Pressure-Velocity Coupled Three-Dimensional CFD on a Massively Parallel Computer

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Outline

- **Pressure-velocity coupled formulation of Navier-Stokes equations**
- **Benchmark problem : 3-D lid driven cavity, differential heated cavity**
- **Multigrid with an Analytical Solution Accelerated (ASA) smoother**
- **3D Domain partition and scalability properties**
- **Application to linear stability analysis**
- **Conclusions**

Incompressible N-S Equations – Numerical Challenge

Continuity - $$\nabla \cdot \boldsymbol{u} = 0$$

Momentum- $$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{u}$$

- No separate equation for pressure
- No boundary conditions for pressure

Incompressible N-S Equations – Numerical Challenge (Cont.)

approaches with pressure-velocity decoupling

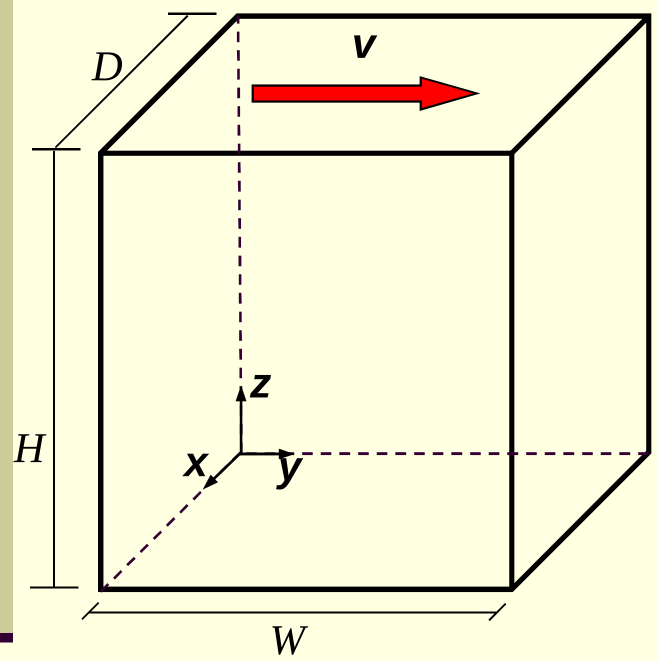
- ✓ High numerical robustness
- ✓ Low memory consumption
- ✘ Slow rate of numerical convergence
- ✘ Non-physical pressure field
- ✘ Not applicable for liquid – solid interface problems

pressure–velocity coupled approaches

- ✓ High rate of numerical convergence
- ✓ The "most natural " way to solve N-S equations
- ✓ Calculated pressure is physical
- ✘ High memory consumption
- ✘ Not as numerically robust as pressure projection methods

Benchmark Problems

Lid-Driven Cubic Cavity



$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{u}$$

✓**Explicit non-linear terms treatment**

$$\left(\boldsymbol{u}^n \cdot \nabla\right)\boldsymbol{u}^n$$

▪**Semi-Implicit non-linear terms treatment**

$$\left(\boldsymbol{u}^n \cdot \nabla\right)\boldsymbol{u}^{n+1}$$

Realistic Boundary Conditions:

$\boldsymbol{u} = 0$ **- at all static walls no slip/no penetration**

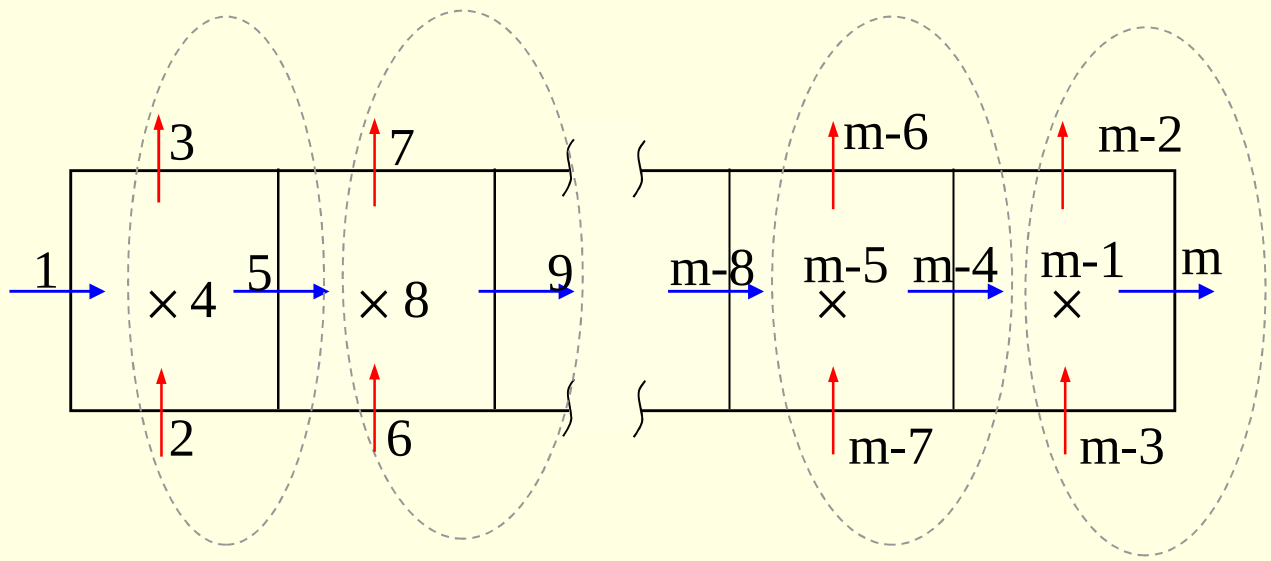
$\boldsymbol{u}\big|_{z=H/W} = v$ **-at the moving wall the flow velocity is equal to that of the moving wall itself**

No boundary condition for pressure is needed

Accelerated Coupled Line Gauss-Seidel Smoother (ASA-CLGS) -2D

Zeng and Wesseling (1993) – CLGS: Horizontal (vertical) sweeping with horizontally (vertically) adjacent pressure linkage

Feldman and Gelfgat (2008) – ASA-CLGS:Horizontal (vertical) sweeping without horizontally (vertically) adjacent pressure linkage



CLGS and ASA-CLGS Efficiency Estimation for 2D

Zeng and Wesseling (1993) – CLGS:

↓

Block 3-diagonal matrix or 7-diagonal matrix

↓

LU decomposition

↓

$\approx O(15M)$

Feldman and Gelfgat (2008) – ASA-CLGS (6-Diagonal Matrix)

↓

$$p'_{k-1} = (c_1^L v'_{k-4} + R_{k-1}^L + \sum_{i=2}^{4} c_i^L R_{k-i}^L) / c_5^L$$

↓

$$\begin{bmatrix} v'_5 \\ u'_2 \\ u'_3 \end{bmatrix} = \begin{bmatrix} c_6^1 \\ c_7^1 \\ c_8^1 \end{bmatrix} \times p'_4 + \begin{bmatrix} c_9^1 R_5^L \\ c_{10}^1 R_2^L \\ c_{11}^1 R_3^L \end{bmatrix}$$

