^{N} \int_{s_k}^{t} e^{2\alpha(s-t)} |C_i g(s)|^2_{Y_i} ds \\
V_U &= 2\alpha(t_k - t)|e_{i_k^*}(t)|^2_{Q_{i_k^*}} + \sum_{i=1, i\ne i_k^*}^{N} \frac{t_k - t}{t_{k+1} - t_k} |e_i(t)|^2_{U_i} \\
& G_i > 0, Y_i > 0, U_i > 0, Q_i > 0, \alpha > 0.
\end{aligned}$$

The term V_G is the stochastic extension of a similar term in [7]. It is introduced to cope with the delay in the reset conditions, which is continuous on $[t_k, t_{k+1})$ and does not grow in reset instant $t = t_{k+1}$ since

$$\begin{aligned}
& V_G(t_{k+1}) - V_G(t_{k+1}^-) \\
&= -\sum_{i=1}^{N} \int_{s_k}^{s_{k+1}} e^{2\alpha(s - t_{k+1})} |C_i f_2(s)|^2_{G_i} ds \\
&\le -\frac{\tilde{\rho}_M}{\tau_M - \eta_m} \sum_{i=1}^{N} |C_i \int_{s_k}^{s_{k+1}} f_2(s)ds|^2_{G_i} \quad (20)
\end{aligned}$$

where we applied Jensen's inequality with

$$\tilde{\rho}_M = e^{-2\alpha\tau_M}. \quad (21)$$

Similarly, for the term V_Y that depends on the stochastic part $g(t)$, via Itô integral property [14], we have

$$\begin{aligned}
& \text{E}\{V_Y(t_{k+1}) - V_Y(t_{k+1}^-)\} \\
&= -\text{E}\{\sum_{i=1}^{N} \int_{s_k}^{s_{k+1}} e^{2\alpha(s - t_{k+1})} |C_i g(s)|^2_{Y_i} ds\} \\
&\le -\text{E}\{\tilde{\rho}_M \sum_{i=1}^{N} |C_i \int_{s_k}^{s_{k+1}} g(s)dw(s)|^2_{Y_i}\}. \quad (22)
\end{aligned}$$

The term V_U is from [6]. Thus, the functional V_2 is piecewise-continuous and stochastically differentiable in the interval $[t_k, t_{k+1})$. The following lemma gives sufficient conditions for the fact that V_2 does not grow in the reset instant t_k, and also for exponential mean-square stability of (18)

Lemma 1: Given scalars $0 \le \eta_m < \tau_M, \alpha > 0$ and $N \ge 2$, let there exist $n_{y_i} \times n_{y_i}$ matrices $G_i > 0$, $U_i > 0$ and $Q_i > 0$ $(i = 1, \ldots, N)$ and V_2 of (19) that satisfy

$$\begin{bmatrix} Q_i - \tilde{\rho}_M G_i & Q_i & Q_i \\ * & Q_i - \tilde{\rho}_M Y_i & Q_i \\ * & * & \frac{2\alpha(\tau_M - \eta_m) - 1}{N-1} Q_i + U_i \end{bmatrix} < 0$$

$$i = 1, \ldots, N \tag{23}$$

with $\tilde{\rho}_M$ given by (21). Then along (17) and (18), V_2 does not grow in the reset instant t_{k+1}

$$\mathrm{E}\{V_2(t_{k+1}) - V_2(t_{k+1}^-)\} \le 0. \tag{24}$$

Assuming additionally that along (18), $\mathrm{E}\{\mathcal{L}V_2 + 2\alpha V_2\} \le 0$ holds for $t \in [t_k, t_{k+1})$, the exponential mean-square stability of (18) is guaranteed.

Proof is given in Appendix B.

By using Lemma 1 and arguments of Theorem 1, we derive LMI conditions for the exponential mean-square stability of system (18).

Theorem 2: Given scalars $0 \le \eta_m < \tau_M$, $\alpha > 0$ and $N \ge 2$, let there exist $n \times n$ matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_2 > 0$, $F_1 > 0$, $F_2 > 0$, W, $n_{y_i} \times n_{y_i}$ matrices $G_i > 0$, $Y_i > 0$, $U_i > 0$, and $Q_i > 0$ $(i = 1, \ldots, N)$ that satisfy (23) and

$$\Omega_i = \Theta_1 + \Omega_1 + \Omega_{2i} < 0, i = 1, \ldots, N$$

$$\Xi = \begin{bmatrix} R_2 & W \\ \star & R_2 \end{bmatrix} \ge 0 \tag{25}$$

where Θ_1 is given by (14) and

$$\begin{aligned} \Omega_1 &= (\tau_M - \eta_m)|D\ell_1|^2_{F_2} + \tilde{\rho}_M[\mathrm{He}\{\ell_3^T(R_2\ell_4 + W\ell_6) \\ &\quad + \ell_5^T R_2 \ell_6\} - \vartheta_1 - \vartheta_2] \\ \Omega_{2i} &= \mathrm{He}\{\ell_1^T P \varkappa_i\} + 2\alpha|\ell_1|^2_P + |\ell_1|^2_{S_1} + |D\ell_1|^2_P \\ &\quad + \sum_{i=1}^{N} |C_i D\ell_1|^2_{Y_i} - \rho_m|\ell_3|^2_{S_1 - S_2} - \tilde{\rho}_M|\ell_7|^2_{S_2} \\ &\quad + \sum_{j=1}^{i-1} |\ell_{j+7}|^2_{\psi_j} + \sum_{j=i}^{N-1} |\ell_{j+7}|^2_{\psi_{j+1}} + |\varkappa_i|^2_H \\ \vartheta_1 &= \begin{bmatrix} \ell_3 - \ell_5 \\ \ell_5 - \ell_7 \end{bmatrix}^T \Xi \begin{bmatrix} \ell_3 - \ell_5 \\ \ell_5 - \ell_7 \end{bmatrix} \\ \vartheta_2 &= \begin{bmatrix} \ell_4 \\ \ell_6 \end{bmatrix}^T \begin{bmatrix} R_2 + F_2 & W + F_2 \\ \star & R_2 + F_2 \end{bmatrix} \begin{bmatrix} \ell_4 \\ \ell_6 \end{bmatrix} \\ \varkappa_i &= A\ell_1 + BKC\ell_5 + \sum_{j=1}^{i-1} BK_j \ell_{j+7} + \sum_{j=i}^{N-1} BK_{j+1}\ell_{j+7} \end{aligned}$$

with ρ_m and $\tilde{\rho}_M$ given by (15) and (21), and

$$\begin{aligned} H &= \eta_m^2 R_1 + (\tau_M - \eta_m)^2 R_2 + (\tau_M - \eta_m) \sum_{j=1}^{N} C_j^T G_j C_j \\ \psi_j &= 2\alpha Q_j - \frac{U_j}{\tau_M - \eta_m}. \end{aligned} \tag{26}$$

