

- [15] P. Apkarian, H. D. Tuan, and J. Bernussou, "Continuous-time analysis, eigenstructure assignment, and H_2 synthesis with enhanced linear matrix inequalities LMI characterizations," *IEEE Trans. Autom. Control*, vol. 46, no. 12, pp. 1941–1946, Dec. 2001.
- [16] M. C. de Oliveira, J. Bernussou, and J. C. Geromel, "A new discrete-time robust stability condition," *Syst. Control Lett.*, vol. 37, pp. 261–265, 1999.
- [17] P. Park, "A delay-dependent stability criterion for systems with uncertain time-invariant delays," *IEEE Trans. Autom. Control*, vol. 44, no. 4, pp. 876–877, Apr. 1999.
- [18] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, no. 14, pp. 1447–1455, 2001.
- [19] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Rev.*, vol. 38, no. 1, pp. 49–95, 1996.
- [20] —, *SP Version 1.1 User's Guide—Software for Semidefinite Programming*. Stanford, CA: Stanford Univ., 1999.
- [21] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox User's Guide*. Natick, MA: MathWorks, 1995.
- [22] P. L. D. Peres and S. R. Souza, " H_∞ decentralized output feedback control for discrete-time uncertain systems," in *Proc. Amer. Control Conf.*, Seattle, WA, 1995, pp. 2926–2930.
- [23] G. D. Howitt and R. Luus, "Simultaneous stabilization of linear single-input systems by linear state feedback control," *Int. J. Control*, vol. 54, no. 4, pp. 1015–1030, 1991.
- [24] Y.-Y. Cao and Y.-X. Sun, "Static output feedback simultaneous stabilization: ILMI approach," *Int. J. Control*, vol. 70, no. 5, pp. 803–814, 1998.

H_∞ Control of Linear Uncertain Time-Delay Systems—A Projection Approach

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Abstract—The issues of stability and H_∞ control of linear systems with time-varying delays are considered. Based on the Lyapunov–Krasovskii approach and on Finsler's projection lemma, delay-dependent sufficient conditions are obtained, in terms of linear matrix inequalities (LMIs), for the stability of these systems. These conditions generalize previous results that were derived using either the descriptor approach or the first and the third model transformations. The obtained criteria are extended to deal with: stabilizability, the bounded real lemma and the H_∞ state-feedback control.

Index Terms—Lyapunov–Krasovskii approach, neutral systems, robust H_∞ control, time-delay systems.

I. INTRODUCTION

During the last decade, a considerable amount of attention has been paid to stability and control of linear systems with uncertain delays (either constant or time-varying) lying in the given segment $[0, h]$ (see, e.g., [1]–[6] and the references therein). The so-called delay-dependent sufficient stability conditions in terms of linear matrix inequalities (LMIs) have been derived by using Lyapunov–Krasovskii functionals or Lyapunov–Razumikhin functions (the latter is usually more conservative). Delay-dependent conditions via Lyapunov–Krasovskii functionals are based on different model transformations. Each model

Manuscript received October 30, 2003; revised April 2, 2004, September 16, 2004, June 23, 2005, and August 31, 2005. Recommended by Associate Editor S.-I. Niculescu. This work was supported by the C&M Maus Chair at Tel Aviv University, Israel

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Digital Object Identifier 10.1109/TAC.2006.872767

transformation leads to a corresponding form of Lyapunov–Krasovskii functional. The third model transformation (according to the classification of [2]), which was applied in [7] and [8], and the most recent and less conservative one, the descriptor representation of the system [4]–[6], lead to the same Lyapunov–Krasovskii functional depending on the derivatives of the state. The derivative of this functional is, however, different in the two approaches, where in the descriptor approach both the state vector and its derivative appear in the expression for the derivative of the Lyapunov–Krasovskii functional along the trajectories of the system.

For systems without delays the LMI stability conditions are obtained either by directly differentiating the quadratic Lyapunov function along the system trajectory [9] or by applying Finsler's lemma [10]. It turns out that the LMIs that are obtained by the latter two methods are equivalent in the case without uncertainty, but since the LMIs that are based on Finsler's lemma possess more degrees of freedom they provide better results in the case where parameter uncertainty is encountered [10].