↓

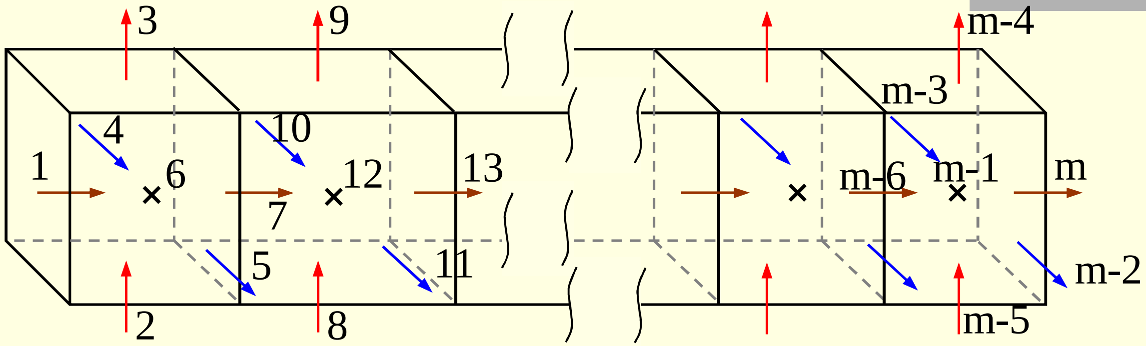
$\approx O(5M)$ ↔ $\approx O(5M)$

Thomas Algorithm (3-Diagonal Matrix)

↓

$\approx O(5M)$

ASA-CLGS -Efficiency Estimation for 3D



$$p'_p = \left(c_1^I w'_d + R_p^I + c_2^I R_e^I + c_3^I R_w^I + c_4^I R_n^I + c_5^I R_5^I + c_6^I R_d^I\right) / c_7^I$$

$$\begin{bmatrix} w'_d \\ v'_s \\ v'_n \\ u'_w \\ u'_e \end{bmatrix} = \begin{bmatrix} c_8^I \\ c_9^I \\ c_{10}^I \\ c_{11}^I \\ c_{12}^I \end{bmatrix} \times p'_p + \begin{bmatrix} c_{13}^I R_d^I \\ c_{14}^I R_s^I \\ c_{15}^I R_n^I \\ c_{16}^I R_w^I \\ c_{17}^I R_e^I \end{bmatrix}$$

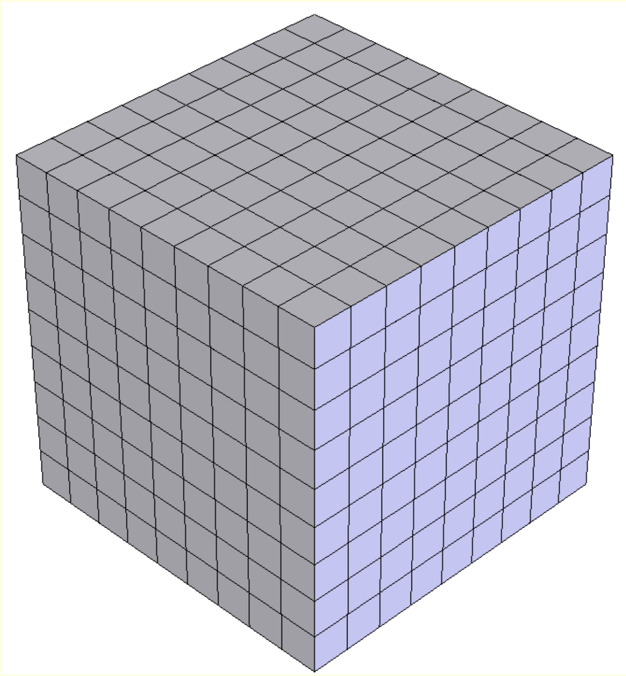
6 corrections for a single volume result in 17 multiplications and divisions and 11 summations

$$\approx O(5M)$$

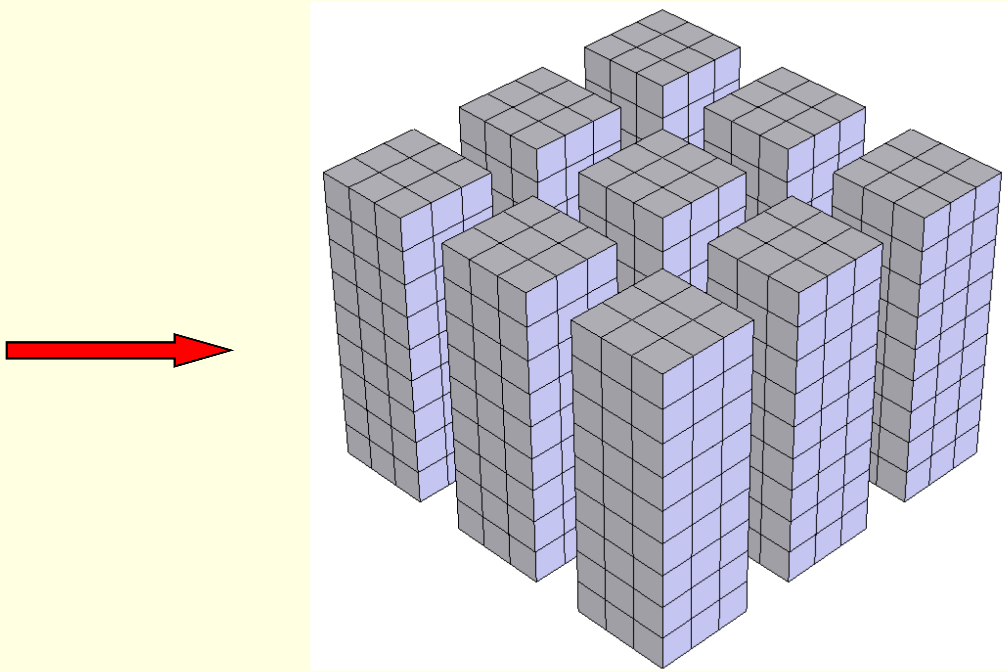
Domain Partition for 3D Configuration

Existence of **analytical solution** for the whole column allows for 2D virtual topology of 3D configuration.