Then system (18) is exponentially mean-square stable with a decay rate $\alpha > 0$.

C. Stability Analysis With I.i.d Protocol

From (9), the mathematical expectations and covariance of $\pi_{\{i_k^*, i\}}$ are obtained as

$$\begin{aligned} &\mathrm{E}\{\pi_{\{i_k^*, i\}}\} = \mathrm{E}\{\pi^2_{\{i_k^*, i\}}\} = \mathrm{Prob}\{i_k^* = i\} = \beta_i \\ &\mathrm{E}\{(\pi_{\{i_k^*, i\}} - \beta_i)(\pi_{\{i_k^*, j\}} - \beta_j)\} = \begin{cases} -\beta_i\beta_j, & i \ne j \\ \beta_i(1 - \beta_i), & i = j. \end{cases} \end{aligned} \tag{27}$$

By utilizing the indicator function $\pi_{\{i_k^*, i\}}$ given by (16), system (18) can be further described by

$$\begin{aligned} dx(t) &= f_3(t)dt + g(t)dw(t), t \ge t_0 \\ f_3(t) &= Ax(t) + BKCx(t - \tau(t)) \\ &\quad + \sum_{i=1}^{N} (1 - \pi_{\{i_k^*, i\}}) BK_i e_i(t). \end{aligned} \tag{28}$$

Let us consider the Lyapunov functional

$$\begin{aligned} V_3 &= \hat{V}_1 + (\tau_M - \eta_m)\hat{V}_G + V_Y + \hat{V}_U \\ &\quad + \sum_{i=1}^{N} |e_i(t)|^2_{Q_i}, t \in [t_k, t_{k+1}) \end{aligned} \tag{29}$$

where $\hat{V}_1$, $\hat{V}_G$, $\hat{V}_U$ are obtained from $\tilde{V}_1$, V_G, V_U given by (19) with $f_2, 2\alpha Q_{i_k^*}$ changed by $f_3, \frac{U_{i_k^*}}{t_{k+1} - t_k}$. Following the proof of Lemma 1, via (17) and (27) we readily formulate the following lemma.

Lemma 2: Given scalars $\tau_M > 0$, $\alpha > 0$ and $N \ge 2$, let there exist $n_{y_i} \times n_{y_i}$ matrices $G_i > 0$, $Y_i > 0$, $U_i > 0$ and $Q_i > 0$ $(i = 1, \ldots, N)$ and V_3 of (29) that satisfy

$$\begin{bmatrix} Q_i - \tilde{\rho}_M G_i & Q_i & (1 - \beta_i)Q_i \\ * & Q_i - \tilde{\rho}_M Y_i & (1 - \beta_i)Q_i \\ * & * & -\beta_i Q_i + U_i \end{bmatrix} < 0$$

$$i = 1, \ldots, N \tag{30}$$

with $\tilde{\rho}_M$ given by (21). Then along (17) and (28), V_3 does not grow in the jumps t_k: (24) holds with V_2 changed by V_3. Assuming additionally that along (28), $\mathrm{E}\{\mathcal{L}V_3 + 2\alpha V_3\} \le 0$ holds for $t \in [t_k, t_{k+1})$, the exponential mean-square stability of (28) is guaranteed.

LMI conditions for the exponential mean-square stability of (28) are derived as follows.

Theorem 3: Given scalars $0 \le \eta_m < \tau_M$, $\alpha > 0$ and $N \ge 2$, let there exist $n \times n$ matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_2 > 0$, $F_1 > 0$, $F_2 > 0$, W, $n_{y_i} \times n_{y_i}$ matrices $G_i > 0$, $Y_i > 0$, $U_i > 0$ and $Q_i > 0$ $(i = 1, \ldots, N)$ that satisfy (30) and

$$\Pi = \Theta_1 + \Omega_1 + \Pi_1 < 0, \Xi \ge 0 \tag{31}$$

where Θ_1, Ω_1 and Ξ are given by (14) and (25), and

$$\begin{aligned} \Pi_1 &= \mathrm{He}\{\ell_1^T P \nu\} + 2\alpha|\ell_1|^2_P + |\ell_1|^2_{S_1} + |D\ell_1|^2_P \\ &\quad + \sum_{i=1}^{N} |C_i D\ell_1|^2_{Y_i} - \rho_m|\ell_3|^2_{S_1 - S_2} - \tilde{\rho}_M|\ell_7|^2_{S_2} \\ &\quad + \sum_{i=1}^{N} |\ell_{i+7}|^2_{\psi_i} + |\nu|^2_H + \sum_{i=1}^{N} \beta_i|\nu_i|^2_H \\ \nu &= A\ell_1 + BKC\ell_5 + \sum_{i=1}^{N} (1 - \beta_i) BK_i \ell_{i+7} \\ \nu_i &= BK_i \ell_{i+7} \end{aligned}$$

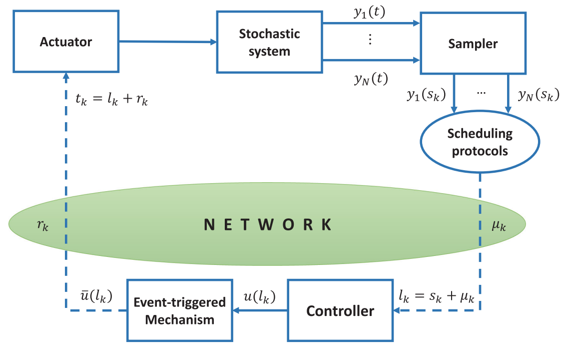

Fig. 2. Networked stochastic systems with scheduling protocols and ET.

with ρ_m and $\tilde{\rho}_M$ from (15), (21), and H and ψ_j from (26). Then system (28) is exponentially mean-square stable with a decay rate $\alpha > 0$.

