Appendix D

The FPCD and multigrid algorithm validation for two-dimensional geometry.

Both FPCD and multigrid algorithms were validated using two-dimensional free convection flow inside differently heated rectangular cavity characterizing by different aspect ratios. Tables D1 and D2 summarizes results obtained by the two algorithms for the three Rayleigh numbers of 10^5,10^6,10^7 and its comparison with the Le Quere and Alziary de Roquefort [5] solution for the flow inside square cavity:

	Ra=10^5		Ra=10^6		Ra=10^7	
	Indep. [5]	Present FPCD	Indep. [5]	Present FPCD	Indep. [5]	Present FPCD
$\Psi(0.5,0.5)$	0.03422	0.034216	0.019451	0.019451	0.01098	0.01102
Ψ_{max} (x,y)	0.036099 (0.285,0.601)	0.036072 (0.285,0.601)	0.019951 (0.151,0.547)	0.19939 (0.151,0.547)	0.01132 (0.086,0.556)	0.01131 (0.086,0.556)
U_{max} $(x=0.5)$ y	0.1304144 0.855	0.128836 0.855	0.076939 0.850	0.07669 0.850	0.05584 0.879	0.05515 0.884
$V_{max}(y=0.5)$ x	0.257601 0.066	0.257508 0.066	0.261757 0.0375	0.261165 0.0375	0.26244 0.0213	0.25900 0.024
Nu (x=0)	4.522	4.5236	8.825	8.8379	16.52	16.6188
Nu (x=1)	----	4.5236	----	8.8379	---	16.6188

Table D1. Comparison between independent results [5] and FPCD solution obtained on 100^2 stretched grid, Pr=0.71.

	Ra =10^5		Ra =10^6		Ra =10^7	
	Indep. [5]	Present Multigrid	Indep. [5]	Present Multigrid	Indep. [5]	Present Multigrid
$\Psi(0.5,0.5)$	0.03422	0.034211	0.019451	0.019446	0.01098	0.011014
Ψ_{max} (x,y)	0.036099 (0.285,0.601)	0.036075 (0.285,0.601)	0.019951 (0.151,0.547)	0.019937 (0.151,0.547)	0.01132 (0.086,0.556)	0.01131 (0.086,0.556)
U_{max} $(x=0.5)$ y	0.1304144 0.855	0.130353 0.855	0.076939 0.850	0.076901 0.850	0.05584 0.879	0.05549 0.879
$V_{max}(y=0.5)$ x	0.257601 0.066	0.257459 0.066	0.261757 0.0375	0.261628 0.0375	0.26244 0.0213	0.26186 0.0213
Nu (x=0)	4.522	4.5232	8.825	8.8354	16.523	16.599
Nu (x=0)	----	4.5232	----	8.8354	---	16.599

Table D2. Comparison between independent results [5] and multigrid solution obtained on 112^2 stretched grid, Pr=0.71

It is seen that results obtained by either FPCD or multigrid approaches are in a good agreement with the independent results. The equal Nu numbers obtained on both isothermal vertical boundaries validate the thermal energy conservation throughout the cavity. Deviations between the obtained and the corresponding independent results are increasing with the increase of the Rayleigh number. Therefore, and additional validation of the obtained results for the Ra=10^7 is necessary and presented in table D3:

	Independent result [32]	Independent result [6]	Independent result [8]	Present FPCD	Present Multigrid
$\Psi(0.5, 0.5)$	29.3614	29.3616		29.3637	29.3477
Ψ_{max} (x,y)	30.1638 (0.086,0.05556)	30.1641 (0.086,0.0556)		30.1364 (0.086,0.556)	30.1364 (0.086,0.556)
U_{max} $(x = 0.5)$ y	148.583 0.87936	148.583 0.879	145.2666 0.8845	146.9517 0.884	147.8577 0.879
$V_{max}(y = 0.5)$ x	699.329 0.0213	699.237 0.021	703.2526 0.0215	690.1269 0.024	697.748 0.0213
Nu $(x=0)$	16.5235	16.523	16.3869	16.6188	16.5994

Table D3. Comparison between independent results [32,6,8] and solution obtained by FPCD and multigrid algorithms Ra=10^7, Pr=0.71.