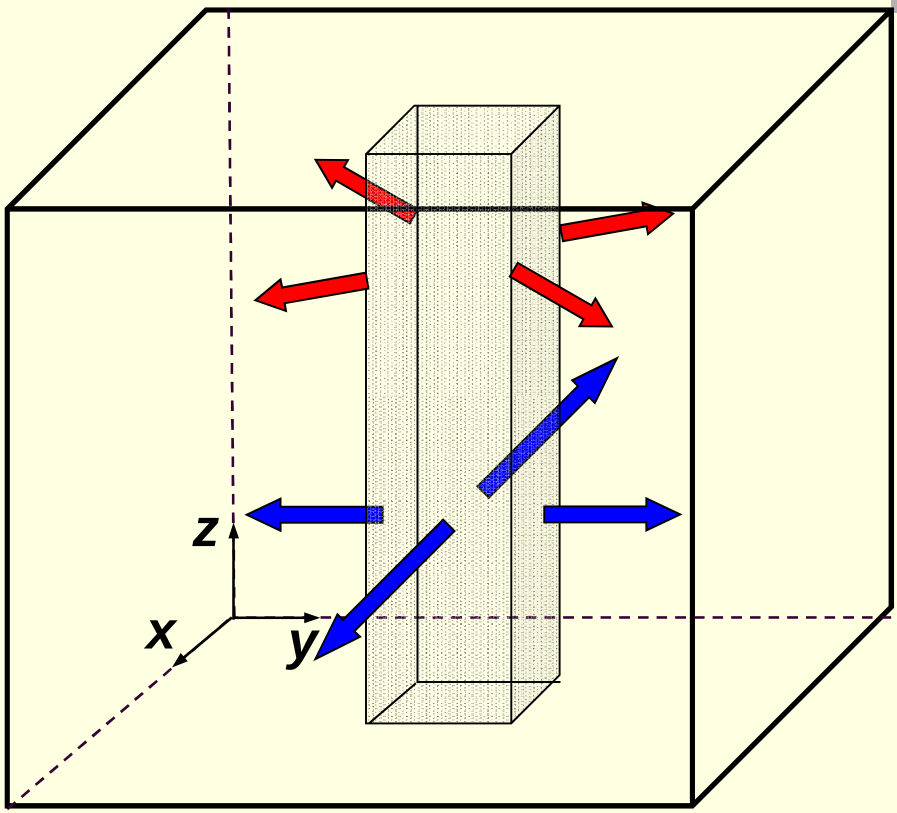
The whole domain



The partitioned domain



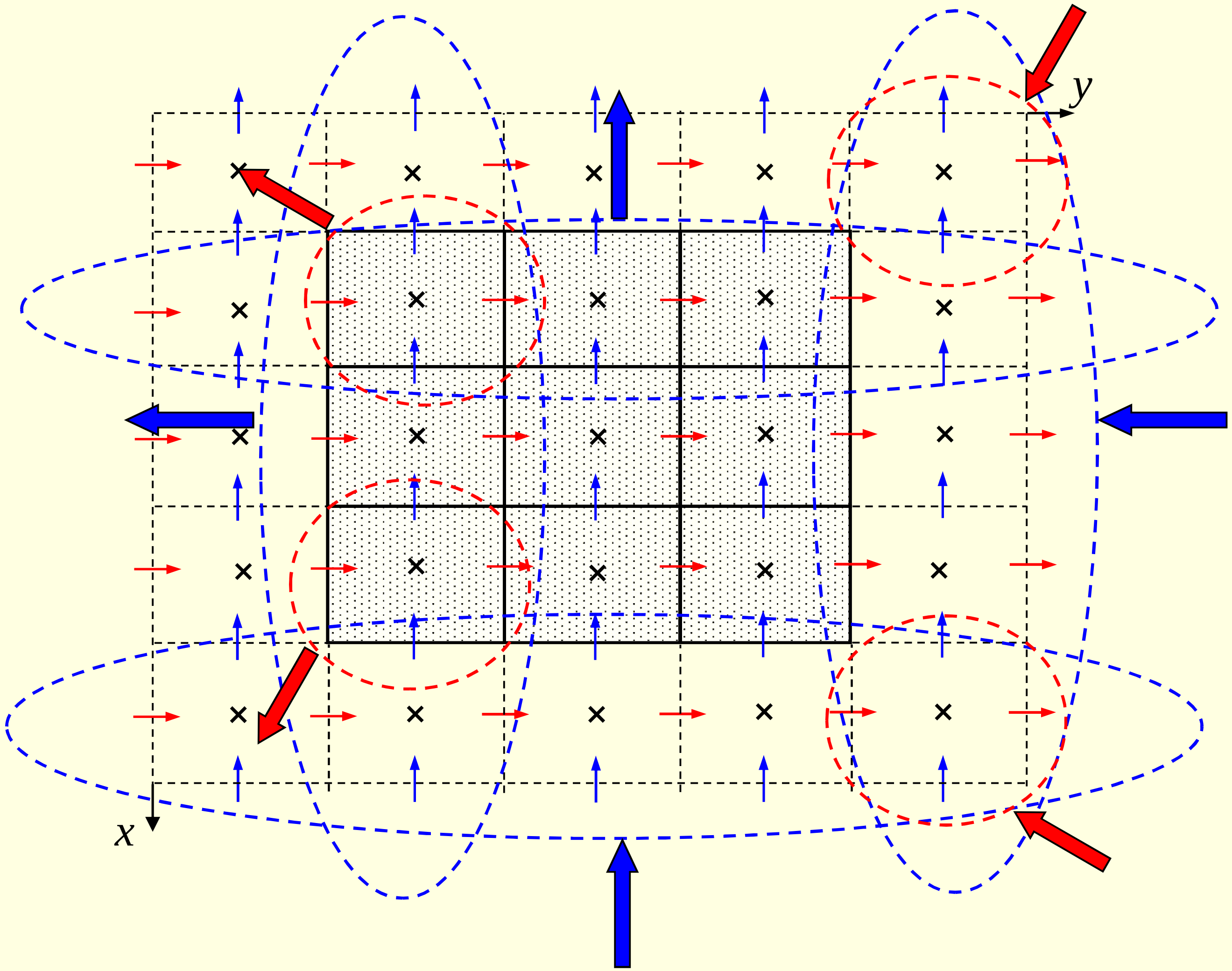
Domain Partition for 3D Configuration (Cont.)



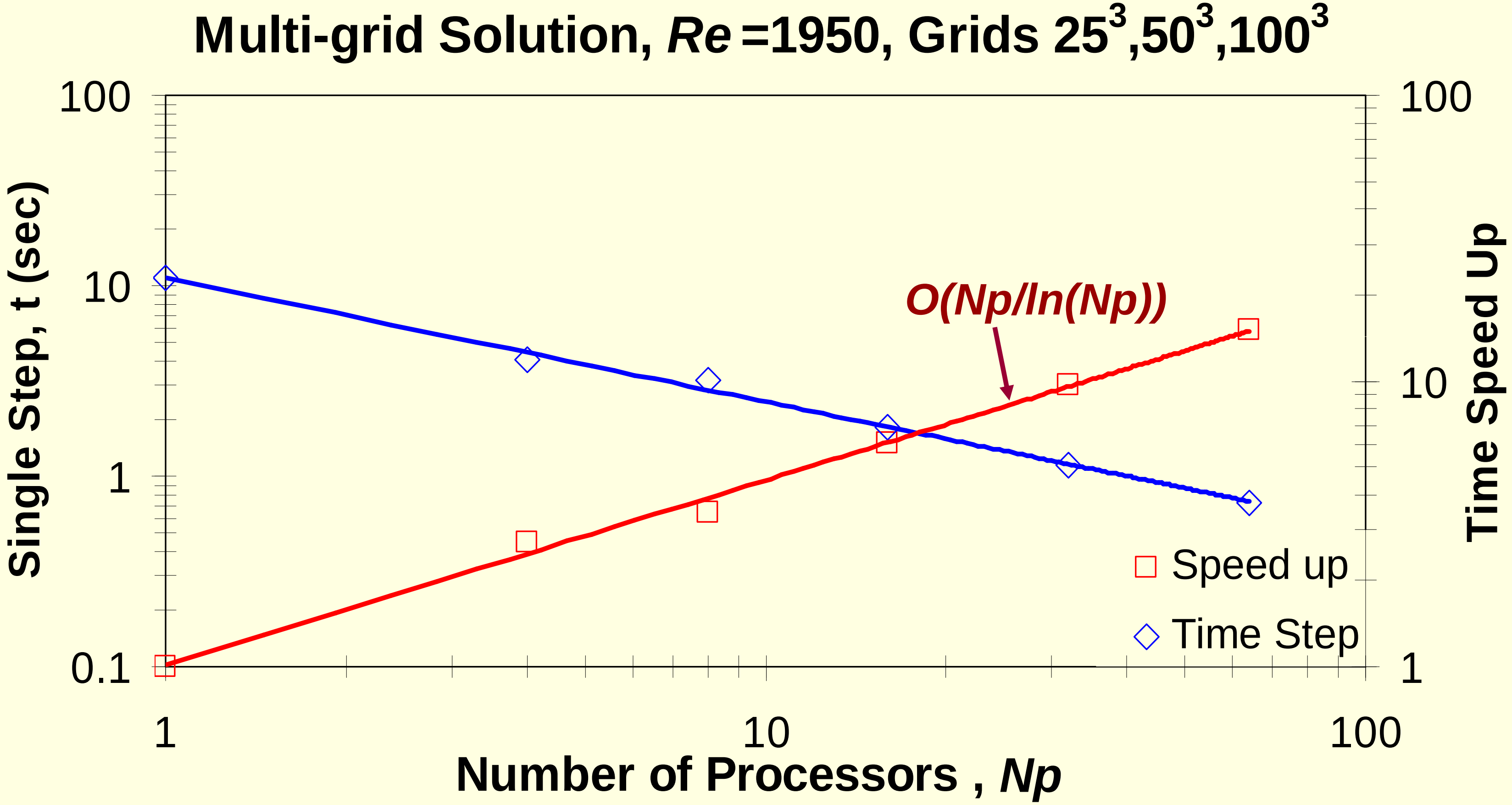
 All volumes located at the sub -volume faces exchange data with neighbors