Remark 2: For small enough stochastic perturbation $|D|$, the assumption of Hurwitz matrix $A + BKC$ implies that there exists a matrix $P > 0$ satisfying $\mathrm{He}\{P(A + BKC)\} + |D|_P^2 < 0$. Note that functionals V_1, V_2, and V_3 are standard Lyapunov functionals for delay-dependent analysis [5]. Therefore, for small enough positive η_M, τ_M, α and $|D|$, the LMIs that preserve the exponential mean-square stability of (11), (18), and (28) are always feasible.

Remark 3: As in [7], our results are applicable to dynamic output-feedback control of (1) with the measurements (2) in the case of one network from sensor to controller. Theorems 1–3 provide here efficient results (see the batch reactor example under RR and TOD protocols in [24]).

IV. Dynamic Event-Triggered Control Based on Control Signal

To reduce the workload of a controller-actuator network, in this section, we consider an ET based on the control signals (see Fig. 2). Inspired by the continuous dynamic ET [22], the idea is to send the control signal $u(l_k)$ that violates the following event-triggering rule:

$$\begin{aligned} &\sigma|u(l_k)|_\Psi^2 - |\delta(l_k)|_\Psi^2 + \gamma\bar{\lambda}(l_k) \geq 0 \\ &\delta(l_k) = u(l_k) - \bar{u}(l_{k-1}) \end{aligned} \tag{32}$$

where $\Psi > 0$ is $n_u \times n_u$ matrix, scalars $\sigma \in (0, 1)$ and $\gamma \geq 0$ are event-triggering parameters, and $\bar{u}(l_{k-1})$ is the last sent control signal before instant l_k

$$\bar{u}(l_k) = \begin{cases} \bar{u}(l_{k-1}), & \text{if (32) is true} \\ u(l_k), & \text{otherwise} \end{cases} \tag{33}$$

with $u(l_{-1}) = 0$. Similar to [23] and [27], $\bar{\lambda}$ is an internal dynamic variable satisfying

$$\dot{\bar{\lambda}}(t) = -\varsigma\bar{\lambda}(t) + \sigma|u(l_k)|_\Psi^2 - |\delta(l_k)|_\Psi^2, t \in [l_k, l_{k+1}) \tag{34}$$

with $\varsigma > 0$ and $\lambda(l_0) = \lambda_0 > 0$. Then we have

$$\begin{aligned} \bar{\lambda}(t) = &\frac{1}{\varsigma}(1 - \mathrm{e}^{-\varsigma(t-l_k)})(\sigma|u(l_k)|_\Psi^2 - |\delta(l_k)|_\Psi^2) \\ &+ \mathrm{e}^{-\varsigma(t-l_k)}\bar{\lambda}(l_k), t \in [l_k, l_{k+1}). \end{aligned}$$

Clearly, $\bar{\lambda}(t)$ is not continuous on $t \in [t_k, t_{k+1})$. It motivates us to introduce an auxiliary function

$$\begin{aligned} \dot{\lambda}(t) = &\frac{l_{k+1} - l_k}{t_{k+1} - t_k}(-\varsigma\lambda(t) + \sigma|u(l_k|_\Psi^2) - |\delta(l_k)|_\Psi^2) \\ &t \in [t_k, t_{k+1}) \end{aligned} \tag{35}$$

and the corresponding solution is given by

$$\begin{aligned} \lambda(t) = &\frac{1}{\varsigma}\left(1 - \mathrm{e}^{-\varsigma\frac{l_{k+1}-l_k}{t_{k+1}-t_k}(t-t_k)}\right)(\sigma|u(l_k)|_\Psi^2 - |\delta(l_k)|_\Psi^2) \\ &+ \mathrm{e}^{-\varsigma\frac{l_{k+1}-l_k}{t_{k+1}-t_k}(t-t_k)}\lambda(t_k), t \in [t_k, t_{k+1}). \end{aligned}$$

Set $\lambda(t_k) = \bar{\lambda}(l_k)$, then $\lambda(t_{k+1}^-) = \bar{\lambda}(l_{k+1}^-)$. From (32), we further have

$$\begin{aligned} \lambda(t) &\geq \frac{1}{\varsigma}((\gamma + \varsigma)\mathrm{e}^{-\varsigma\frac{l_{k+1}-l_k}{t_{k+1}-t_k}(t-t_k)} - \gamma)\lambda(t_k) \\ &\geq \frac{1}{\varsigma}((\gamma + \varsigma)\mathrm{e}^{-\varsigma\tilde{\mu}_M} - \gamma)\lambda(t_k), t \in [t_k, t_{k+1}) \end{aligned}$$

since

$$l_{k+1} - l_k \leq \mathrm{MATI} + \mu_M - \mu_m \triangleq \tilde{\mu}_M.$$

Noting that if $(\gamma + \varsigma)\mathrm{e}^{-\varsigma\tilde{\mu}_M} - \gamma > 0$ holds, we have $\lambda(t) > 0$, $t \in [t_0, t_1)$. Via mathematical induction, we can conclude that $\lambda(t)$ of (35) is positive for $t \geq t_0$ if there exist scalars ς, γ and $\tilde{\mu}_M$ such that $\gamma < \frac{\varsigma}{\mathrm{e}^{\varsigma\tilde{\mu}_M} - 1}$. We assume $s_{k+1} - s_k \geq h > 0$ implying

$$\begin{aligned} &l_{k+1} - l_k \geq h + \mu_m - \mu_M \triangleq \tilde{\mu}_m \\ &t_{k+1} - t_k \geq \tilde{\mu}_m + r_m - r_M \triangleq \tilde{r}_m. \end{aligned}$$