It can be seen that the general agreement at Ra=10^7 is rather good (given the fact that some solutions were obtained more than 10 years ago) with maximal discrepancy not exceeding 2%.

At the next validation step, the FPCD and the multigrid algorithm are tested by comparison with a non-steady periodical solutions obtained for the differentially heated cavity of aspect ratio 8:1 and Ra=3.4×10^5, obtained by Xin and Le Quere [21]:

	Grid resolution 48×180			Uniform Grid, resolution 100×800		
	Steps per period 2024.37[21]			Steps per period 849.51455 (Δt=0.004)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.6345×10^{-2}	5.4768×10^{-2}	3.4115	6.06445×10^{-2}	8.02494×10^{-2}	3.3981
v_1	0.46188	7.7125×10^{-2}	3.4115	0.46150	10.44785×10^{-2}	3.3981
θ_1	0.26548	4.2690×10^{-2}	3.4115	0.26567	6.1737×10^{-2}	3.3981
ΔP_1	-1.8536×10^{-3}	2.0355×10^{-2}	3.4115	-1.7021×10^{-3}	2.8001×10^{-2}	3.3981
ΔP_{51}	-0.53486	2.2442×10^{-2}	3.4115	-0.53620	2.97847×10^{-2}	3.3981
ΔP_{35}	0.53671	1.0057×10^{-2}	3.4115	0.53791	1.4105×10^{-2}	3.3981
Nu_0	-4.57946	7.0921×10^{-3}	3.4115	-4.59299	10.2769×10^{-3}	3.3981
Nu_w	-4.57946	7.0921×10^{-3}	3.4115	-4.59299	10.2769×10^{-3}	3.3981

	Uniform Grid, resolution 100×800			Uniform Grid, resolution 100×800		
	Steps per period 1707.3171 (Δt=0.002)			Steps per period 3414.6342 (Δt=0.001)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.7490×10^{-2}	6.2236×10^{-2}	3.4146	5.6095×10^{-2}	5.2232×10^{-2}	3.4146
v_1	0.46165	8.5893×10^{-2}	3.4146	0.46177	7.3937×10^{-2}	3.4146
θ_1	0.26565	4.8475×10^{-2}	3.4146	0.26561	4.0901×10^{-2}	3.4146
ΔP_{14}	-1.7841×10^{-3}	2.2811×10^{-2}	3.4146	-1.85305×10^{-3}	1.9522×10^{-2}	3.4146
ΔP_{51}	-0.53575	2.4893×10^{-2}	3.4146	-0.53546	2.1564×10^{-2}	3.4146
ΔP_{35}	0.53753	1.1393×10^{-2}	3.4146	0.53732	0.9764×10^{-2}	3.4146
Nu_0	-4.579232	6.7021×10^{-3}	3.4146	-4.59220	6.9334×10^{-3}	3.4146
Nu_w	-4.579232	6.7021×10^{-3}	3.4146	-4.59220	6.9334×10^{-3}	3.4146

Table D4. Local and global results comparison between the FPCD algorithm and the independent solution [21] at Ra=3.4×10^5, Pr=0.71, uniform grid and time duration 700. Nu_0 and Nu_w represent the hot and the cold wall Nusselt numbers respectively.

	Grid resolution 48×180 Steps per period 2024.37 [21]			Stretched Grid, resolution 100×800 Steps per period 849.51455 (Δt=0.004)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.6345 × 10^{-2}	5.4768 × 10^{-2}	3.4115	5.9705 × 10^{-2}	7.7847 × 10^{-2}	3.3981
v_1	0.46188	7.7125 × 10^{-2}	3.4115	0.46097	10.2494 × 10^{-2}	3.3981
θ_1	0.26548	4.2690 × 10^{-2}	3.4115	0.26555	6.0080 × 10^{-2}	3.3981
ΔP_1	-1.8536 × 10^{-3}	2.0355 × 10^{-2}	3.4115	-1.7401 × 10^{-3}	2.7336 × 10^{-2}	3.3981
ΔP_{51}	-0.53486	2.2442 × 10^{-2}	3.4115	-0.53601	2.9344 × 10^{-2}	3.3981
ΔP_{35}	0.53671	1.0057 × 10^{-2}	3.4115	0.53775	1.3790 × 10^{-2}	3.3981
Nu_0	-4.57946	7.0921 × 10^{-3}	3.4115	-4.57999	10.0974 × 10^{-3}	3.3981
Nu_w	-4.57946	7.0921 × 10^{-3}	3.4115	-4.59999	10.0974 × 10^{-3}	3.3981