 All volumes located at the sub -volume vertical edges exchange data with diagonal neighbors

3D Configuration- Data Exchange Principle

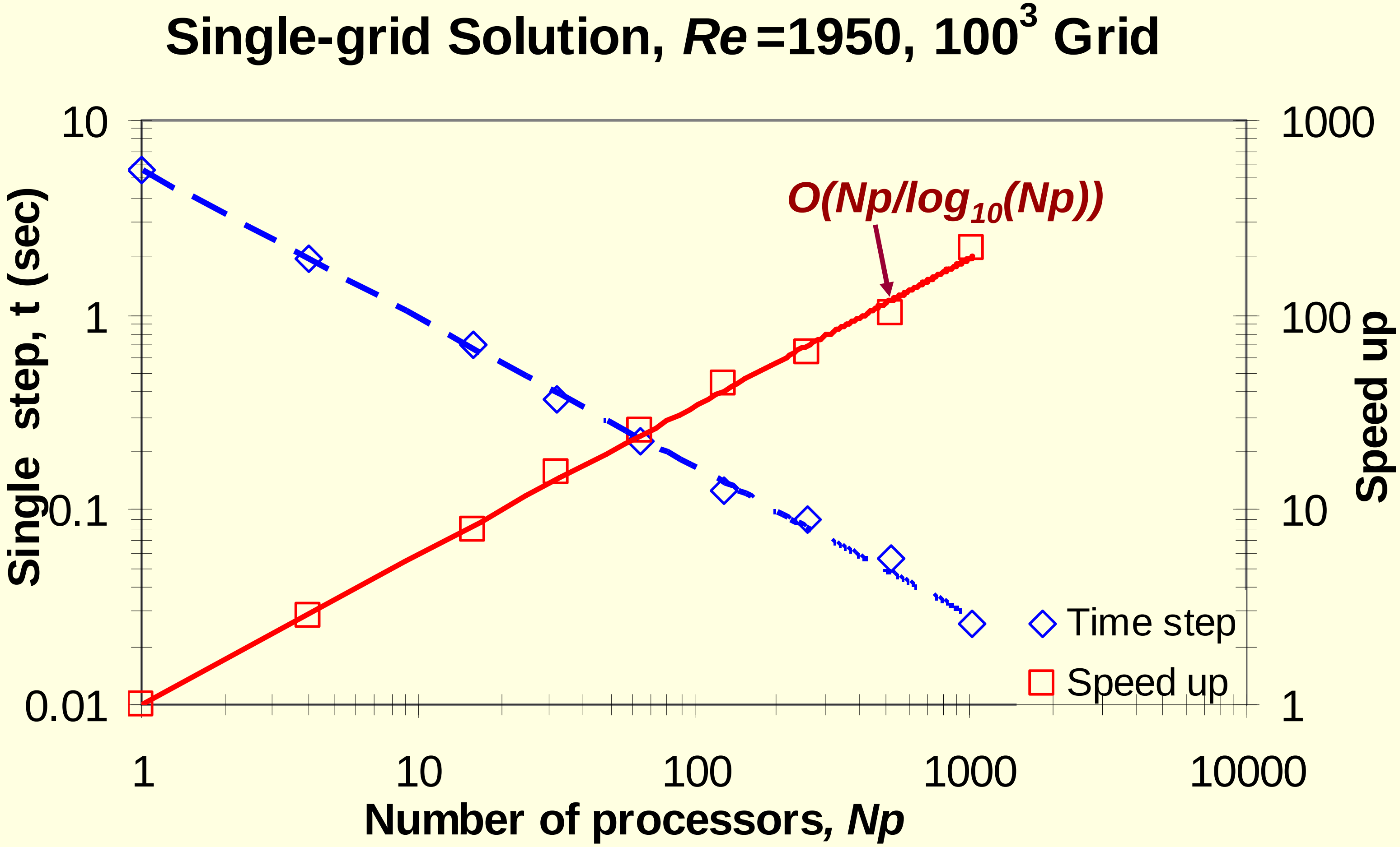


Scalability Characteristics of Multi-Grid



Number of CPU is restricted by the coarsest level (8x8=64 CPU)

Scalability Characteristics of Single-Grid



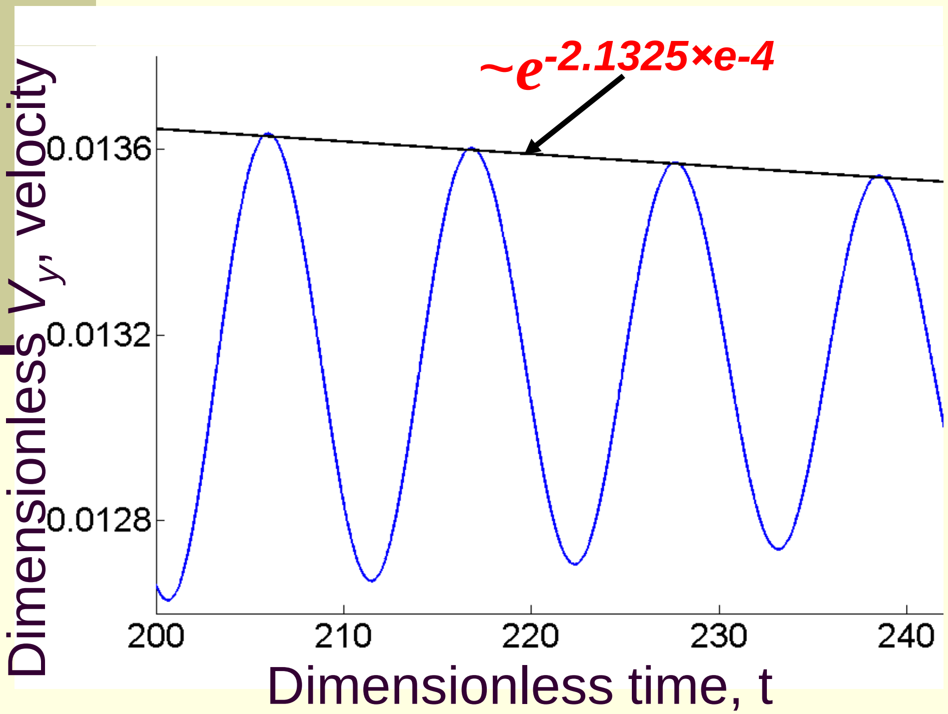
1024 CPU 25.7 msec per time step only 8 hours for 10^6 time steps