Denote

$$\varrho_1 \triangleq \frac{\sigma\tilde{\mu}_M}{\tilde{r}_m}, \varrho_2 \triangleq e^{-2\alpha(\tau_M - \eta_m)}\frac{\tilde{\mu}_m}{\tau_M - \eta_m}. \tag{36}$$

Remark 4: In comparison to the static ET [20], [21], the dynamic ET (32) has an additional tuning parameter $\gamma \geq 0$ that may allow to further reduce a workload. In the worst case (32) with $\gamma = 0$ coincides with the static ET. However, as mentioned in [22], a positive γ may improve the result leading to a smaller number of sent signals. Our example below shows essential improvement by the dynamic ET with appropriate values of $\gamma > 0$ over the static ET.

A. Dynamic Event-Triggered Control With RR Protocol

The stochastic system (6) under RR protocol (7) and ET (32) takes the form [cf., (11)]

$$dx(t) = f_4(t)dt + g(t)dw(t), t \in [t_k, t_{k+1}) \tag{37}$$

where $f_4(t) = f_1(t) - B\delta(l_k)$ with f_1 given by (11). We now establish the following LMI conditions.

Theorem 4: Given scalars $0 \leq \eta_m < \eta_M$, $\alpha > 0$, $N \geq 2$, ϱ_1 and ϱ_2 given by (36), let there exist $n \times n$ matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_{2i} > 0$, $F_1 > 0$, $F_{2i} > 0$, W_i $(i = 1, \ldots, N)$ and $n_u \times n_u$ matrix $\Psi > 0$ that satisfy

$$\tilde{\Theta} + \hat{\Theta} < 0, \Xi_i \geq 0, i = 1, \ldots, N \tag{38}$$

where $\tilde{\Theta}$ is obtained from Θ given by (14) with χ changed by $\tilde{\chi} = \chi - B\ell_{3N+5}$, Ξ_i $(i = 1, \ldots, N)$ are given by (14) and

$$\hat{\Theta} = \varrho_1|\sum_{i=1}^{N} K_iC_i\ell_{3i+2}|_\Psi^2 - \varrho_2|\ell_{3N+5}|_\Psi^2. \tag{39}$$

Then system (37) is exponentially mean-square stable with a decay rate $\alpha > 0$.

Proof is given in Appendix C.

B. Dynamic Event-Triggered Control With TOD Protocol

The stochastic system (6) under TOD protocol (8) and ET (32) takes the form [cf., (18)]

$$dx(t) = f_5(t)dt + g(t)dw(t), t \in [t_k, t_{k+1}), \tag{40}$$

where $f_5(t) = f_2(t) - B\delta(l_k)$ with f_2 given by (18). Let us consider the Lyapunov functional

$$V_5 = \bar{V}_2 + e^{-2\alpha(t-t_k)}\lambda, t \in [t_k, t_{k+1}) \tag{41}$$

where $\bar{V}_2$ is obtained from V_2 given by (19) with f_2 changed by f_5. Then following the proof of Theorems 2 and 4, we obtain the following LMI conditions.

Theorem 5: Given scalars $0 \le \eta_m < \tau_M, \alpha > 0, N \ge 2$, ϱ_1 and ϱ_2 given by (36), let there exist $n \times n$ matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_2 > 0$, $F_1 > 0$, $F_2 > 0$, W, $n_{y_i} \times n_{y_i}$ matrices $G_i > 0$, $Y_i > 0$, $U_i > 0$, $Q_i > 0$ $(i = 1, \dots, N)$ and $n_u \times n_u$ matrix $\Psi > 0$ that satisfy (23) and

$$\tilde{\Omega}_i + \hat{\Omega} < 0, i = 1, \dots, N, \Xi \ge 0 \tag{42}$$

where $\tilde{\Omega}_i$ is obtained from Ω_i given by (25) with $\varkappa_i$ changed by $\tilde{\varkappa}_i = \varkappa_i - B\ell_{N+7}$, Ξ is given by (25) and

$$\hat{\Omega} = \varrho_1 |KC\ell_5|^2_\Psi - \varrho_2 |\ell_{N+7}|^2_\Psi. \tag{43}$$

Then system (40) is exponentially mean-square stable with a decay rate $\alpha > 0$.

C. Dynamic Event-Triggered Control With I.i.d Protocol

The stochastic system (6) under the i.i.d protocol (9) and ET (32) takes the form [cf., (28)]

$$dx(t) = f_6(t)dt + g(t)dw(t), t \in [t_k, t_{k+1}) \tag{44}$$

where $f_6(t) = f_3(t) - B\delta(l_k)$ with f_3 given by (28). Choosing Lyapunov functional

$$V_6 = \bar{V}_3 + e^{-2\alpha(t-t_k)}\lambda, t \in [t_k, t_{k+1}) \tag{45}$$

where $\bar{V}_3$ is obtained from V_3 given by (29) with f_3 changed by f_6, we arrive at the following result.

Theorem 6: Given scalars $0 \le \eta_m < \tau_M, \alpha > 0, N \ge 2$, ϱ_1 and ϱ_2 given by (36), let there exist $n \times n$ matrices $P > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, $R_2 > 0$, $F_1 > 0$, $F_2 > 0$, W, $n_{y_i} \times n_{y_i}$ matrices $G_i > 0$, $Y_i > 0, U_i > 0$ and $Q_i > 0$ $(i = 1, \dots, N)$ and $n_u \times n_u$ matrix $\Psi > 0$ that satisfy (30) and

$$\tilde{\Pi} + \hat{\Pi} < 0, \Xi \ge 0 \tag{46}$$

where $\tilde{\Pi}$ is obtained from Π given by (31) with ν changed by $\tilde{\nu} = \nu - B\ell_{N+8}$, Ξ is given by (25) and

$$\hat{\Pi} = \varrho_1 |KC\ell_5|^2_\Psi - \varrho_2 |\ell_{N+8}|^2_\Psi. \tag{47}$$

Then system (44) is exponentially mean-square stable with a decay rate $\alpha > 0$.