	Stretched Grid, resolution 100×800 Steps per period 1707.3171 (Δt=0.002)			Stretched Grid, resolution 100×800 Steps per period 3414.6342 (Δt=0.001)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.7535 × 10^{-2}	7.4267 × 10^{-2}	3.4146	5.7985 × 10^{-2}	5.7985 × 10^{-2}	3.4146
v_1	0.46220	8.4425× 10^{-2}	3.4146	0.46119	8.1097 × 10^{-2}	3.4146
θ_1	0.26524	4.9038× 10^{-2}	3.4146	0.26547	4.5300 × 10^{-2}	3.4146
ΔP_{14}	-1.8602× 10^{-3}	2.4578× 10^{-2}	3.4146	-1.8967 × 10^{-3}	2.1381 × 10^{-2}	3.4146
ΔP_{51}	-0.53528	2.5371× 10^{-2}	3.4146	-0.53527	2.3609 × 10^{-2}	3.4146
ΔP_{35}	0.53735	1.1157× 10^{-2}	3.4146	0.53769	1.0762 × 10^{-2}	3.4146
Nu_0	-4.57957	8.0646× 10^{-3}	3.4146	-4.57923	6.7021 × 10^{-3}	3.4146
Nu_w	-4.57957	8.0646× 10^{-3}	3.4146	-4.57923	6.7021 × 10^{-3}	3.4146

Table D4. Local and global results comparison between the FPCD algorithm and the independent solution [21] at Ra=3.4×10^5, Pr=0.71, stretched grid and time duration 700. Nu_0 and Nu_w represent the hot and the cold wall Nusselt numbers respectively.

	Grid resolution 48×180 Steps per period 2024.37 [21]			Uniform Grid, resolution 100×800 Steps per period 849.51455 (Δt=0.004)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.6345 × 10^{-2}	5.4768 × 10^{-2}	3.4115	6.09918 × 10^{-2}	8.2313 ×10^{-2}	3.3816
v_1	0.46188	7.7125 × 10^{-2}	3.4115	0.46148	10.6226 ×10^{-2}	3.3816
θ_1	0.26548	4.2690 × 10^{-2}	3.4115	0.26568	6.3087 × 10^{-2}	3.3816
ΔP_1	-1.8536 × 10^{-3}	2.0355 × 10^{-2}	3.4115	-1.6777 × 10^{-3}	2.8480 × 10^{-2}	3.3816
ΔP_{51}	-0.53486	2.2442 × 10^{-2}	3.4115	-0.53614	3.0176 × 10^{-2}	3.3816
ΔP_{35}	0.53671	1.0057 × 10^{-2}	3.4115	0.53780	1.4288 × 10^{-2}	3.3816
Nu_0	-4.57946	7.0921 × 10^{-3}	3.4115	-4.59053	9.4669 × 10^{-3}	3.3816
Nu_w	-4.57946	7.0921 × 10^{-3}	3.4115	-4.59051	10.4001 × 10^{-3}	3.3816

Uniform Grid, resolution 100×800 Steps per period 1707.3171 (Δt=0.002)				Uniform Grid, resolution 100×800 Steps per period 3414.6342 (Δt=0.001)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.7846×10^{-2}	6.4487×10^{-2}	3.4146	5.64156×10^{-2}	5.4671×10^{-2}	3.4146
v_1	0.46168	8.8484×10^{-2}	3.4146	0.46179	7.6954×10^{-2}	3.4146
θ_1	0.26558	5.0139×10^{-2}	3.4146	0.26558	4.2729×10^{-2}	3.4146
ΔP_{14}	-1.8054×10^{-3}	2.3524×10^{-2}	3.4146	-1.84985×10^{-3}	2.0334×10^{-2}	3.4146
ΔP_{51}	-0.53565	2.5613×10^{-2}	3.4146	-0.53539	2.2402×10^{-2}	3.4146
ΔP_{35}	0.53745	1.1698×10^{-2}	3.4146	0.53725	1.0121×10^{-2}	3.4146
Nu_0	-4.58996	8.0165×10^{-3}	3.4146	-4.58969	7.0867×10^{-3}	3.4146
Nu_w	-4.58995	8.3944×10^{-3}	3.4146	-4.58969	7.2008×10^{-3}	3.4146