Linear Stability Analysis for the Lid Driven Cavity

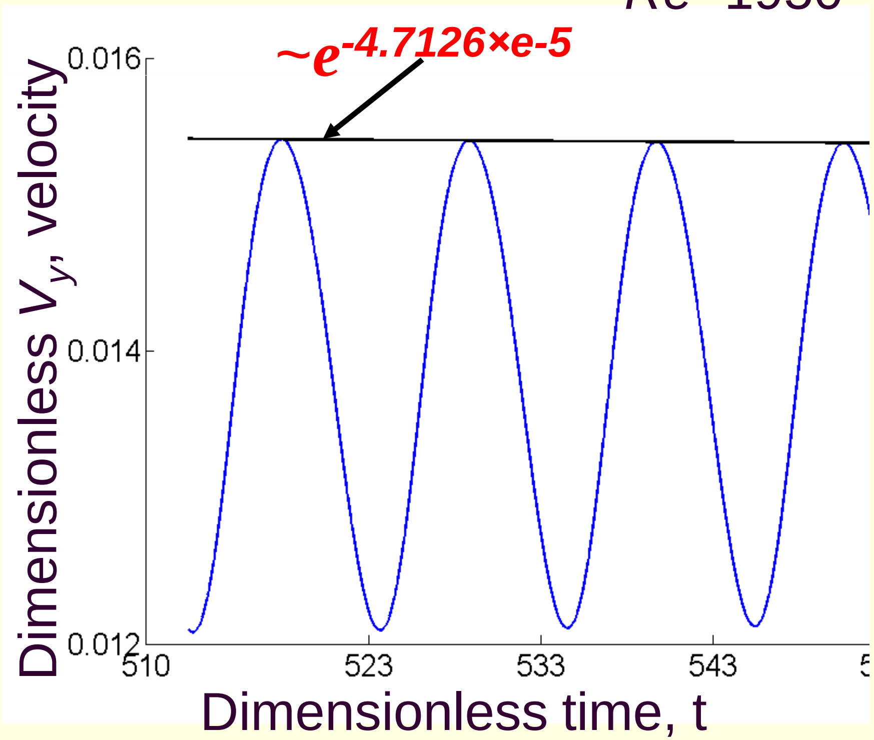
Dimensionless V_y velocity at the left corner of the middle plane (0.5, 0.1, 0.9), 100^3 Grid