Remark 5: Note that due to ET (32), LMIs (38), (42), and (46) have additional terms that are proportional to the triggering parameter $\sigma \in (0, 1)$. As in Remark 2, we can conclude that LMIs (38), (42), and (46) are always feasible for small enough positive σ, η_M, τ_M, α, and $|D|$.

TABLE I
MAXIMUM VALUES OF $\mu_M + r_M$ FOR DIFFERENT d

d	0.1	0.2	0.5	1	2	3
Th. 1 (RR)	0.82	0.81	0.76	0.66	0.39	0.16
Th. 2 (TOD)	0.18	0.18	0.17	0.16	0.07	–
Th. 3 (i.i.d, $\beta_0 = 0$)	0.20	0.20	0.19	0.18	0.11	0.02
Th. 3 (i.i.d, $\beta_0 = 0.2$)	0.14	0.13	0.13	0.12	0.05	–

V. NUMERICAL EXAMPLE

Consider an air vehicle system taken from [13]

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = (-5x_2 + u)dt \\ dx_3 = 5(x_1 - x_3)dt + 0.5x_1 dw(t) \\ dx_4 = 5(x_2 - x_4)dt. \end{cases} \tag{48}$$

We assume that x_3 is affected by state multiplicative noise [17], [21]. Then the corresponding matrices described in (1) are given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 5 & 0 & -5 & 0 \\ 0 & 5 & 0 & -5 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{49}$$
$$D = [0_{4\times1}, 0_{4\times1}, \bar{D}^T, 0_{4\times1}], \bar{D} = [d, 0, 0, 0].$$

We choose $K = -[3.2938, 1.3576, 0.5706, 0.5706] = [k_1, k_2, k_3, k_4]$ and $C = I_4$ such that $A + BKC$ is Hurwitz. In the case of $N = 2$, we consider $K_1 = [k_1, k_2]$, $K_2 = [k_3, k_4]$ and

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In the case of $N = 4$, $K_1, \dots, K_4$ are the entries of K and $C_1, \dots, C_4$ are the rows of I_4. Let $\mu_m = r_m = 0.005$, $s_{k+1} - s_k = 0.05$, and $\alpha = 0.01$.

1) For the values of stochastic perturbation d given in Table I, by applying Theorems 1–3 with $N = 2$, we obtain the maximum values of $\mu_M + r_M$ that preserve the stability (see Table I), where under the i.i.d protocol we consider $\beta_0 = 0$, $\beta_1 = \beta_2 = 0.5$ in the case of no collisions and $\beta_0 = 0.2$, $\beta_1 = \beta_2 = 0.4$ in the case of collisions.

The results under i.i.d protocol with $\beta_0 = 0$ are more conservative than those under RR protocol for two reasons: 1) i.i.d protocol includes essentially more possibilities (not just periodic transmission of measurements); 2) as in the deterministic case [4], [6], [7], the Lyapunov-Krasovskii method for hybrid delayed systems (the case of i.i.d protocol) is usually less efficient than that for continuous-time delayed systems (the case of RR protocol). Moreover, if the probability of collisions β_0 grows, TOD protocol may lead to better results than those under i.i.d protocol.

2) Next, we perform the simulations to illustrate the advantage of dynamic ET (32) over the static ET. Consider $N = 4$. Let $d = 1$ and $\mu_M = r_M = 0.01$. Solving LMI conditions of Theorems 4–6 leads to the maximum values of triggering parameter σ (which is the main interest under ET control) and the corresponding values of Ψ (see Table II), where under the i.i.d protocol we choose $\beta_0 = 0$ and $\beta_i = 0.25$ $(i = 1, \dots, 4)$.

Given the initial state $x(0) = [-0.8, 0.5, -0.3, 0.2]^T$ and triggering parameters $\gamma = 0.5$, $\lambda_0 = 0.2$, and $\varsigma = 1$, the state trajectories, release intervals with RR, TOD, and i.i.d protocols are shown in Fig. 3. Clearly, the stability is preserved in the presence of scheduling protocols, dynamic ET, and state multiplicative noise. As shown in Table II, dynamic ET (32) allows to reduce the workload of the controller-actuator

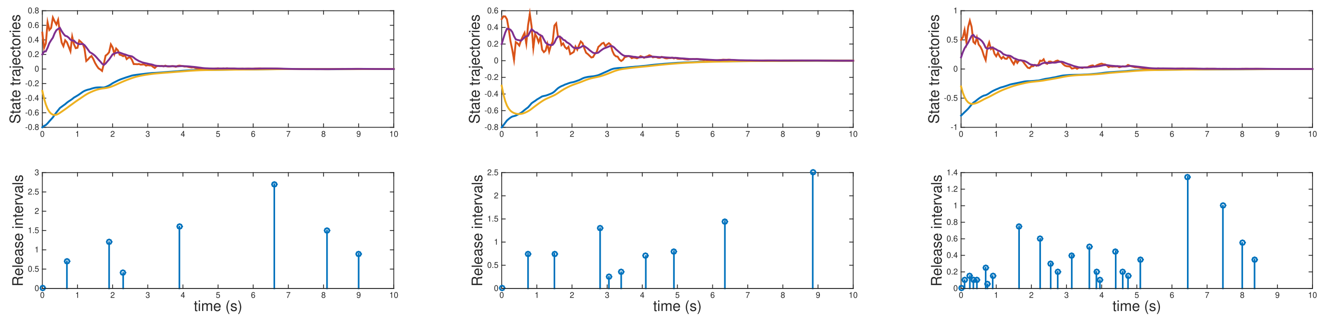

Fig. 3. State trajectories and release intervals of air vehicle system (48) with RR (left), TOD (middle), and i.i.d (right) protocols.