It is shown in the present note that similar improvement is achieved when applying Finsler's lemma to the robust analysis and design of retarded and neutral systems. The sufficient conditions that are obtained for testing stability and for the bounded real lemma (BRL) are more general than the results achieved based on the descriptor approach or on the first and the third model transformations (see the classification of [2]). The latter results are obtained as a special case of the new conditions by taking few of the additional free matrix parameters to be zero. Moreover, for the first time, it is theoretically proved that the descriptor approach conditions are generalization of the conditions based on the first and the third model transformations and, thus, less conservative. Utilizing the geometric structure of the resulting inequalities, for these special cases, the results of [4]–[6] are obtained by solving LMIs with fewer decision variables, where there is no longer a need to find all the matrix blocks of the Lyapunov's kernel matrix explicitly.

A new effective method for state-feedback design is introduced. The merit of the new results lies not only in the fact that it provides another geometric approach to the analysis and the synthesis of retarded systems and that it reduces the complexity of the resulting LMIs. The main merit of the proposed method is the fact that it provides additional degrees of freedom which, similar to the case without delay, lead to less conservative results when uncertainty of the polytopic type is encountered. Some effort of applying Finsler's lemma to the case of systems with time delay has been recently made. A generalization of [8] was obtained in [11] where the elimination lemma was used to generalize the results of [8] that are based on the third model transformation. A delay-independent stability conditions via Finsler's lemma have been derived recently in [12].

II. STABILITY

Consider the following system (the system can be extended to include more delays):

$$\begin{aligned} \dot{x}(t) - F\dot{x}(t-g) &= A_0x(t) + A_1x(t-\tau(t)), \quad t \geq t_0 \\ x(\theta) &= \phi(\theta), \quad \theta \in \mathcal{E}_{t_0} \end{aligned} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the system state, A_0 , A_1 and F are constant $n \times n$ -matrices, t_0 is a given initial time, ϕ is a continuously differentiable initial function and $\mathcal{E}_{t_0} = \{\theta \in \mathcal{R} : \theta = \eta - \tau(\eta) \leq t_0, \eta \geq t_0\} \cup [t_0 - g, t_0]$. It is assumed that g is a known constant delay and that the delay $\tau(t)$ is a bounded differentiable function that satisfies

$$0 \leq \tau \leq h, \quad \dot{\tau}(t) \leq d < 1. \quad (2)$$

Moreover, it is assumed that all the eigenvalues of F are inside the unit circle. The latter guarantees that the difference equation $x(t) - Fx(t -$

$g) = 0$ is asymptotically stable for all g [13]. Similar to [3, Sec. 5.5.], results will be delay-independent in g and dependent in h and d . For $g = \tau$ (usually such models appear in the applications), one can apply the results with $d = 0$.

Consider also the Lyapunov–Krasovskii functional

$$V(t) = x^T(t)P_1x(t) + V_2 + V_3 + V_4 \quad (3)$$

where

$$V_2 = \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s)R\dot{x}(s)dsd\theta, \quad V_3 = \int_{t-g}^t \dot{x}^T(s)U\dot{x}(s)ds$$

$$V_4 = \int_{t-\tau}^t x^T(s)Sx(s)ds$$

and where $P_1, R, U,$ and S are positive-definite matrices. Differentiating (3), with respect to t , we require

$$\dot{V} = [x^T(t) \quad \dot{x}^T(t)]^T \begin{bmatrix} 0 & P_1 \\ P_1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 < 0 \quad (4)$$

for all $x(t)$ that satisfy (1). If (4) holds, then (1) is asymptotically stable (see [13, pp. 336–337]). We define: $\xi = \text{col}\{x(t), \dot{x}(t), x(t-\tau), \dot{x}(t-g)\}$ and rewrite (4) in the following form:

$$\dot{V} = \xi^T \begin{bmatrix} 0 & P_1 & 0 & 0 \\ P_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 < 0$$

$$\forall \xi \in \mathcal{R}^{4n} \text{ s.t. } [A_0, -I, A_1, F] \xi = 0. \quad (5)$$

To (5), the following version of Finsler's lemma is applied.

Lemma 1: [9] The following statement holds:

$x^T Q x + f(x) < 0, \forall \bar{B}x = 0, x \neq 0$, where Q is a symmetric matrix, $\bar{B} \in \mathcal{R}^{m \times n}$ and $f(x)$ is a scalar function, if there exists $X \in \mathcal{R}^{n \times m}$ such that: $x^T [Q + X\bar{B} + \bar{B}^T X^T] x + f(x) < 0, \forall x \neq 0$.

In the sequel, we also use the following bounding result [8].