Reynolds number is increasing

Re=1900



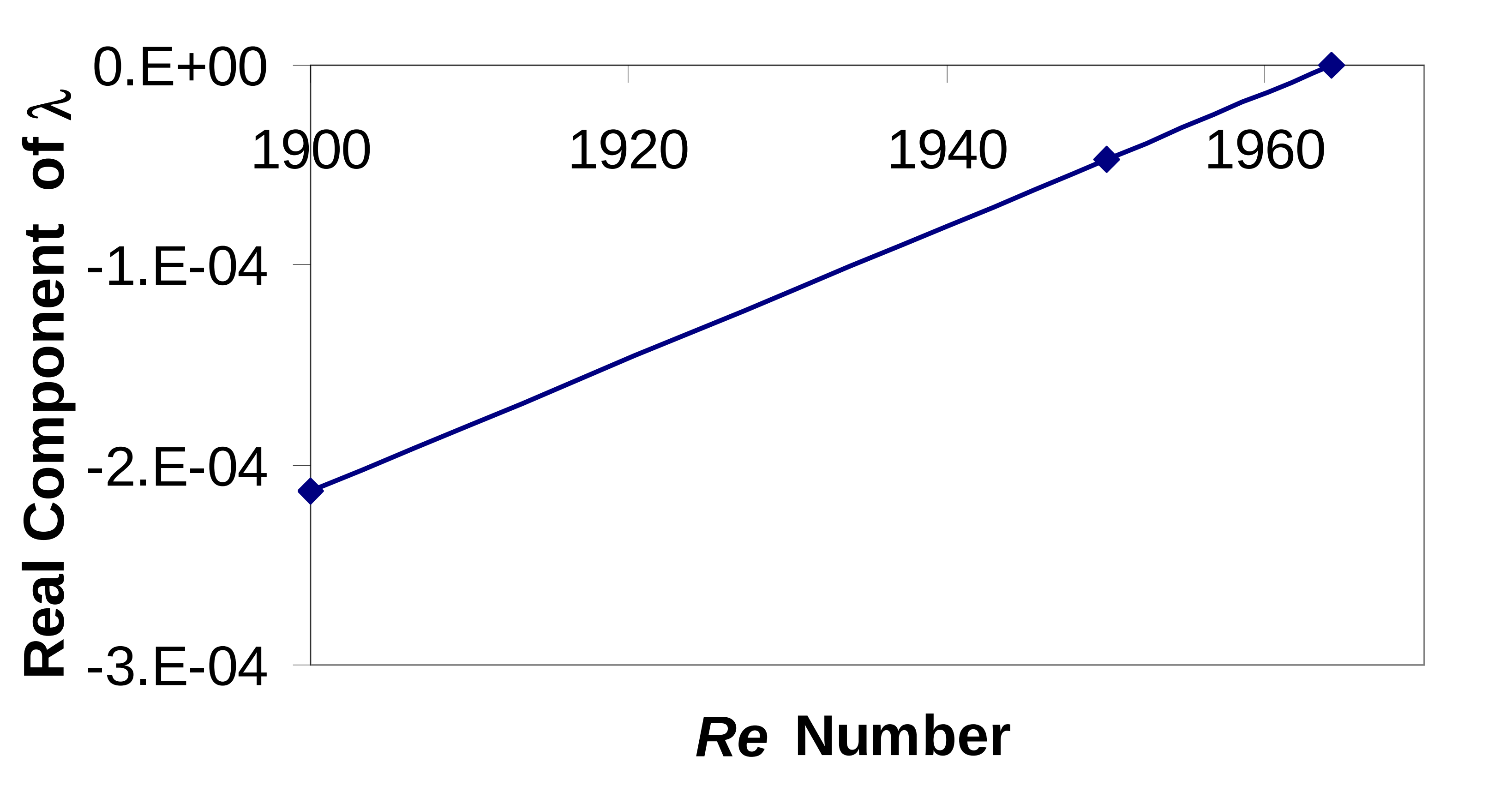
Re=1950



Estimation of the critical *Re* number for Hopf bifurcation

100^3 Grid

$\mathrm{Re}_{cr} \approx 1964$



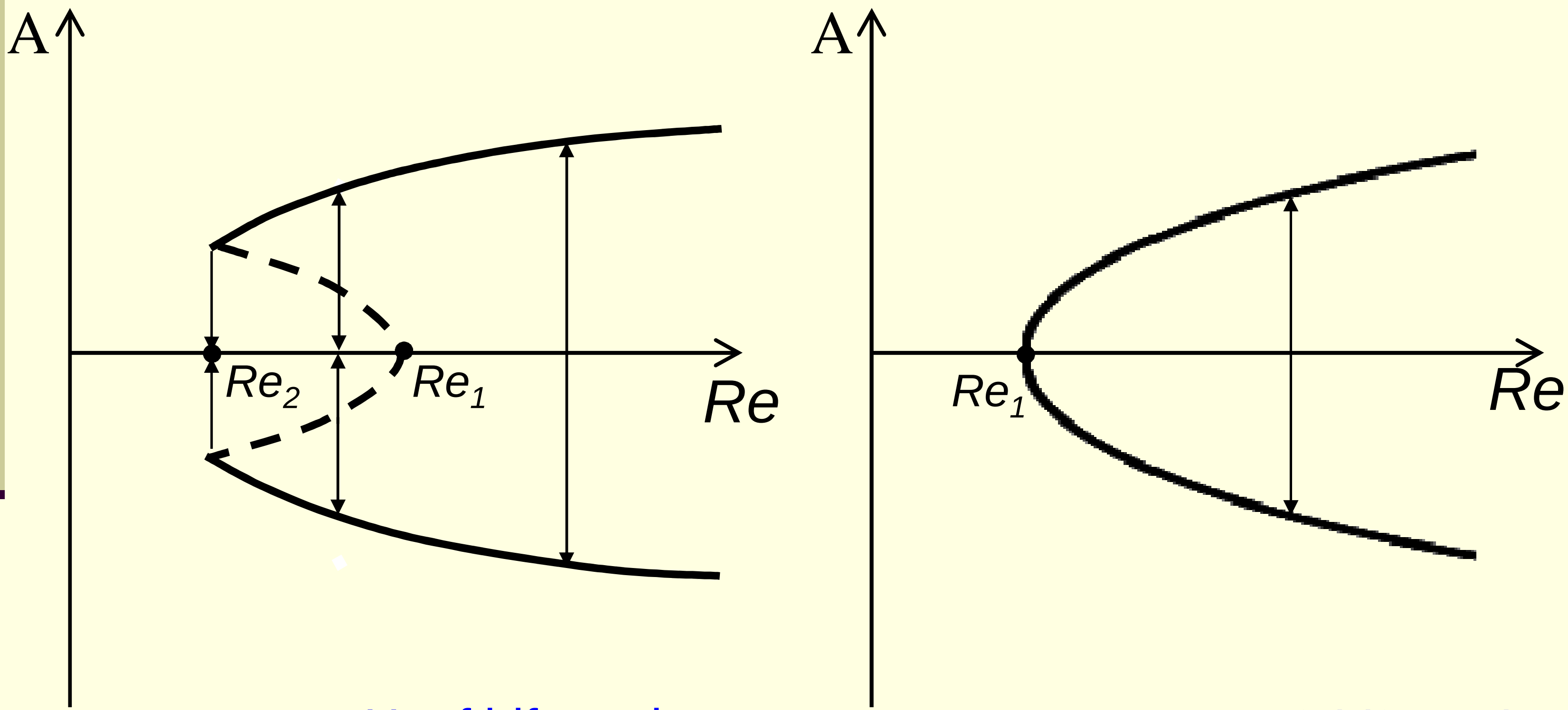
Estimation of the critical *Re* number for Hopf bifurcation (Cont.)

Grid	*Re*=1900	*Re*=1925	Re_{cr}	Richardson Extrapolation
100^3	$\lambda = -7.0256\times 10^{-3}$ ω=0.575	$\lambda = -3.8756\times 10^{-3}$ ω=0.575	1964	Re_{cr}=1917
150^3	$\lambda = -4.3438\times 10^{-3}$ ω=0.575	$\lambda = -1.2417\times 10^{-3}$ ω=0.575	1935	Re_{cr}=1916
200^3	$\lambda = -3.3512\times 10^{-3}$ ω=0.575	$\lambda = -2.8473\times 10^{-4}$ ω=0.575	1927	