Table D4. Local and global results comparison between the multigrid algorithm and the independent solution [21] at Ra=3.4×10^5, Pr=0.71, uniform grid and time duration 700. Nu_0 and Nu_w represent the hot and the cold wall Nusselt numbers respectively.

Grid resolution 48×180 Steps per period 2024.37[21]				Stretched Grid, resolution 100×800 Steps per period 849.51455 (Δt=0.004)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.6345×10^{-2}	5.4768×10^{-2}	3.4115	6.03474×10^{-2}	8.0805×10^{-2}	3.3981
v_1	0.46188	7.7125×10^{-2}	3.4115	0.46113	10.5111×10^{-2}	3.3981
θ_1	0.26548	4.2690×10^{-2}	3.4115	0.26553	6.2098×10^{-2}	3.3981
ΔP_1	-1.8536×10^{-3}	2.0355×10^{-2}	3.4115	-1.7476×10^{-3}	2.8085×10^{-2}	3.3981
ΔP_{51}	-0.53486	2.2442×10^{-2}	3.4115	-0.53595	3.0125×10^{-2}	3.3981
ΔP_{35}	0.53671	1.0057×10^{-2}	3.4115	0.53768	1.4096×10^{-2}	3.3981
Nu_0	-4.57946	7.0921×10^{-3}	3.4115	-4.58009	10.0341×10^{-3}	3.3981
Nu_w	-4.57946	7.0921×10^{-3}	3.4115	-4.58009	10.0313×10^{-3}	3.3981

Stretched Grid, resolution 100×800 Steps per period 1707.3171 (Δt=0.002)				Stretched Grid, resolution 100×800 Steps per period 3414.6342 (Δt=0.001)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period
u_1	5.7326×10^{-2}	6.3663×10^{-2}	3.4146	5.5869×10^{-2}	5.3652×10^{-2}	3.4146
v_1	0.46123	8.7693×10^{-2}	3.4146	0.46128	7.5847×10^{-2}	3.4146
θ_1	0.26552	4.9546×10^{-2}	3.4146	0.26547	4.1990×10^{-2}	3.4146
ΔP_{14}	-1.8137×10^{-3}	2.3258×10^{-2}	3.4146	-1.88024×10^{-3}	1.9981×10^{-2}	3.4146
ΔP_{51}	-0.53553	2.5521×10^{-2}	3.4146	-0.53526	2.2116×10^{-2}	3.4146
ΔP_{35}	0.53735	1.1578×10^{-2}	3.4146	0.53714	0.9984×10^{-2}	3.4146
Nu_0	-4.57949	8.1449×10^{-3}	3.4146	-4.57925	6.9967×10^{-3}	3.4146
Nu_w	-4.57950	8.4378×10^{-3}	3.4146	-4.57925	7.1579×10^{-3}	3.4146

Table D4. Local and global results comparison between the multigrid algorithm and the independent solution [21] at Ra=3.4×10^5, Pr=0.71, stretched grid and time duration 700. ***Nu_0* and *Nu_w* represent the hot and the cold wall Nusselt numbers respectively**.

Solutions obtained by the both algorithms are in a good agreement with the benchmark results. The results accuracy tends to increase with the decreasing of the time step. The grid stretching slightly improves the accuracy obtained for the large time step but its effect diminishes with the time step decreasing. The hot and cold wall Nusselt number amplitudes obtained by the multigrid solution slightly differ from each other that should not occur. The possible reason for such behavior is inaccuracy introduced by the insufficient numerical convergence of the temperature field to the asymptotic oscillatory solution, making the Nu number the most sensitive quantity. The average values of both Nusselt number however are equal to each other, so that thermal energy conservation throughout the cavity holds.