Lemma 2: For any $a \in \mathcal{R}^n, b \in \mathcal{R}^{2n}, \mathcal{N} \in \mathcal{R}^{2n \times n}, R \in \mathcal{R}^{n \times n}, Y \in \mathcal{R}^{n \times 2n}, Z \in \mathcal{R}^{2n \times 2n}$, the following holds:

$$-2b^T \mathcal{N} a \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} R & Y - \mathcal{N}^T \\ Y^T - \mathcal{N} & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} R & Y \\ Y^T & Z \end{bmatrix} \geq 0. \quad (6)$$

Lemma 1 can be used with

$$x = \xi \quad Q = \begin{bmatrix} 0 & P_1 & 0 & 0 \\ P_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{B} = [A_0, -I, A_1, F] \quad X = [P_2, P_3, P_4, P_5]^T.$$

Carrying out the multiplications and substituting for $\dot{V}_2, \dot{V}_3, \dot{V}_4$ (7a), as shown at the bottom of the page, is obtained, where

$$\mu(t) \triangleq \xi^T \begin{bmatrix} P_2^T \\ P_3^T \\ 0 \\ 0 \end{bmatrix} [0 \quad 0 \quad A_1 \quad 0] \xi$$

$$+ \xi^T \begin{bmatrix} 0 \\ 0 \\ A_1^T \\ 0 \end{bmatrix} [P_2 \quad P_3 \quad 0 \quad 0] \xi - \int_{t-h}^t \dot{x}(s)^T R \dot{x}(s) ds. \quad (7b)$$

Since

$$\xi^T \begin{bmatrix} P_2^T \\ P_3^T \\ 0 \\ 0 \end{bmatrix} [0 \quad 0 \quad A_1 \quad 0] \xi$$

$$= \xi^T \begin{bmatrix} P_2^T \\ P_3^T \\ 0 \\ 0 \end{bmatrix} A_1 x(t) - \int_{t-\tau}^t \xi(t)^T \begin{bmatrix} P_2^T \\ P_3^T \\ 0 \\ 0 \end{bmatrix} A_1 \dot{x}(s) ds$$

we have that

$$\mu(t) = 2\xi^T \begin{bmatrix} P_2^T \\ P_3^T \\ 0 \\ 0 \end{bmatrix} A_1 x(t) - 2 \int_{t-\tau}^t \xi(t)^T \begin{bmatrix} P_2^T \\ P_3^T \\ 0 \\ 0 \end{bmatrix} A_1 \dot{x}(s) ds$$

$$- \int_{t-h}^t \dot{x}(s)^T R \dot{x}(s) ds. \quad (8)$$

We apply Lemma 2 to the expression we have obtained above for $\mu(t)$. This is done by taking in (6) $\mathcal{N} = [P_2 \quad P_3]^T A_1, a = \dot{x}(s)$ and $b = \text{col}\{x(t), \dot{x}(t)\}$. We obtain

$$\mu(t) \leq 2x^T(t)Y \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - 2x^T(t-\tau) \left[Y - A_1^T [P_2 \quad P_3] \right]$$

$$\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + h [x^T(t) \quad \dot{x}^T(t)] Z \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}. \quad (9)$$

Denoting $Y = [Y_1 \quad Y_2]$ and $Z = \begin{bmatrix} Z_1 & Z_2 \\ Z_2^T & Z_3 \end{bmatrix}$ the following is then obtained.

Theorem 1: System (1) is uniformly (with respect to t_0) asymptotically stable for a delay that satisfies (2) if there exist $n \times n$ matrices $0 < P_1, 0 < S, P_i, i = 2, \dots, 5, Y_1, Y_2, Z_1, Z_2, Z_3, 0 < U$ and $0 < R$ that satisfy the LMIs shown in (10a) at the bottom of the page, and

$$\begin{bmatrix} R & Y_1 & Y_2 \\ * & Z_1 & Z_2 \\ * & * & Z_3 \end{bmatrix} > 0. \quad (10b)$$

$$\xi^T \begin{bmatrix} P_2^T A_0 + A_0^T P_2 + S & P_1 - P_2^T + A_0^T P_3 & A_0^T P_4 & P_2^T F + A_0^T P_5 \\ * & -P_3^T - P_3 + U + hR & -P_4 & P_3^T F - P_5 \\ * & * & -S(1-d) + A_1^T P_4 + P_4^T A_1 & P_4^T F + A_1^T P_5 \\ * & * & * & P_5^T F + F^T P_5 - U \end{bmatrix} \xi + \mu < 0 \quad (7a)$$

$$\begin{bmatrix} P_2^T A_0 + A_0^T P_2 + Y_1 + Y_1^T + S + hZ_1 & P_1 - P_2^T + A_0^T P_3 + Y_2 + hZ_2 & -Y_1^T + P_2^T A_1 + A_0^T P_4 & P_2^T F + A_0^T P_5 \\ * & -P_3^T - P_3 + U + h(R + Z_3) & -Y_2^T + P_3^T A_1 - P_4 & P_3^T F - P_5 \\ * & * & -S(1-d) + A_1^T P_4 + P_4^T A_1 & P_4^T F + A_1^T P_5 \\ * & * & * & P_5^T F + F^T P_5 - U \end{bmatrix} < 0 \quad (10a)$$