TABLE II
NUMBER OF SENT CONTROL SIGNALS (SCSS) UNDER VARIOUS ETS

	σ	Ψ	Numbers of SCSs	
			Dynamic ET	Static ET
Th. 4 (RR)	0.42	0.4268	8	16
Th. 5 (TOD)	0.19	0.4592	10	24
Th. 6 (i.i.d, $\beta_0 = 0$)	0.02	2.0511	24	61

network by 96.02%, 95.02%, and 88.06%, and improves the results under the static ET. Here TOD protocol leads to a smaller number of sent control signals than that under i.i.d protocol, whereas (as in 1)) RR protocol leads to the best result.

VI. CONCLUSION

In this article, the time-delay approach has been developed for networked systems with state multiplicative noise, where RR, TOD, and i.i.d protocols on the sensor side, and dynamic ET on the controller side are taken into account. Sufficient conditions for the exponential mean-square stability have been obtained in terms of LMIs. The efficiency of the presented approach has been demonstrated by the air vehicle system. Future work will involve improvements by using advanced Lyapunov-based methods.

APPENDIX

A. Proof of Theorem 1

Denote

$$\zeta = [x^T(t), \int_{t-\eta_m}^{t} g^T(s)dw(s), x^T(t-\eta_m), \nu_1^T \ldots, \nu_N^T, x^T(t-\eta_M)]^T$$

$$\nu_i = [\int_{t-\tau_i(t)}^{t-\eta_m} g^T(s)dw(s), x^T(t-\tau_i(t)) \int_{t-\eta_M}^{t-\tau_i(t)} g^T(s)dw(s)]^T.$$

For $t \geq t_0$ we have

$$\mathcal{L}V_P + 2\alpha V_P = \zeta^T(\mathrm{He}\{\ell_1^T P\chi\} + 2\alpha|\ell_1|_P^2 + |D\ell_1|_P^2)\zeta$$

$$\mathcal{L}V_{S_1} + 2\alpha V_{S_1} = \zeta^T(|\ell_1|_{S_1}^2 - \rho_m|\ell_3|_{S_1}^2)\zeta$$

$$\mathcal{L}V_{S_2} + 2\alpha V_{S_2} = \zeta^T(\rho_m|\ell_3|_{S_2}^2 - \rho_M|\ell_{3N+4}|_{S_2})\zeta$$

$$\mathcal{L}V_{R_1} + 2\alpha V_{R_1} = \eta_m|\chi\zeta|_{R_1}^2 - \int_{t-\eta_m}^{t} e^{2\alpha(s-t)}|f_1(s)|_{R_1}^2 ds$$

$$\mathcal{L}V_{R_{2i}} + 2\alpha V_{R_{2i}} = (\eta_M - \eta_m)|\chi\zeta|_{R_{2i}}^2 - \int_{t-\eta_M}^{t-\eta_m} e^{2\alpha(s-t)}|f_1(s)|_{R_{2i}}^2 ds$$

$$\mathcal{L}V_{F_1} + 2\alpha V_{F_1} = \eta_m|D\ell_1\zeta|_{F_1}^2 - \int_{t-\eta_m}^{t} e^{2\alpha(s-t)}|g(s)|_{F_1}^2 ds$$

$$\mathcal{L}V_{F_{2i}} + 2\alpha V_{F_{2i}} = (\eta_M - \eta_m)|D\ell_1\zeta|_{F_{2i}}^2 - \int_{t-\eta_M}^{t-\eta_m} e^{2\alpha(s-t)}|g(s)|_{F_{2i}}^2 ds. \tag{50}$$

Via Jensen's inequality [25]

$$\frac{\eta_m}{\rho_m}\int_{t-\eta_m}^{t} e^{2\alpha(s-t)}|f_1(s)|_{R_1}^2 ds \geq |\int_{t-\eta_m}^{t} f_1(s)ds|_{R_1}^2 \tag{51}$$

whereas under (14) by arguments of [26]

$$\frac{\eta_M - \eta_m}{\rho_M}\int_{t-\eta_M}^{t-\eta_m} e^{2\alpha(s-t)}|f_1(s)|_{R_{2i}}^2 ds \geq \begin{bmatrix} \int_{t-\tau_i(t)}^{t-\eta_m} f_1(s)ds \\ \int_{t-\eta_M}^{t-\tau_i(t)} f_1(s)ds \end{bmatrix}^T \Xi_i \begin{bmatrix} \int_{t-\tau_i(t)}^{t-\eta_m} f_1(s)ds \\ \int_{t-\eta_M}^{t-\tau_i(t)} f_1(s)ds \end{bmatrix} \tag{52}$$

where Ξ_i $(i = 1, \ldots, N)$ are given by (14), and ρ_m and ρ_M are given by (15). From (11), we have

$$\int_{t-\eta_m}^{t} f_1(s)ds = (\ell_1 - \ell_2 - \ell_3)\zeta$$

$$\int_{t-\tau_i(t)}^{t-\eta_m} f_1(s)ds = (\ell_3 - \ell_{3i+1} - \ell_{3i+2})\zeta$$

$$\int_{t-\eta_M}^{t-\tau_i(t)} f_1(s)ds = (\ell_{3i+2} - \ell_{3i+3} - \ell_{3N+4})\zeta. \tag{53}$$

By using Itô integral property [14], we have

$$\mathrm{E}\{\int_{t-\eta_m}^{t} e^{2\alpha(s-t)}|g(s)|_{F_1}^2 ds\} \geq \mathrm{E}\{\rho_m|\ell_2\zeta|_{F_1}^2\} \tag{54}$$

and

$$\mathrm{E}\{\int_{t-\eta_M}^{t-\eta_m} e^{2\alpha(s-t)}|g(s)|_{F_{2i}}^2 ds\} \geq \mathrm{E}\{\rho_M|(\ell_{3i+1} + \ell_{3i+3})\zeta|_{F_{2i}}^2\}. \tag{55}$$