Re_{cr}=1916 **ω_{cr}=0.575**

Type of the obtained bifurcation

Subcritical | **Supercritical**



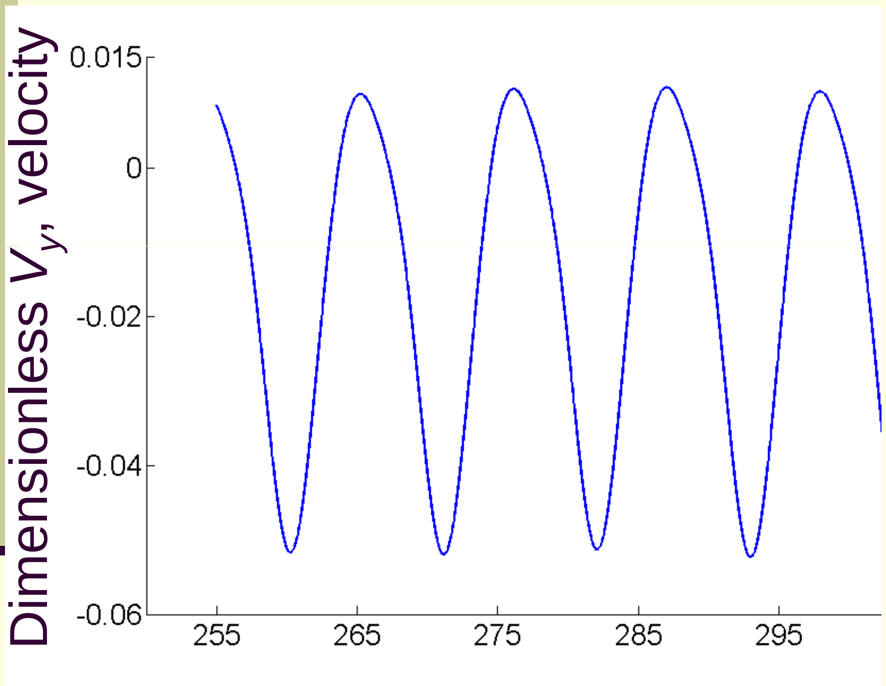
Re_1 ➡ Hopf bifurcation

Re_2 ➡ Saddle node bifurcation

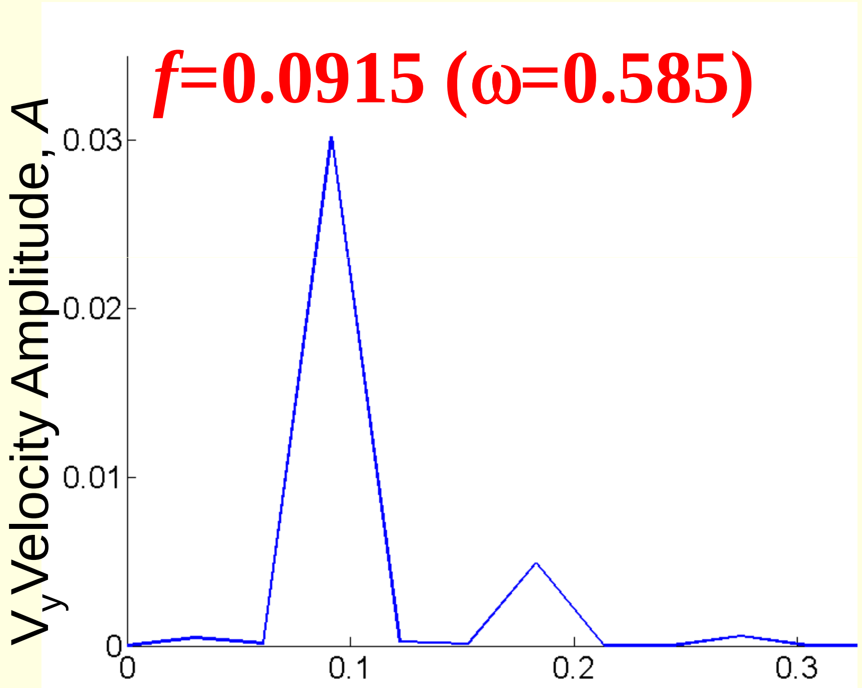
Re_1 ➡ Hopf bifurcation

The Character of the obtained bifurcation (Cont.)

Decreasing from *Re*=2000
leads to an oscillatory flow for *Re*=1950



Dimensionless time, t

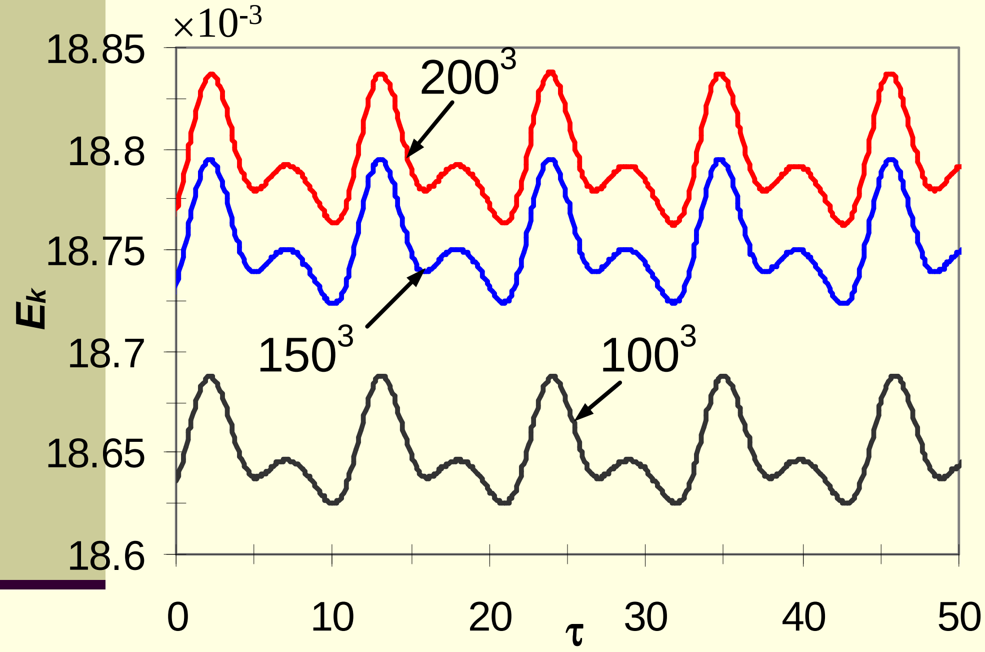


Frequency, f

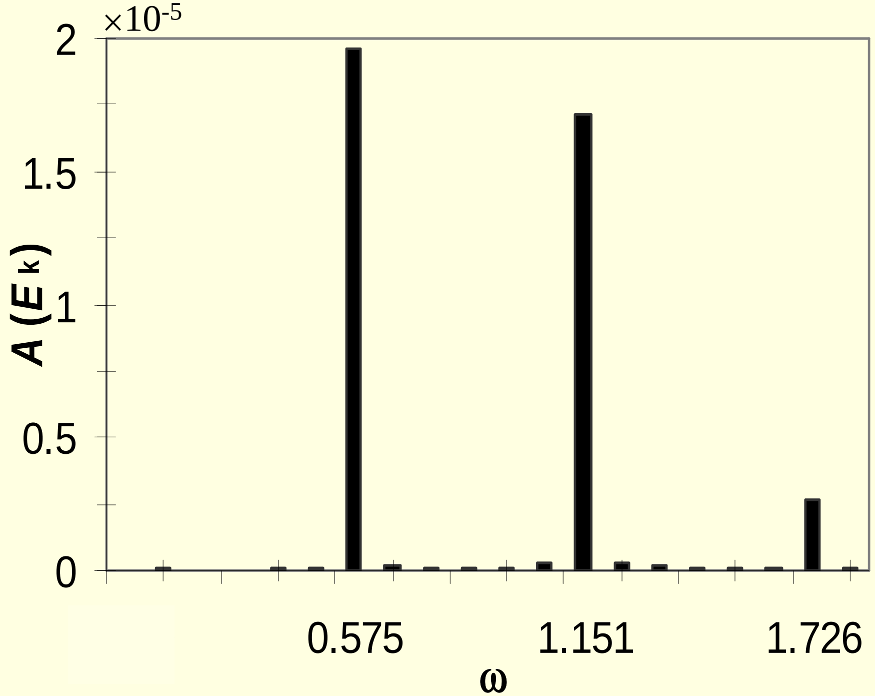
Re_{cr}=1964 is a subcritical Hopf bifurcation

The Flow Total Kinetic Energy

Time Evolution



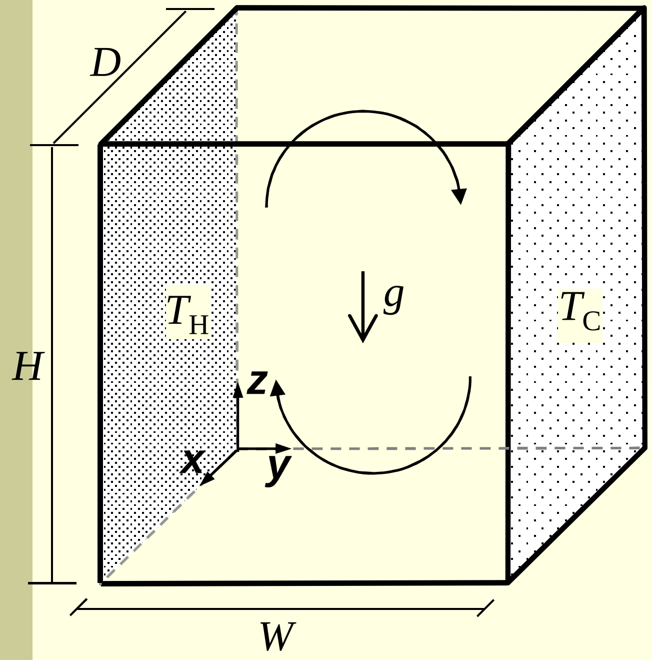
Fourier Transform



The maximum deviation between the kinetic energy values calculated on the 152^3 and 200^3 grids does not exceed 1%

Benchmark Problems (Cont.)