Remark 1: The case where $\tau(t)$ is a continuous arbitrarily time-varying function, satisfying for all $t \geq 0$, $0 \leq \tau(t) \leq h$, is solved by choosing, in Theorem 1, $S \rightarrow 0$.

Remark 2: The two stability conditions in Theorem 1 can be joined into one LMI by applying Schur complements formula to (10b), replacing the term hZ_1 in (10a) by $h[Y_1 \ Y_2]^T R^{-1}[Y_1 \ Y_2]$. The latter can then be transformed by Schur formula to additional column and row in (10a) thus producing a single LMI.

Inequality (10a) can be written as

$$\Xi + \begin{bmatrix} A_0^T \\ -I \\ A_1^T \\ F^T \end{bmatrix} [P_2 \ P_3 \ P_4 \ P_5] + \begin{bmatrix} P_2^T \\ P_3^T \\ P_4^T \\ P_5^T \end{bmatrix} [A_0 \ -I \ A_1 \ F] < 0 \quad (11)$$

where

$$\Xi = \begin{bmatrix} Y_1 + Y_1^T + S + hZ_1 & P_1 + Y_2 + hZ_2 & -Y_1^T & 0 \\ * & U + h(R + Z_3) & -Y_2^T & 0 \\ * & * & -S(1-d) & 0 \\ * & * & * & -U \end{bmatrix}$$

Using Lemma 1 it is readily seen that there exist matrices P_2, P_3, P_4, P_5 that solve (10a) iff the following LMI has a solution:

$$\mathcal{N}_A^T \Xi \mathcal{N}_A < 0 \quad (12)$$

where we denote the full-rank matrix representations of the right annihilator of $[A_0 \ -I \ A_1 \ F]$ by \mathcal{N}_A . Since

$$\mathcal{N}_A = \begin{bmatrix} I & 0 & 0 \\ A_0 & A_1 & F \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

we obtain the following.

Theorem 2: The conditions of Theorem 1 are satisfied iff there exist $n \times n$ matrices $0 < P_1$, $0 < S$, $Y_1, Y_2, Z_1, Z_3, X, \bar{U}$ and R that satisfy the LMIs shown in (13a)–(13b) at the bottom of the page, where $\bar{U} \triangleq U + h(R + Z_3)$ and $X \triangleq P_1 + Y_2 + hZ_2$.

Remark 3: The LMIs of the latter theorem are independent of P_i , $i = 2, \dots, 5$. They thus involve a smaller number of decision variables.

A. Comparison With Other Methods

The above approach generalizes the main existing methods for delay-dependent stability such as: model transformation 1 [1], model transformation 3 [7], [8] and the descriptor approach [4]–[6]. In the sequel we show that the results of the latter three approaches are special cases of Theorems 1 and 2. We also discuss what are the extra degrees of freedom offered by the new approach.

The Descriptor Model Transformation: It is readily seen that by choosing $P_4 = P_5 = 0$, the LMIs of Theorem 1 are identical to those obtained in [4]. The question arises, however, to what an extent the introduction of the additional decision variables P_4 and P_5 in Theorem 1

lead to results that are less conservative than those obtained by the descriptor approach (where P_4 and P_5 are zero). Rewriting the stability condition of [4] in the following way:

$$\Xi + \begin{bmatrix} A_0^T \\ -I \\ A_1^T \\ F^T \end{bmatrix} [P_2 \ P_3 \ P_4 \ P_5] \text{diag}\{I, I, 0, 0\} + \text{diag}\{I, I, 0, 0\} \begin{bmatrix} P_2^T \\ P_3^T \\ P_4^T \\ P_5^T \end{bmatrix} [A_0 \ -I \ A_1 \ F] < 0 \quad (14)$$

where Ξ is as in (11), we use Lemma 1 and find that there exist matrices P_2, P_3, P_4, P_5 that solve (14) iff (10b) and the following LMIs in $Y_1, Y_2, Z_i, i = 1, \dots, 3, U$, and R have a solution

$$\mathcal{N}_A^T \Xi \mathcal{N}_A < 0 \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix}^T \Xi \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} < 0. \quad (15a)$$