Applying Theorem 5.9, [14, p. 22] and in view of (50)–(55), we have

$$\mathrm{E}\{\mathcal{L}V_1 + 2\alpha V_1\} \leq \mathrm{E}\{\zeta^T\Theta\zeta\} \leq 0 \tag{56}$$

where the second inequality is based on (14). It yields that $\mathrm{E}\{\mathcal{L}(e^{2\alpha t}V_1)\} \leq 0$. By integrating from t_0 to t, we obtain

$$\mathrm{E}\{V_1(t)\} \leq e^{2\alpha(t_0-t)}\mathrm{E}\{V_1(t_0)\}. \tag{57}$$

Since $\lambda_{\min}\{P\}\mathrm{E}\{|x(t)|^2\} \leq \mathrm{E}\{V_1(t)\}$ and $\mathrm{E}\{V_1(t_0)\} \leq \mu\mathrm{E}\{|x(t_0)|^2\}$ for some $\mu > 0$, the exponential mean-square stability of (11) is guaranteed. □

B. Proof of Lemma 1

First, we show that V_2 does not grow in the reset instant t_{k+1}. From (19), we obtain

$$\begin{aligned} V_2(t_{k+1}) - V_2(t_{k+1}^-) &= \sum_{i=1}^{N}[|e_i(t_{k+1})|^2_{Q_i} - |e_i(t_k)|^2_{Q_i}] \\ &+ 2\alpha(t_{k+1}-t_k)|e_{i_k^*}(t_k)|^2_{Q_{i_k^*}} \\ &+ (\tau_M - \eta_m)[V_G(t_{k+1}) - V_G(t_{k+1}^-)] \\ &+ V_Y(t_{k+1}) - V_Y(t_{k+1}^-) + \sum_{i=1, i\neq i_k^*}^{N}|e_i(t_k)|^2_{U_i}. \end{aligned} \tag{58}$$

Since $t_{k+1} - t_k \leq \tau_M - \eta_m$, from (20), (22), and (58), we further obtain

$$\begin{aligned} &\mathrm{E}\{V_2(t_{k+1}) - V_2(t_{k+1}^-)\} \\ &\leq \mathrm{E}\{\sum_{i=1, i\neq i_k^*}^{N}[|e_i(t_{k+1})|^2_{Q_i} + |e_i(t_k)|^2_{U_i - Q_i}] + |e_{i_k^*}(t_{k+1})|^2_{Q_{i_k^*}} \\ &+ [2\alpha(\tau_M - \eta_m) - 1]|e_{i_k^*}(t_k)|^2_{Q_{i_k^*}} \\ &- \tilde{\rho}_M\sum_{i=1}^{N}[|C_i\int_{s_k}^{s_{k+1}} f_2(s)ds|^2_{G_i} + |C_i\int_{s_k}^{s_{k+1}} g(s)dw(s)|^2_{Y_i}]\}. \end{aligned} \tag{59}$$

Noticing that

$$x(s_{k+1}) - x(s_k) = \int_{s_k}^{s_{k+1}} f_2(s)ds + \int_{s_k}^{s_{k+1}} g(s)dw(s)$$

and applying (8), (17) with $2\alpha(\tau_M - \eta_m) - 1 < 0$, we have

$$\mathrm{E}\{V_2(t_{k+1}) - V_2(t_{k+1}^-)\} \leq \mathrm{E}\{|\tilde{\kappa}_{i_k^*}|^2_{\tilde{\Lambda}_{i_k^*}} + \sum_{i=1, i\neq i_k^*}^{N}|\kappa_i|^2_{\Lambda_i}\} \tag{60}$$

where $\tilde{\kappa}_i = [\int_{s_k}^{s_{k+1}} f_2^T(s)C_i^T ds, \int_{s_k}^{s_{k+1}} g^T(s)C_i^T dw(s)]^T$, $\kappa_i = [\tilde{\kappa}_i^T, e_i^T(t_k)]^T$ and $\tilde{\Lambda}_i$ is obtained from Λ_i by taking away the last block-column and block-row. It is clear that (23) provides (24).

Next, we show the exponential mean-square stability of (18). From $e^{2\alpha t}[\mathcal{L}V_2 + 2\alpha V_2] \leq 0$, it follows that

$$\mathrm{E}\{V_2(t)\} \leq \mathrm{E}\{e^{-2\alpha(t-t_k)}V_2(t_k)\}, t \in [t_k, t_{k+1}). \tag{61}$$

By using mathematical induction with (24), we arrive at

$$\mathrm{E}\{V_2(t)\} \leq \mathrm{E}\{e^{-2\alpha(t-t_0)}V_2(t_0)\}, t \geq t_0. \tag{62}$$

The end of the proof is similar to that of Theorem 1. □

C. Proof of Theorem 4

From (32) and (35), we have

$$\dot{\lambda}(t) \leq \frac{l_{k+1} - l_k}{t_{k+1} - t_k}(\sigma|u(l_k)|^2_\Psi - |\delta(l_k)|^2_\Psi), t \in [t_k, t_{k+1}).$$

By using the notations (36), we have

$$\begin{aligned} \dot{\lambda}(t) &\leq \varrho_1|u(l_k)|^2_\Psi - \varrho_2|\delta(l_k)|^2_\Psi \\ &= \tilde{\zeta}^T(t)\hat{\Theta}\tilde{\zeta}(t), t \in [t_k, t_{k+1}). \end{aligned} \tag{63}$$

Here $\tilde{\zeta}(t) = [\zeta^T(t), \delta^T(l_k)]^T$ and $\hat{\Theta}$ is given by (39). We consider the Lyapunov functional

$$V_4 = \bar{V}_1 + e^{-2\alpha(t-t_k)}\lambda, t \in [t_k, t_{k+1}) \tag{64}$$

where $\bar{V}_1$ is obtained from V_1 given by (12) with f_1 changed by f_4. It is clear that functional V_4 is positive definite since $\lambda(t) > 0$ for $t \geq t_0$. Applying the generator $\mathcal{L}$ to V_4, we obtain

$$\begin{aligned} \mathcal{L}V_4 &= \mathcal{L}\bar{V}_1 - 2\alpha e^{-2\alpha(t-t_k)}\lambda(t) + e^{-2\alpha(t-t_k)}\dot{\lambda}(t) \\ &\leq \mathcal{L}\bar{V}_1 - 2\alpha e^{-2\alpha(t-t_k)}\lambda(t) + \tilde{\zeta}^T(t)\hat{\Theta}\tilde{\zeta}(t) \\ &t \in [t_k, t_{k+1}). \end{aligned} \tag{65}$$