Differently Heated Rectangular and Cubic Cavity (Boussinesq Approximation)



$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \sqrt{\frac{1}{\mathrm{Gr}}}\nabla^2 \boldsymbol{u} + \theta \vec{e}_z$$

$$\frac{\partial \theta}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta = \frac{1}{\mathrm{Pr}\sqrt{\mathrm{Gr}}}\nabla^2 \theta$$

✓**Explicit Discretization**

$$(\boldsymbol{u}^n \cdot \nabla)\boldsymbol{u}^n \quad (\boldsymbol{u}^n \cdot \nabla)\theta^{n+1}$$

▪**Semi-Implicit Discretization**

$$(\boldsymbol{u}^n \cdot \nabla)\boldsymbol{u}^{n+1} \quad (\boldsymbol{u}^n \cdot \nabla)\theta^{n+1}$$

Boundary Conditions:

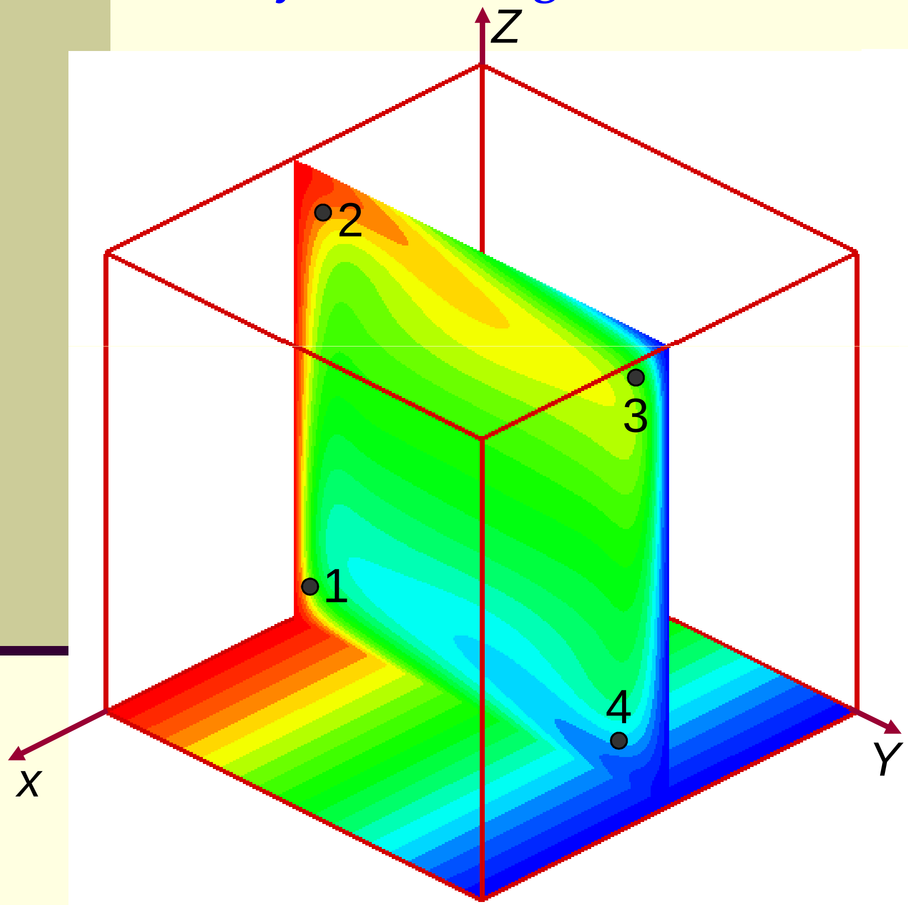
$\theta|_{y=0} = 1,\ \theta|_{y=H/W} = 0$ **-isothermal vertical walls,** $\frac{\partial \theta}{\partial \boldsymbol{n}} = 0 \ or \ \theta = 1 - y$ **-horizontal and lateral walls**

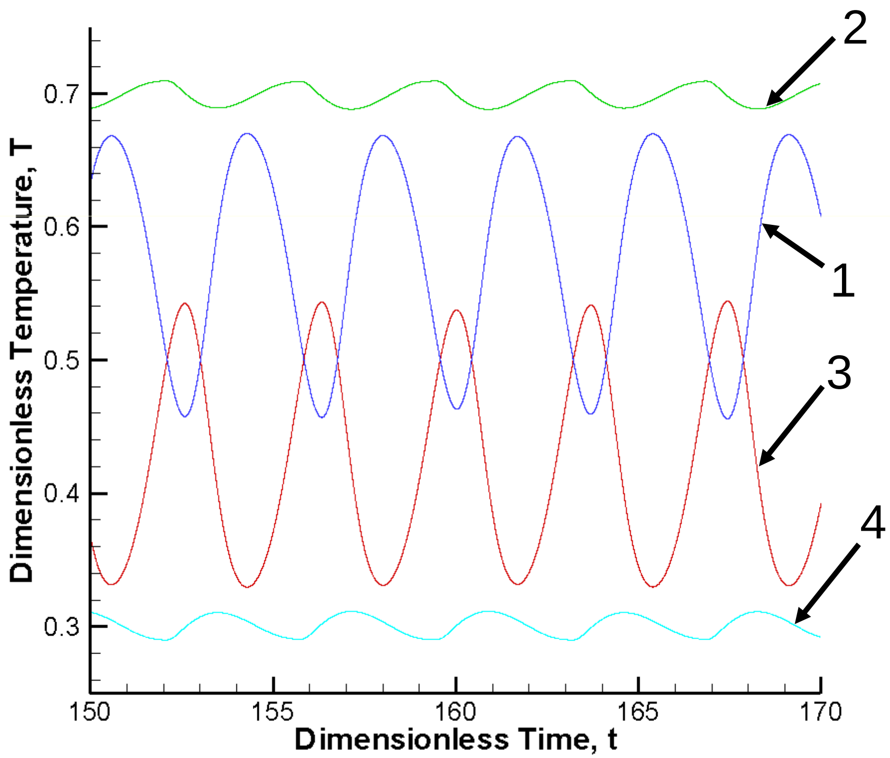
$\boldsymbol{u} = 0$ **-at all walls,**

No boundary condition for pressure is needed

Differentially Heated Cavity, $Gr=3.5\times10^6$

Perfectly conducting lateral walls **DNS results for midplane points**





Central symmetry is preserved (opposite phases for opposite corners)

Conclusions

- ✓ **An Accelerated Semi-Analytical Coupled Line Implicit Gauss-Seidel Smoother (ASA-CLGS) was developed and implemented for the solution of incompressible N-S equations.**
- ✓ **The Navier-Stokes equations are solved without pressure-velocity decoupling.**
- ✓ **The code was successfully parallelized for running on massively parallel supercomputers. The overall obtained speed up reaches 200 for 1024 processors.**
- ✓ **The multi- and single-grid approaches are scalable as $O(Np/\ln(Np))$ and $O(Np/\log_{10}(Np))$ respectively.**
- ✓ **The potential implementation of the developed parallelized time marching solver to the linear stability analysis was studied.**