Carrying out multiplication in (15b), we obtain

$$\begin{bmatrix} -(1-d)S & 0 \\ 0 & -U \end{bmatrix} < 0.$$

This inequality is redundant because it is a part of the LMI in (15a). Since (15a) is equivalent to the LMI in Theorem 1 no improvement can thus be obtained by the new method in the case where the parameters of the system are all known. It is shown, however, in Example 1 below that the new approach has a considerable advantage in the case where polytopic type parameter uncertainty is encountered.

Model Transformations 1 and 3: We choose [8] to be one of the recent publication that is based on the third model transformation. In order to allow comparison with the results of [8], we consider retarded system with constant unknown delay (namely, we take $F = 0$ and $d = 0$ in (1)). Choosing $Y_2 = 0, Z_2 = 0, X = P_1, U = hR$ and $Z_3 = 0$ in (13), we obtain the following LMI condition for stability that is identical to the one that appears in [8, Th. 1]:

$$\begin{bmatrix} A_0^T P_1 + P_1 A_0 + Y_1 + Y_1^T + S & -Y_1 + P_1 A_1 & h A_0^T R & h Y_1^T \\ * & -S & h A_1^T R & 0 \\ * & * & -h R & 0 \\ * & * & * & -h R \end{bmatrix} < 0. \quad (16)$$

It is shown in [18] that the results of the first model transformation, as expressed in [1], are a special case of the third model transformation results of [7]. Since [8] is a generalization of [7], we conclude that Theorem 1 and Theorem 2 of the present note are generalizations of the results obtained by the first and the third model transformations.

III. THE BRL

We consider the following system:

$$\begin{aligned} \dot{x} - F \dot{x}(t-g) &= A_0 x(t) + A_1 x(t-\tau(t)) + B u(t) + B_1 w(t) \\ x(t) &= 0, \quad t \leq 0 \\ z(t) &= C x(t) \end{aligned} \quad (17ab)$$

$$\begin{bmatrix} Y_1 + Y_1^T + S + hZ_1 + X A_0 + A_0^T X^T & -Y_1^T + X A_1 - A_0^T Y_2^T & X F & A_0^T \bar{U} \\ * & -S(1-d) - Y_2 A_1 - A_1^T Y_2^T & -Y_2 F & A_1^T \bar{U} \\ * & * & -\bar{U} + h(R + Z_3) & F^T \bar{U} \\ * & * & * & -\bar{U} \end{bmatrix} < 0 \quad (13a)$$

$$\begin{bmatrix} R & Y_1 & h Y_2 \\ * & Z_1 & X - P_1 - Y_2 \\ * & * & h^2 Z_3 \end{bmatrix} > 0 \quad (13b)$$

values of $h = 0.408$ and $h = 0.263$ are obtained for $d = 0.1$ and $d = 0.5$, respectively.

B. Example 2

Given two systems that are described by (17). The matrices of the first system are

$$A_0 = \begin{bmatrix} -1.3 & 0.2 \\ 0.2 & -1 \end{bmatrix} \quad A_1 = \begin{bmatrix} -0.6 & -0.5 \\ -0.5 & -0.6 \end{bmatrix} \quad F = 0 \\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad D_{12} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}.$$

The matrices of the second system are identical to the first, except for A_1 that is given by

$$A_1 = \begin{bmatrix} -2.3 & 0 \\ 0 & -0.8 \end{bmatrix}.$$

It is required to find a state-feedback controller $u = Kx(t)$ that minimizes an upper bound on the disturbance attenuation γ over all convex combinations of the two systems. Applying the descriptor method of [5] to the above system a minimum value of $\gamma = 33.861$ is obtained for $\epsilon = -0.9923$. Using Theorem 4 with $\epsilon_2 = \epsilon_4 = 0$ (i.e., descriptor method with a design procedure of Theorem 4, where $P_3 = \epsilon_1 P_2$), a minimum bound of $\gamma = 8.2641$ is obtained. On the other hand, applying Theorem 4 with no restrictions on ϵ_2 and ϵ_4 , a better bound of $\gamma = 6.87$ is obtained for $\epsilon_1 = 0.4153$, $\epsilon_2 = 0.6068$ and $\epsilon_4 = 0.0021$. The corresponding state-feedback gain matrix is $K = [-754.41 \quad -236.15]$.