Following the proof of Theorem 1, we arrive at

$$\mathrm{E}\{\mathcal{L}\bar{V}_1 + 2\alpha\bar{V}_1\} \leq \mathrm{E}\{\tilde{\zeta}^T(t)\tilde{\Theta}\tilde{\zeta}(t)\}, t \geq t_0 \tag{66}$$

where $\tilde{\Theta}$ is given by (38). Combining (65) and (66) yields

$$\begin{aligned} &\mathrm{E}\{\mathcal{L}V_4 + 2\alpha V_4\} \leq \mathrm{E}\{\tilde{\zeta}^T(t)(\tilde{\Theta} + \hat{\Theta})\tilde{\zeta}(t)\} \leq 0 \\ &t \in [t_k, t_{k+1}). \end{aligned} \tag{67}$$

Similar to the end of the proof of Theorem 1, we can conclude that the exponential mean-square stability of (37) is guaranteed. □

References

[1] G. C. Walsh, H. Ye, and L. Bushnell, "Stability analysis of networked control systems," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 3, pp. 438–446, May 2002.

[2] D. Nesic and A. R. Teel, "Input-output stability properties of networked control systems," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1650–1667, Oct. 2004.

[3] W. M. H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nesic, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.

[4] K. Liu, E. Fridman, and L. Hetel, "Stability and L_2-gain analysis of networked control systems under round-robin scheduling: A time-delay approach," *Syst. Control Lett.*, vol. 61, no. 5, pp. 666–675, 2012.

[5] E. Fridman, *Introduction to Time-Delay Systems: Analysis and Control*. Cambridge, MA, USA: Birkhäuser, 2014.

[6] D. Freirich and E. Fridman, "Decentralized networked control of systems with local networks: A time-delay approach," *Automatica*, vol. 69, pp. 201–209, 2016.

[7] K. Liu, E. Fridman, and K. H. Johansson, "Networked control with stochastic scheduling," *IEEE Trans. Autom. Control*, vol. 60, no. 99, pp. 3071–3076, Nov. 2015.

[8] K. Liu and E. Fridman, "Discrete-time network-based control under scheduling and actuator constraints," *Int. J. Robust Nonlinear Control*, vol. 25, no. 12, pp. 1816–1830, 2015.

[9] J. Zhang and C. Peng, "Networked H_∞ filtering under a weighted TOD protocol," *Automatica*, vol. 107, pp. 333–341, 2019.

[10] K. Liu, A. Selivanov, and E. Fridman, "Survey on time-delay approach to networked control," *Annu. Rev. Control*, vol. 48, pp. 57–79, 2019.

[11] E. Gershon, U. Shaked, and I. Yaesh, *H_∞ Control and Estimation of State-Multiplicative Linear Systems*. New York, NY, USA: Springer Science & Business Media, vol. 318, 2005.

[12] L. Shaikhet, *Lyapunov Functionals and Stability of Stochastic Functional Differential Equations*. New York, NY, USA: Springer Science & Business Media, 2013.

[13] I. Yaesh, U. Shaked, and T. Yossef, "Simplified adaptive control of F16 aircraft pitch and angle-of-attack loops," in *Proc. Isr. Annu. Conf. Aerosp. Sci.*, pp. 25–26, 2004.

[14] X. Mao, *Stochastic Differential Equations and Applications*. Chichester: Horwood, 2007.

[15] F. Cacace, A. Germani, C. Manes, and M. Papi, “Predictor-based control of stochastic systems with nonlinear diffusions and input delay,” *Automatica*, vol. 107, pp. 43–51, 2019.
[16] E. Fridman and L. Shaikhet, “Simple LMIs for stability of stochastic systems with delay term given by Stieltjes integral or with stabilizing delay,” *Syst. Control Lett.*, vol. 124, pp. 83–91, 2019.
[17] X. Meng, J. Lam, and H. Gao, “Network-based H_∞ control for stochastic systems,” *Int. J. Robust Nonlinear Control*, vol. 19, no. 3, pp. 295–312, 2009.
[18] Z. Hu, F. Deng, M. Xing, and J. Li, “Modeling and control of itô stochastic networked control systems with random packet dropouts subject to time-varying sampling,” *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4194–4201, Mar. 2017.
[19] P. Tabuada, “Event-triggered real-time scheduling of stabilizing control tasks,” *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, Oct. 2007.
[20] D. Yue, E. Tian, and Q. Han, “A delay system method for designing event-triggered controllers of networked control systems,” *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.
[21] S. Hu, D. Yue, X. Xie, and Z. Du, “Event-triggered H_∞ stabilization for networked stochastic systems with multiplicative noise and network-induced delays,” *Inf. Sci.*, vol. 299, pp. 178–197, 2015.
[22] A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1992–1997, Jul. 2015.
[23] S. Luo, F. Deng, and W.-H. Chen, “Dynamic event-triggered control for linear stochastic systems with sporadic measurements and communication delays,” *Automatica*, vol. 107, pp. 86–94, 2019.
[24] J. Zhang and E. Fridman, “Networked control of stochastic systems with scheduling protocols,” in *Proc. Eur. Control Conf.*, 2019, pp. 3002–3007.
[25] K. Gu, J. Chen, and V. L. Kharitonov, *Stability of Time Delay Systems*. Cambridge, MA, USA: Birkhäuser, 2003.
[26] P. Park, J. W. Ko, and C. Jeong, “Reciprocally convex approach to stability of systems with time-varying delays,” *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.
[27] D. Liu and G.-H. Yang, “Dynamic event-triggered control for linear time-invariant systems with L_2-gain performance,” *Int. J. Robust Nonlinear Control*, vol. 25, no. 2, pp. 507–518, 2019.