Applying Theorem 5, for $\epsilon_1 = 0.3442$, $\epsilon_2 = -0.1047$ and $\epsilon_4 = 0.1788$ a minimum value of $\gamma = 4.5157$ is obtained with a corresponding state-feedback gain matrix $K = [-49.8490 \quad -15.6345]$. A clear advantage of the method that is based on the adjoint system is evident.

VI. CONCLUSION

A generalization of the results obtained by using either the descriptor approach or the conditions found by the first or the third model transformation to the analysis and synthesis of time-delay systems is presented. Extra degrees of freedom are introduced which allow solutions to problems with polytopic uncertainty that are less conservative than those obtained in the past. The new approach also simplifies the inequalities that have to be solved by the descriptor approach in cases with no uncertainty. It leads to new effective design procedures.

The LMIs that have to be solved by the new approach are of larger size and their solution may require longer computation time. The benefit of these LMIs will thus be in the robust control case where the improvement they introduce compensate for the larger computation complexity.

REFERENCES

- [1] X. Li and C. de Souza, "Criteria for robust stability and stabilization of uncertain linear systems with state delay," *Automatica*, vol. 33, pp. 1657–1662, 1997.
- [2] V. Kolmanovskii and J.-P. Richard, "Stability of some linear systems with delays," *IEEE Trans. Autom. Control*, vol. 44, no. 5, pp. 984–989, May 1999.
- [3] S.-I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*. London, U.K.: Springer-Verlag, 2001, vol. 269, Lecture Notes in Control and Information Sciences.
- [4] E. Fridman, "New Lyapunov–Krasovskii functionals for stability of linear retarded and neutral type systems," *Syste. Control Lett.*, vol. 43, pp. 309–319, 2001.
- [5] E. Fridman and U. Shaked, "A descriptor system approach to H_∞ control of time-delay systems," *IEEE Trans. Autom. Control*, vol. 47, no. 2, pp. 253–270, Feb. 2002.
- [6] —, "An improved stabilization method for linear systems with time-delay," *IEEE Trans. Autom. Control*, vol. 47, no. 11, pp. 1931–1937, Nov. 2002.
- [7] P. Park, "A delay-dependent stability criterion for systems with uncertain time-invariant delays," *IEEE Trans. Autom. Control*, vol. 44, no. 4, pp. 876–877, Apr. 1999.
- [8] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, pp. 1447–1455, 2001.
- [9] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequality in Systems and Control Theory*. Philadelphia, PA: SIAM, 1994.
- [10] M. C. de Oliveira and R. E. Skelton, "Stability test for constrained linear systems," in *Perspectives in Robust Control*, S. O. Reza Moheimani, Ed. London, U.K.: Springer-Verlag, 2001, vol. 268, Lecture Notes in Control and Information Sciences.
- [11] D. Mehdi and E. K. Boukas, "Stability and stabilizability of dynamical systems with multiple time-varying delays: Delay dependent criteria," *Les Cahiers vdu GERAD*, G-2002-27, 2002.
- [12] E. Castelan, I. Queinnec, and S. Tarbouriech, "Delay-independent robust stability conditions of neutral linear time-delay systems," presented at the IFAC Workshop on Time-delay Systems, Rocquencourt, France, Sep. 2003.
- [13] V. Kolmanovskii and A. Myshkis, *Applied Theory of Functional Differential Equations*. Norwell, MA: Kluwer, 1999.
- [14] A. Bensoussan, G. Do Prato, M. C. Delfour, and S. K. Mitter, *Representation and Control of Infinite Dimensional Systems*. Boston, MA: Birkhäuser, 1992, vol. 1.
- [15] J. Hale, *Functional Differential Equations*. New York: Springer-Verlag, 1977.
- [16] M. Green and D. J. N. Limebeer, *Linear Robust Control*. Upper Saddle River, NJ: Prentice-Hall, 1995.
- [17] E. Fridman and U. Shaked, "Parameter dependent stability and stabilization of uncertain time-delay systems," *IEEE Trans. Autom. Control*, vol. 48, no. 5, pp. 861–866, May 2003.
- [18] J. Zhang, C. R. Knospe, and P. Tsiotras, "Stability of time-delay systems: Equivalence between Lyapunov and scaled small-gain conditions," *IEEE Trans. Autom. Control*, vol. 46, no. 3, pp. 482–486, Mar. 2001.